Complex Time and Relativity

Carl Andrew Brannen

lastgreat@meatphysicist.com

firstgreat@toasterphysicist.com

210 NW Anthony, Apt 1, Pullman WA 99163

Submission: March 5, 2026

Abstract

We propose a reformulation of relativistic physics based on complex time and discrete structure. Relativistic kinematics is derived as the infrared limit of a single cubic lattice with local unitary dynamics, where Lorentz symmetry emerges as an accidental long-wavelength symmetry. The same lattice, governed by full octahedral (O_h) symmetry, organizes fermionic degrees of freedom through representation structure rather than continuum assumptions. Gravitation is then reconsidered by reorganizing the physical degrees of freedom of general relativity on a flat background, representing gravitational effects through modified propagation dynamics without adding new dynamical content. The framework preserves the verified predictions of special and general relativity while suggesting that spacetime symmetry and particle structure arise from a common discrete foundation.

Essay written for the Gravity Research Foundation 2026 Awards for Essays on Gravitation.

Part I: Foundations

1 Why Time Is the Problem

Time plays incompatible roles across quantum mechanics, relativity, and measurement. In quantum mechanics, time appears as an external parameter governing unitary evolution. In special relativity, time is promoted to a coordinate mixed with space under Lorentz transformations. In measurement and thermodynamics, time functions as an irreversible ordering principle tied to entropy and outcome selection. These rules appear to be structurally incompatible within their standard formulations.

This conflict arises prior to any attempt to quantize gravity or introduce microscopic spacetime structure. Most unification programs modify dynamics or geometry while leaving the underlying conception of time untouched. The persistence of the conflict across otherwise successful theories suggests that the difficulty lies instead in a shared assumption: that time is fundamentally real-valued and single-component.

2 Complex Time: One Extension, Many Consequences

The inconsistency identified above suggests relaxing the assumption that time is real-valued. Allowing time to be complex provides a minimal extension that reorganizes existing formalisms without introducing new degrees of freedom. The imaginary component governs reversible, unitary evolution, while the real component governs irreversible processes familiar from statistical mechanics and measurement.

Both forms of evolution already appear in standard physics through exponential operators generated by the Hamiltonian. Imaginary-time evolution is routinely used as a projection method onto stable or ground states, while real-time evolution governs dynamical change.[1] What changes here is not the operator structure but its interpretation: these are understood as propagations along orthogonal directions of a single complex time. This proposal does not introduce new dynamical rules; it reinterprets familiar Hamiltonian exponentials as distinct components of one underlying temporal structure.

3 Why Ensembles Come First

Treating time as complex naturally shifts the focus of quantum theory from pure state vectors to ensembles. Reversible evolution acts most naturally on collections of systems, while irreversible evolution refines ensemble weights and generically selects stable configurations. Density matrices, or more generally positive ensemble operators, are therefore taken as ontologically primary similar to[2].

Pure states arise only in limiting cases, such as the zero-temperature limit of ensemble refinement, with higher-rank limits appearing only in symmetry- or degeneracy-protected situations. This ensemble-first perspective eliminates the need for a separate collapse postulate and aligns naturally with the operational structure of quantum mechanics, where predictions are expressed in terms of expectation values and correlations.

The present framework shares a superficial similarity with ensemble interpretations of quantum mechanics, in which the wavefunction is understood as describing statistical properties of an ensemble rather than an individual system.[3] It differs fundamentally in that ensemble structure is taken to be ontologically primary and dynamically active, with irreversible real-time evolution providing a physical account of measurement and definiteness.

4 Measurement Without Collapse

In this framework, measurement is not an added dynamical rule but a physical process associated with evolution along the real component of time. Just as imaginary-time evolution projects onto energetically stable configurations, real-time refinement reshapes ensemble weights under the same Hamiltonian, selecting configurations stable under continued propagation. Irreversibility arises from this refinement process rather than from an independent collapse postulate.

The dynamical treatment of measurement adopted here is consistent with established approaches in which decoherence and environmental coupling render classical outcomes stable under continued evolution. The present framework differs in that the refinement mechanism is attributed directly to the temporal structure of Hamiltonian propagation rather than to additional stochastic or dissipative terms, but it shares the core idea that measurement can be understood as dynamical rather than axiomatic.

The operator foundations required for this approach find a historical precedent in the work of Julian Schwinger. In the 1950s, Schwinger set out to rigorously define the foundations of quantum field theory.[4] Recognizing that the conceptual difficulties of the theory were distinct from its well-understood relativistic kinematics, he developed a finite, non-relativistic formulation that came to be known as the Measurement Algebra. He utilized these ideas to teach introductory quantum mechanics, later collected into textbooks.[5] While structurally a matrix theory similar to the mixed density matrix formalism, it differs crucially in its normalization. For example, the unit matrix represents the high-temperature limit corresponding to the absence of measurement. This specific algebraic structure provides precisely the normalizations required to make complex time work consistently.[6]

Part II: Discrete Special Relativity

Einstein elevated Lorentz symmetry to a fundamental principle exact at all scales. By relaxing this assumption, we introduce a preferred frame in the ultraviolet from which Lorentz invariance emerges at long wavelengths. While similar possibilities have been explored in approaches permitting ultraviolet Lorentz violation,[7, 8] here the empirical success of special relativity remains untouched; only its logical foundation is reorganized.

We show a lattice symmetry compatible with special relativity, the gauge principle, and the quark–lepton structure of the Standard Model. Because complex time makes density matrices fundamental, symmetries act on the operator algebra, which carries more degrees of freedom than pure states and reveals structures not manifest in the state-vector formulation.

A viable microscopic model must preserve locality and unitarity while suppressing directional imprints under coarse-graining. Its long-wavelength dispersion relation must approach an isotropic relativistic form near the origin of the Brillouin zone. These conditions sharply restrict the admissible discrete symmetries before any detailed dynamics are specified. Rather than constructing models case by case, we therefore proceed by symmetry filtering, eliminating incompatible structures at the outset.

Nonsymmorphic space groups[9] contain screw axes or glide planes that embed fractional translations and sublattice structure directly into the symmetry. Because such built-in offsets persist microscopically and complicate the requirement that all spatial points be treated equivalently, it is natural to restrict attention to symmorphic space groups. Generated purely by point-group operations and unit cell translations, they provide a neutral ultraviolet background from which isotropic continuum behavior can cleanly emerge.

Considering crystal systems, only the cubic system avoids privileging specific axes or planes in the infrared. Lower-symmetry crystal systems retain directional distinctions that survive coarse-graining. If relativistic isotropy is to emerge as an effective continuum property, the microscopic symmetry must suppress such residual anisotropies.

At the operator level, a color triplet is represented not by a three-component state vector but by a 3×3 density matrix with eight traceless degrees of freedom. This matches the adjoint structure of SU(3), suggesting that continuous gauge symmetry may emerge from relational redundancy at the operator level[10] rather than being assumed fundamental. When neighboring lattice sites are compared, additional labeling freedom appears that leaves observable quantities unchanged. This redundancy is precisely what is encoded by SU(3) gauge symmetry.

For a single fermion generation, the quark–lepton spectrum requires four color-singlet representations (leptons) and four color-triplet representations (quarks) under SU(3), including antiparticles. In character-theoretic language, the values $1, 1, 1, 1$ and $3, 3, 3, 3$ are the characters of the identity element for these irreducible representations. Because each row of a character table labels a distinct particle species, any discrete symmetry intended to organize internal degrees of freedom must accommodate exactly four inequivalent singlets and four inequivalent triplets in a manner compatible with spatial symmetry and parity.

Within the cubic system, the full octahedral group O_h ($m\bar{3}m$) uniquely realizes this triplet and parity structure.[11] Its representation theory contains four inequivalent three-dimensional irreducible representations, together with singlets and doublets arranged with natural parity grading. No other cubic point group provides this combination of triplet multiplicity and symmetry between spatial directions. The simultaneous demands of infrared isotropy and operator-level organization therefore single out O_h without additional assumptions.

Bialynicki-Birula derived the Weyl equation for massless spin-1/2 fermions from a cubic lattice with O_h symmetry, showing explicitly that a discrete propagation rule can reproduce relativistic kinematics in the infrared.[12] In numerical implementations of lattice fermions, however, one generically encounters additional unwanted species—the familiar "fermion doubling" problem.[7] Bialynicki-Birula's construction arranges the propagation rule so that these mirror modes are projected out, yielding a single Weyl fermion in the long-wavelength limit.

In the present framework, however, fermion doubling is not an obstacle but a structural resource. The full octahedral group O_h contains 48 elements, which may be represented as all signed permutations of the coordinate axes. Concretely, there are $3! = 6$ permutations of (x, y, z) together with $2^3 = 8$ independent sign choices corresponding to reflections across the coordinate planes. These combine to produce $6 \times 8 = 48$ symmetry operations. In lattice formulations, the sign changes are precisely those operations that generate mirror-related species, while the permutations reorganize them among the coordinate directions. The degrees of freedom ordinarily interpreted as "doublers" therefore correspond to the reflection subgroup of O_h and are required if the full signed-permutation symmetry is to be realized. Rather than eliminating these mirror-related modes, we retain them and reorganize the enlarged species space using products of spin-projection operators, as developed in our formulation of Feynman path integrals over spin.[13] In that construction, spin evolution is expressed as a sequence of projection operations whose combinatorics naturally generate the 48-dimensional operator space associated with the signed-permutation structure of O_h.

The resulting 48-dimensional operator structure aligns with the irreducible content of O_h, allowing fermion doubling and permutation to be interpreted as discrete symmetry realization rather than lattice artifact. In the infrared limit, Bialynicki-Birula's analysis guarantees that the dispersion relation approaches the relativistic form, while the additional species reorganize into the triplet and singlet structure required by one fermion generation. Special relativity is therefore recovered dynamically, and the apparent excess degrees of freedom acquire physical interpretation within the discrete symmetry framework.

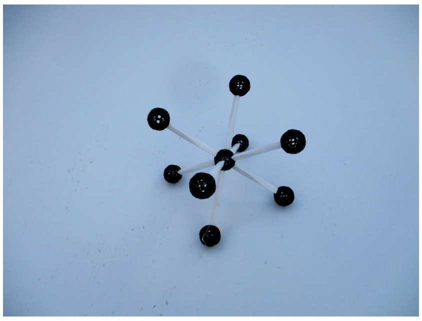
In Bialynicki-Birula's formulation, propagation occurs along the eight body diagonals of the cubic lattice, as illustrated in Fig. 1 (left). To each step labeled by $(\pm, \pm, \pm)$ corresponding to independent sign choices in the x, y, and z directions—he associates a weight which may be written in factored form as

$$W(\pm, \pm, \pm) = \frac{1 \pm \sigma_x}{2} \frac{1 \pm \sigma_y}{2} \frac{1 \pm \sigma_z}{2}, \tag{1}$$

that is, his weights can be expressed as products of three spin-1/2 projection operators. The Weyl equation emerges from the coherent superposition of these eight projected steps in the long-wavelength limit.

Seeing Bialynicki-Birula's weights written in factored form makes it natural to consider arbitrary products of the individual spin-projection operators. This viewpoint was developed in the work Spin Path Integrals and Generations[13], where spin evolution is formulated directly in terms of ordered products of spin-1/2 projection operators. In that framework, propagation is not expressed as a sum over paths in configuration space alone, but as a combinatorial structure built from successive projections in spin space. From this perspective, Bialynicki-Birula's weights appear not as special constructions, but as particular elements within a larger algebra generated by the same projection operators.

The motivation for considering such products follows directly from the logic of the Feynman path integral, where propagation is constructed by inserting resolutions of the identity $\int dx \ |x\rangle\langle x|$ in between mutually incompatible bases such as position and mo-



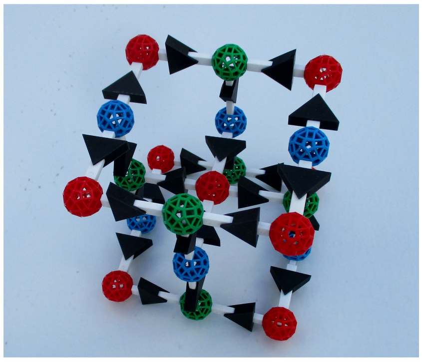


Figure 1: Left: The eight body-diagonal propagation directions. Right: An RGB cyclic generation structure. The node colors represent x, y, and z step movements, while the black triangular indicators represent the directed spin-projection operators associated with each step.

mentum. Each infinitesimal step amounts to a transition between mutually unbiased bases,[1] which for spin-1/2 are explicitly the eigenstates of σ_x, σ_y, and σ_z. Generalizing this methodology from spatial degrees of freedom directly to internal spin space naturally leads to ordered products of spin-1/2 projection operators. The noncommuting product structure of these successive projections generates the enlarged operator space discussed above.

Because products of spin-1/2 projection operators are intrinsically discontinuous at the microscopic level, it is natural to ask what structure emerges in the long-wavelength limit. In Spin Path Integrals and Generations,[13] it was shown that successive projection dynamics reorganize into three effective spin-1/2 states in the continuum limit. These states naturally suggest a generational structure and admit mass relations of Koide type for the charged leptons. The same analysis revealed a discrete phase structure, yielding a factor of 2/9 for the charged leptons[13] and, in later extensions, 2/27 and 4/27 for the up- and down-type quarks.[15, 16] Thus the combinatorics of projection operators do not merely reproduce relativistic dispersion; they generate internal multiplicities and phase relations reminiscent of the observed fermion mass spectrum.

Although the Bialynicki-Birula lattice supports the quark–lepton representation content, it does not by itself explain the origin of three generations. One natural possibility is

[1]Two bases are mutually unbiased if all transition amplitudes are equal in magnitude.[14]

that the unit cell contains three classes of symmetry-related lattice sites. An illustration of this idea is suggested in Fig. 1 (right), where propagation cycles through three node types: a particle exiting a red node in the $\pm x$ direction arrives at a green node, exits in the $\pm y$ direction to reach a blue node, and then exits in the $\pm z$ direction to return to the red class. Such a three-step cyclic structure is compatible with O_h symmetry while providing a minimal discrete mechanism for generation multiplicity.

Part III: Propagation Based Gravity

The previous part discussed how special relativity can arise from a lattice propagation rule. This part continues the same program for gravitation: we sketch how a lattice-friendly, flat-background description can reproduce the correct counting of degrees of freedom and we give an exact stationary rotating black-hole (Kerr) example. Detailed derivations and extensions beyond the stationary sector are left for future work.

5 Flat-Background Gauge Formulation

In tetrad or gauge formulations, the metric's six physical degrees of freedom are equivalently described by a local Lorentz structure containing three boosts and three rotations at each point.[17] While only two combinations propagate as radiative modes in the weak-field limit, the full boost–rotation description is particularly natural for a lattice-based framework where velocity is primitive. Rather than treating curvature as fundamental, one may regard gravity as specifying how local propagation directions and velocities are modified relative to a background frame.

6 Rotating Black Holes in River Form

The rotating (Kerr) black hole[18] admits a representation in which the gravitational field is described by a velocity–twist structure on a flat spatial background.[19] In this river formulation, spacetime is characterized by a spatial flow field together with a local rotation (twist) field associated with frame dragging. Schematically, the gravitational data may be decomposed into river velocity (boost) $\mathbf{v}(\mathbf{x})$ and rotation pseudovector (twist) $\mathbf{u}(\mathbf{x})$. In a black hole, the event horizon occurs where the inward component of $\mathbf{v}$ reaches the speed of light relative to the background frame.

In the lattice-based formulation of this paper, null propagation is expressed in terms of left- and right-handed modes. The boost and twist fields of the rotating black hole can therefore be reorganized into effective chiral velocity fields,

$$\mathbf{v}_L = \mathbf{v} + \mathbf{u}, \qquad \mathbf{v}_R = \mathbf{v} - \mathbf{u}. \tag{2}$$

Reversing the spin of the black hole exchanges $\mathbf{v}_L$ and $\mathbf{v}_R$, while the non-rotating limit corresponds to $\mathbf{u} = 0$ and $\mathbf{v}_L = \mathbf{v}_R$. In this manner, the six local Lorentz parameters (three boosts and three rotations) of the stationary Kerr solution are repackaged into two chiral propagation environments without introducing additional structure.

In this formulation, curvature is not eliminated but reorganized: general relativity emerges as the infrared description of how local boost and rotation fields modify null propagation relative to a background frame.

Part IV: Discussion

7 Implications and Open Problems

The present framework also suggests a natural perspective on the gauge principle. In a relational formulation of quantum mechanics, physical information resides in interactions between systems rather than in absolute state vectors.[20] When internal degrees of freedom are treated at the operator level and compared between neighboring lattice sites, additional descriptive variables are introduced. The resulting redundancy is precisely of the type removed by gauge symmetry.[10] In this sense, gauge structure need not be postulated as a separate principle but may arise from the relational comparison of local operator degrees of freedom. A detailed development of this idea is left for future work.

The reorganization proposed here suggests multiple directions for further investigation. A more mathematical and interactive introduction to these ideas, tentatively titled *Oh Physics: A Prompt Introduction to Discrete Space*, is currently in preparation. The "prompts" are pdf sentences that the reader can click to open an AI chat about an idea, for example, on dark matter[2, 21]:

Prompt

AI Exploration Prompt At a first-year physics graduate student level, explain how superoperator symmetry relates Steven Weinberg's paper *"Quantum mechanics without state vectors"* to Carl Brannen's paper *"Group Geometric Algebra and the Standard Model."* Run this prompt using:

ChatGPT (prompt preloaded)

Or copy paste into your favorite AI.

Particularly intriguing is the behavior of spacetime singularities. If local propagation is fundamentally governed by combinations of spin-1/2 projection operators, gravitational degrees of freedom may themselves be subject to exclusion principles. This aligns with observations of "tripled Pauli statistics" in the quasinormal modes of non-rotating black holes [22]. The discrete spacetime lattice may therefore regularize classical singularities via an effective degeneracy pressure inherent to the geometric degrees of freedom.

The central claim of this paper is that many long-standing tensions in fundamental physics arise not from dynamics or geometry but from assumptions about time and space.

References

[1] A. Zee. *Quantum Field Theory in a Nutshell: Second Edition*. In a Nutshell. Princeton University Press, 2010. ISBN 9781400835324. URL https://books.google.com/books?id=n8Mmbjtco78C.

[2] S. Weinberg. Quantum mechanics without state vectors. *PRA*, 90(4):042102, Octo-

ber 2014. doi: 10.1103/PhysRevA.90.042102.

[3] L. E. Ballentine. The Statistical Interpretation of Quantum Mechanics. *Reviews of Modern Physics*, 42(4):358–381, October 1970. doi: 10.1103/RevModPhys.42.358.

[4] J. Schwinger. *Quantum Kinematics And Dynamics*. Advanced Books Classics. Avalon Publishing, 2000. ISBN 9780813346892.

[5] J. Schwinger and B.G. Englert. *Quantum Mechanics: Symbolism of Atomic Measurements*. Physics and astronomy online library. Springer, 2001. ISBN 9783540414087.

[6] C. Brannen. Algebra of Classical and Quantum Binary Measurements. *Journal Mod Phys*, 9:628–650, 2018.

[7] H. B. Nielsen and M. Ninomiya. A no-go theorem for regularizing chiral fermions. *Physics Letters B*, 105(2-3):219–223, October 1981. doi: 10.1016/0370-2693(81) 91026-1.

[8] Grigory E. Volovik. *The Universe in a Helium Droplet*, volume 117 of *International Series of Monographs on Physics*. Oxford University Press, Oxford, UK, 2003. ISBN 9780198507826.

[9] N. W. Ashcroft and N. D. Mermin. *Solid State Physics*. Holt-Saunders, 1976.

[10] Carlo Rovelli. Why Gauge? *Foundations of Physics*, 44(1):91–104, January 2014. doi: 10.1007/s10701-013-9768-7.

[11] Morton Hamermesh. *Group Theory and its Application to Physical Problems*. Dover, 1962. ISBN 0486661814.

[12] Iwo Bialynicki-Birula. Dirac and Weyl equations on a lattice as quantum cellular automata. *Phys. Rev.*, D49:6920–6927, 1994. doi: 10.1103/PhysRevD.49.6920.

[13] Carl A. Brannen. Spin path integrals and generations. *Found. Phys.*, 40:1681–1699, 2010. doi: 10.1007/s10701-010-9465-8.

[14] George Svetlichny. Feynman's integral is about mutually unbiased bases, 2008. URL `https://arxiv.org/abs/0708.3079`.

[15] P. Żenczykowski. Remark on Koide's Z_3-symmetric parametrization of quark masses. *PRD*, 86(11):117303, December 2012. doi: 10.1103/PhysRevD.86.117303.

[16] P. Żenczykowski. Koide's Z_3-symmetric parametrization, quark masses, and mixings. *PRD*, 87(7):077302, April 2013. doi: 10.1103/PhysRevD.87.077302.

[17] Charles W. Misner, Kip S. Thorne, and John Archibald Wheeler. *Gravitation [by] Charles W. Misner, Kip S. Thorne [and] John Archibald Wheeler.* W. H. Freeman, San Francisco, 1973. ISBN 0716703343.

[18] Roy P. Kerr. Gravitational field of a spinning mass as an example of algebraically special metrics. *Phys. Rev. Lett.*, 11:237–238, Sep 1963. doi: 10.1103/PhysRevLett.11.237. URL `https://link.aps.org/doi/10.1103/PhysRevLett.11.237`.

[19] A. J. S. Hamilton and J. P. Lisle. The river model of black holes. *American Journal of Physics*, 76:519–532, June 2008. doi: 10.1119/1.2830526.

[20] Carlo Rovelli. Relational quantum mechanics. *International Journal of Theoretical Physics*, 35(8):1637–1678, Aug 1996. ISSN 1572-9575. doi: 10.1007/bf02302261. URL `http://dx.doi.org/10.1007/BF02302261`.

[21] C. Brannen. Group Geometric Algebra and the Standard Model. *Journal Mod Phys*, 11:1193–1214, 2020.

[22] Carl A. Brannen. Tripled pauli statistics and the internal structure of black holes. *viXra*, May 2025. URL `https://ai.vixra.org/abs/2505.0192`.