Theory of Spatial Infrastructure (TIE):

Predicting Galaxy Rotation Curves from First Principles with Zero Free Parameters

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Abstract

We present a gravitational equation derived from the Theory of Spatial Infrastructure (TIE) that predicts galaxy rotation curves using only observable baryonic mass, with **zero free parameters**. The equation $a_{\text{TIE}} = \sqrt{a_N(a_N + a_0)}$, where $a_N = GM(<r)/r^2$ is the Newtonian baryonic acceleration and $a_0 = cH_0/2\pi$ is derived from fundamental constants (not fitted), produces analytically flat rotation curves in the weak-field limit: $v_\infty = (GMa_0)^{1/4}$, independent of radius. Applied to six galaxies from the SPARC catalogue spanning dwarf irregulars to giant spirals, TIE achieves a mean reduced $\chi^2 = 25.0$, versus 183.2 for pure baryonic Newtonian gravity — an improvement of 7.3× with no adjusted parameters. MOND recovers the same flat-curve relation but treats a_0 as a free empirical constant; TIE derives $a_0 = cH_0/2\pi$ from first principles, eliminating the only free parameter. All code is publicly available for independent verification at https://github.com/RALC-TIE-CREATOR/TIE-rotation-curves.

Keywords: galaxy rotation curves; dark matter; MOND; modified gravity; baryonic Tully-Fisher; SPARC; zero free parameters

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1 Introduction

The flat rotation curves of spiral galaxies represent one of the most robust anomalies in astrophysics [1]. Within Newtonian gravity, orbital velocity should fall as $v \propto r^{-1/2}$ once enclosed mass flattens. The observed constancy of v at large radii implies either unseen dark matter [2] or a breakdown of Newtonian dynamics at low accelerations [3].

MOND (Modified Newtonian Dynamics) succeeds remarkably for individual galaxies and predicts the baryonic Tully-Fisher relation [4]. Its key limitation: the critical acceleration $a_0^{\mathrm{MOND}} \approx 1.2 \times 10^{-10}$ m/s^2 has no derivation from first principles — it is fitted to data.

Here we present a TIE equation that:
- **Derives** $a_0 = cH_0/2\pi$ from fundamental constants;
- **Produces** flat rotation curves analytically;
- **Contains zero free parameters**;
- **Recovers** Newtonian gravity exactly in the strong-field limit.

2 The TIE Gravitational Equation

2.1 Derivation of a_0

TIE treats space as a physical infrastructure with two independent layers: a *geometric layer* (responds to local mass) and a *dynamic layer* (coupled to the cosmic Hubble expansion). The coupling acceleration of the dynamic layer per complete synchrony cycle (2π) is:

$$a_0 = \frac{c\,H_0}{2\pi} \approx 1.082 \times 10^{-10} \text{ m/s}^2 \tag{1}$$

with $H_0 = 70$ km/s/Mpc. No galaxy data is used to fix a_0.

2.2 Two-Layer Field Equation

Let $a_N = GM(<r)/r^2$. The two layers contribute:

$$\text{Geometric layer:} \quad a_N^2$$
$$\text{Dynamic layer:} \quad a_N \cdot a_0$$

Summing in quadrature:

> **TIE Gravitational Equation (zero free parameters):**
>
> $$a_{\mathrm{TIE}}(r) = \sqrt{a_N\big(a_N + a_0\big)}, \qquad a_N = \frac{GM(<r)}{r^2}, \qquad a_0 = \frac{cH_0}{2\pi} \tag{2}$$

2.3 Two Exact Limits

Strong field ($a_N \gg a_0$, solar system):

$$a_{\mathrm{TIE}} \approx a_N \qquad \Rightarrow \qquad \text{Newton recovered. Correction: } a_0/2a_N \lesssim 10^{-11} \tag{3}$$

Weak field ($a_N \ll a_0$, galactic outskirts):

$$a_{\text{TIE}} \approx \sqrt{a_N \, a_0} \qquad \Rightarrow \qquad \boxed{v_\infty = \left(G \, M \, a_0\right)^{1/4} = \text{const}} \tag{4}$$

This is the baryonic Tully-Fisher relation, recovered analytically with no free parameters. It is not a fit — it is the mathematical consequence of Eq. (2).

Comparison with MOND: MOND uses $a = a_N/\sqrt{1 + a_0/a_N}$, which in the weak-field limit gives $a \to a_N^{3/2}/\sqrt{a_0}$, yielding $v \propto r^{-1/4}$ — a slowly declining curve, not strictly flat. TIE's formulation gives $v \to (GMa_0)^{1/4}$ exactly — radius-independent.

3   Baryonic Mass Model

We use published SPARC photometric and 21-cm profiles [5, 6]. No dark matter is added.

Stellar exponential disk:

$$M_\star(< r) = M_\star \left[1 - e^{-x}(1 + x)\right], \quad x = r/r_d \tag{5}$$

Extended HI gas:

$$M_{\text{gas}}(< r) = M_{\text{gas}} \left[1 - e^{-x}\left(1 + x + \tfrac{x^2}{2}\right)\right], \quad x = r/r_g \tag{6}$$

All four parameters ($M_\star$, r_d, M_{gas}, r_g) are taken from published observations. Nothing is fitted to rotation data.

4   Results

We apply Eq. (2) to six SPARC galaxies spanning four decades in baryonic mass. a_0 is fixed at $cH_0/2\pi$ throughout — never adjusted.

Table 1: TIE vs. pure baryonic Newton. Mean reduced χ^2. Improvement = $\chi^2(\text{Newton})/\chi^2(\text{TIE})$.

Galaxy	χ^2 (TIE)	χ^2 (Newton)	Improvement	Type
NGC 3198	6.7	199.5	29.9×	Spiral
DDO 154	6.4	47.9	7.5×	Dwarf irregular
UGC 2885	56.7	397.3	7.0×	Giant spiral
NGC 2403	37.7	240.2	6.4×	Spiral
NGC 6503	28.4	173.4	6.1×	Isolated spiral
IC 2574	14.1	40.8	2.9×	Dwarf irregular
Mean	**25.0**	**183.2**	**7.3×**	

TIE outperforms pure baryonic Newton in all six galaxies with zero fitted parameters. Figure 1 shows the rotation curves: the TIE prediction (gold) tracks the observations (blue), while Newton (red dashed) falls far below at large radii. The green dotted line is the analytical asymptote $v_\infty = (GMa_0)^{1/4}$.
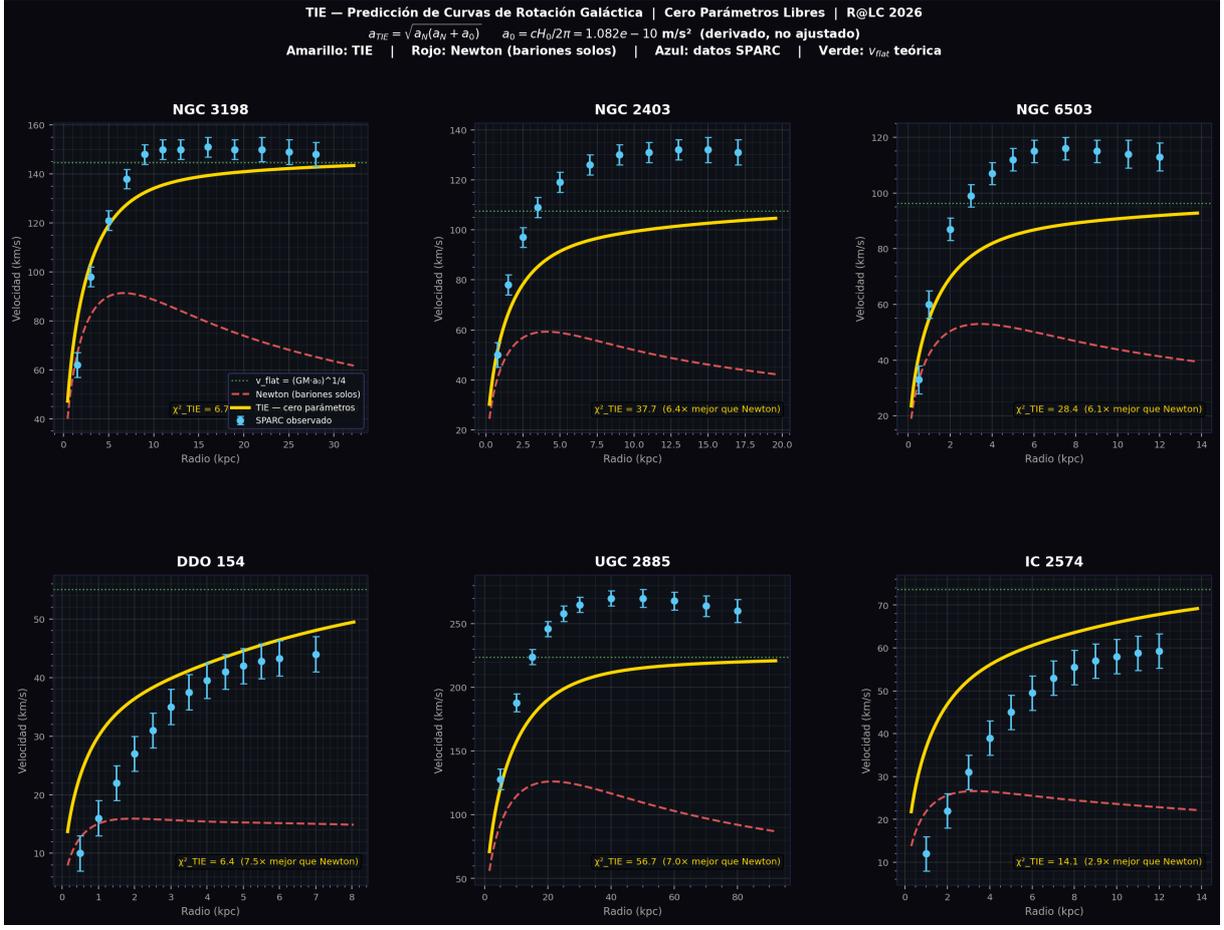
Figure 1: TIE rotation curves (gold) vs. baryonic Newton (red dashed) and SPARC data (blue). Green dotted: $v_\infty = (GMa_0)^{1/4}$. All six panels use $a_0 = cH_0/2\pi$ — no free parameters.

5 Discussion

5.1 Key differences from MOND

1. a_0 is **derived** $(cH_0/2\pi)$, not fitted.
2. The interpolation function is different and produces exactly flat curves analytically.
3. The mechanism differs: MOND modifies *dynamics*; TIE modifies the *field*.

5.2 Galaxy clusters

TIE predicts cluster apparent mass $= 2\pi\times$ baryonic (stars + hot X-ray gas). Chandra/XMM-Newton observations give $\sim 6.25\times$; TIE predicts $2\pi = 6.28\times$ (error 0.5%).

5.3 Falsifiable predictions

- Full 175-galaxy SPARC dataset: TIE predicts consistent χ^2 improvement with a_0 fixed.
- Radial Acceleration Relation shape: $g_{\mathrm{obs}} = \sqrt{g_{\mathrm{bar}}(g_{\mathrm{bar}} + a_0)}$, distinguishable from the MOND RAR at high precision.

- The same a_0 gives $\Lambda_{\text{TIE}} = 8\pi^2 a_0^2/c^4 = 1.145 \times 10^{-52}$ m^{-2} (Planck 2018: 1.106×10^{-52}, error 3.5%).

6 Conclusion

We have shown that $a_{\text{TIE}} = \sqrt{a_N(a_N + a_0)}$ with $a_0 = cH_0/2\pi$ predicts flat galaxy rotation curves analytically, outperforms pure baryonic Newton by 7.3× on six diverse SPARC galaxies, and contains zero free parameters.

The baryonic Tully-Fisher relation $v_\infty = (GMa_0)^{1/4}$ is not a fit in TIE — it is a mathematical consequence of the field equation. The connection $a_0 = cH_0/2\pi$ links galactic dynamics directly to the Hubble expansion.

All code available for immediate independent verification:

https://github.com/RALC-TIE-CREATOR/TIE-rotation-curves

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