F-Term Structure of the sBootstrap: Three-Regime Meson Mass Splittings from Volkov–Akulov Nonlinear Supersymmetry

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March 2026

Abstract

We investigate the internal structure of the meson mass spectrum within the sBootstrap framework, which interprets the six charged pseudoscalar mesons of the Standard Model as scalar superpartners of the three charged leptons in an emergent, spontaneously broken $\mathcal{N} = 1$ supersymmetry realized à la Volkov–Akulov in the confined phase of QCD. Starting from the established relation $m_\pi^2 - m_\mu^2 = f_\pi^2$ (accurate to 2%), we decompose the mass-squared matrix of each meson–lepton row into a universal soft mass m^2_{soft} and a splitting parameter F_i, and discover that the three rows of the sBootstrap generation table exhibit distinct scaling behaviors corresponding to the three regimes of the theory. The lightest row yields the known pion–muon relation with residual $\delta_1 = f_\pi^2$. The intermediate row produces a new relation: the D–D_s splitting satisfies $F_2 = f_{B_c}^2$ to 0.5%, where $f_{B_c} = 434$ MeV is the independently measured B_c decay constant. The heaviest row gives $F_3 \approx m_c m_b$ to 8%, the perturbative quark mass product. We introduce the concept of *bridge mesons* to explain why the F-term for each row equals the decay constant squared of the meson connecting that row to its neighbors, with the perturbative quark mass product as the limiting case when the bridge meson cannot form. We show that the electron mass, long an open problem of the sBootstrap, is determined by the Koide formula applied to the F-term–generated muon and tau masses, closing the $6 \to 3$ map: all three charged lepton masses emerge from QCD inputs with no free electroweak parameters. We further propose a graded nilpotency for the Volkov–Akulov superfield constraint across the three rows. Throughout, we discuss the foundations in classical SUSY breaking, the distinction between F-term and D-term mechanisms, and the problem of fermion mass protection in emergent supersymmetry.

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1 Introduction and Context

The Standard Model of particle physics contains, among its low-energy hadronic degrees of freedom, exactly six electrically charged pseudoscalar mesons: $\pi^\pm$, $K^\pm$, $D^\pm$, $D_s^\pm$, $B^\pm$, and $B_c^\pm$. It also contains three charged leptons: e, μ, and τ. The sBootstrap hypothesis [11, 12] proposes that this $6 = 2 \times 3$ counting is not accidental: the six mesons are the scalar degrees of freedom of three chiral supermultiplets whose fermionic components are the charged leptons. If each $\mathcal{N} = 1$ chiral superfield Φ_i contains a Weyl fermion and a complex scalar—hence two real scalar mass eigenstates—then three such superfields produce exactly three fermions and six scalars.

This hypothesis requires supersymmetry to be *emergent* in QCD's confined phase, spontaneously broken by the chiral condensate $\langle \bar{q}q \rangle \neq 0$, and realized nonlinearly in the sense of Volkov and Akulov [1]. The foundational numerical evidence is the mass relation

$$m_\pi^2 - m_\mu^2 = f_\pi^2 , \tag{1}$$

which holds to approximately 2%: numerically, $m_\pi^2 - m_\mu^2 = 8316$ MeV^2 versus $f_\pi^2 = 8538$ MeV^2 (using $f_\pi = 92.4$ MeV, $m_\pi = 139.57$ MeV, $m_\mu = 105.66$ MeV). This relation was first noted by Rivero and identifies the pion decay constant—the order parameter of chiral symmetry breaking—as the supersymmetry-breaking F-term that splits the scalar (pion) from its fermionic partner (muon) in a Volkov–Akulov nonlinear realization [12].

The purpose of this paper is to extend the F-term analysis to the full three-generation structure of the sBootstrap. We ask: what governs the mass splittings within each generation, and can the F-term mechanism predict relations beyond the pion–muon system? We find that the answer involves three distinct regimes—chiral, mixed, and perturbative—and that the second regime yields a new, quantitatively precise relation connecting the D–D_s mass splitting to the B_c decay constant.

The paper is organized as follows. In Section 2, we review the classical theory of SUSY breaking via F-terms and D-terms and establish notation. Section 3 discusses the Volkov–Akulov realization and its distinction from standard soft breaking. Section 4 introduces the sBootstrap generation table and the constraints that fix the non-standard quark–lepton assignments. Section 5 performs the mass-squared decomposition for each row. Section 6 presents the three-regime structure and derives the new relations. Section 7 traces these results to the Kähler and superpotential structure. Section 8 addresses the problem of fermion mass protection. Section 9 discusses the connection to Koide mass relations. Section 10 resolves the electron mass problem through Koide applied to the F-term–generated heavy lepton masses. Section 11 introduces the bridge meson concept and derives the coefficient $c = 1$ in $F_2 = f_{B_c}^2$. Section 12 proposes the graded nilpotency of the superfield constraint across the three rows. Section 13 provides a critical assessment, and Section 14 concludes.

2 Classical SUSY Breaking: F-Terms and D-Terms

We begin with a self-contained review of spontaneous SUSY breaking in $\mathcal{N} = 1$ global supersymmetry, following the treatments in Wess and Bagger [7], Weinberg [8], and Martin [9].

2.1 The chiral superfield and auxiliary fields

An $\mathcal{N} = 1$ chiral superfield Φ contains three component fields: a complex scalar ϕ, a Weyl spinor ψ_α, and a complex auxiliary field F:

$$\Phi(y) = \phi(y) + \sqrt{2}\,\theta\psi(y) + \theta\theta\,F(y)\,, \qquad y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}\,. \tag{2}$$

The SUSY transformations, parameterized by a constant Grassmann spinor ξ_α, act as

$$\delta_\xi\phi = \sqrt{2}\,\xi^\alpha\psi_\alpha\,, \qquad \delta_\xi\psi_\alpha = i\sqrt{2}\,\sigma^\mu_{\alpha\dot{\beta}}\bar{\xi}^{\dot{\beta}}\partial_\mu\phi + \sqrt{2}\,\xi_\alpha F\,, \qquad \delta_\xi F = i\sqrt{2}\,\bar{\xi}_{\dot{\beta}}\bar{\sigma}^{\mu\dot{\beta}\alpha}\partial_\mu\psi_\alpha\,. \tag{3}$$

The key observation is that $\delta_\xi F$ is a total derivative. Consequently, a vacuum expectation value $\langle F\rangle \neq 0$ does not break translational invariance but does break supersymmetry, since $\delta_\xi\psi_\alpha$ acquires a nonzero vacuum value proportional to ξ_α.

2.2 F-term breaking

Consider a theory of chiral superfields Φ_i with Kähler potential $K(\Phi^\dagger, \Phi)$ and superpotential $W(\Phi)$. The scalar potential is

$$V = \sum_i |F_i|^2 + \frac{1}{2}\sum_a (D^a)^2\,, \tag{4}$$

where the auxiliary fields are determined by their equations of motion:

$$F_i = -\frac{\partial W}{\partial \phi_i}\,, \qquad D^a = -g\,\phi^\dagger T^a\phi\,. \tag{5}$$

Supersymmetry is unbroken if and only if $V = 0$, which requires all $F_i = 0$ and all $D^a = 0$ simultaneously. If no field configuration achieves this, SUSY is spontaneously broken.

F-term breaking occurs when $\langle F_i\rangle \neq 0$ for at least one field. The classic example is the O'Raifeartaigh model [3], where the superpotential $W = \lambda\Phi_0(\Phi_1^2 - \mu^2) + m\Phi_2\Phi_1$ leads to $F_0 = -\lambda(\phi_1^2 - \mu^2)$ and $F_2 = -m\phi_1$, which cannot both vanish. The resulting mass spectrum exhibits the *supertrace sum rule*:

$$\mathrm{STr}\,\mathcal{M}^2 = \sum_j (-1)^{2j}(2j+1)m_j^2 = 0\,, \tag{6}$$

which states that the mass-squared sum over bosons equals that over fermions, weighted by spin multiplicities, at tree level. For a single chiral multiplet with fermion mass m_f and two real scalar masses m_1, m_2:

$$m_1^2 + m_2^2 = 2m_f^2\,. \tag{7}$$

In models with explicit soft SUSY-breaking terms (as appear in the MSSM upon integrating out heavy mediators), the scalar mass-squared matrix for a chiral multiplet takes the form

$$m_{1,2}^2 = m_f^2 + m_{\text{soft}}^2 \pm |F|\,, \tag{8}$$

where m_{soft}^2 is a universal soft mass shift and $\pm|F|$ splits the two real scalar components. The fermion mass m_f, set by the superpotential, is unaffected at tree level by both F-term and D-term contributions—a point of central importance for the sBootstrap, where the lepton masses are fixed inputs and the meson masses shift around them.

2.3 D-term breaking

D-term breaking occurs when $\langle D^a \rangle \neq 0$ for a gauge generator T^a. The Fayet–Iliopoulos mechanism [4] adds a term ξD for a $U(1)$ factor, forcing $\langle D \rangle \neq 0$ if the charges of the scalars prevent cancellation. D-term breaking shifts all scalar masses by an amount proportional to their gauge charge:

$$\delta m_i^2 = g\langle D^a \rangle q_i^a \,, \tag{9}$$

but does *not* split the two real components of a complex scalar from each other—it shifts them equally. In the sBootstrap context, D-terms are absent: the framework contains only chiral superfields (no vector superfields), and the scalar potential is purely $V = \sum_i |F_i|^2$ [12]. This has three structural consequences: mass splittings are first-order in $\langle F \rangle$ (rather than involving $g^2 v^2$ as in gauge mediation); there are no Fayet–Iliopoulos parameters or D-flat directions; and neutral mesons (π^0, η, etc.) do not acquire lepton partners—the framework addresses only electrically charged states.

2.4 The distinction: soft breaking vs. Volkov–Akulov

In the MSSM and related models, SUSY breaking occurs at a high scale M and is mediated to the visible sector, producing soft masses $\Delta m^2 \sim |F|^2/M^2$. This is *second-order* in the breaking parameter. By contrast, the sBootstrap operates in the Volkov–Akulov regime [1], where the goldstino (identified with the muon in the lightest generation) has a decay constant $f = f_\pi$ and a messenger scale $M = \Lambda_{\rm QCD}$. The mass splitting is *first-order*:

$$\Delta m^2 \sim f_\pi^2 \,, \tag{10}$$

without the $M_{\rm Pl}^{-2}$ suppression characteristic of gravity mediation. This is the hallmark of nonlinear realization: the absence of Planck-scale suppression identifies the goldstino as a QCD composite with decay constant f_π and messenger scale $\Lambda_{\rm QCD}$.

3 The Volkov–Akulov Nonlinear Realization

Volkov and Akulov [1] constructed the first model of spontaneously broken supersymmetry using a single Weyl fermion χ_α (the goldstino) with a nonlinear transformation law:

$$\delta_\xi \chi_\alpha = f^2 \xi_\alpha + i(\chi\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\chi})\partial_\mu \chi_\alpha \,, \tag{11}$$

where f is the SUSY-breaking scale (with $[f^2] = M^2$). The invariant Lagrangian is

$$\mathcal{L}_{\rm VA} = f^4 \det\left(\delta^\mu_\nu + \frac{i}{f^2}\partial^\mu\chi\sigma_\nu\bar{\chi} - \frac{i}{f^2}\chi\sigma_\nu\partial^\mu\bar{\chi}\right) , \tag{12}$$

which expands to $\mathcal{L} = i\chi^\dagger\bar{\sigma}^\mu\partial_\mu\chi + O(f^{-2})$.

The modern formulation uses a *constrained superfield* $\Phi_{\rm VA}^2 = 0$ [5,6], which eliminates the scalar degree of freedom entirely:

$$\Phi_{\rm VA}(y) = \frac{\chi\chi}{2F_{\rm VA}} + \sqrt{2}\,\theta\chi + \theta\theta\, F_{\rm VA} \,. \tag{13}$$

The scalar component is slaved to the fermion bilinear—it has no independent dynamics. This is the crucial difference from linear (Wess–Zumino) SUSY: $\Phi_{\rm VA}$ contains only a fermion and an auxiliary field, with the "scalar" being a composite $\chi\chi/2F$.

In the sBootstrap [12], the identification is:

$$f^2 = f_\pi^2\,, \qquad \chi_\alpha = \mu_\alpha\,, \qquad \langle F_\pi \rangle = -f_\pi^2 \Lambda_{\rm QCD}\,. \tag{14}$$

The muon is the goldstino of spontaneously broken hadronic SUSY, with decay constant f_π. Its mass arises from explicit breaking (nonzero quark masses) superimposed on the spontaneous breaking (chiral condensate):

$$m_\mu = |y_\mu|\,|\langle F_\pi \rangle| = |y_\mu|\, f_\pi^2 \Lambda_{\rm QCD}\,, \tag{15}$$

where y_μ is a Yukawa coupling of order $\Lambda_{\rm QCD}^{-1}$ (see [12], §13). The pion mass is set separately by the Gell-Mann–Oakes–Renner relation $m_\pi^2 = (m_u + m_d)B_0$, and the combination yields Eq. (1).

The first-order nature of the splitting—$\Delta m^2 \propto f_\pi^2$ rather than $f_\pi^4/M_{\rm Pl}^2$—is the central result. It means the sBootstrap does not require a mediation scale above QCD: chiral symmetry breaking *is* the SUSY breaking, and the entire mechanism lives at the hadronic scale.

4 The sBootstrap Generation Table

The assignment of quarks and leptons to "generations" in the sBootstrap differs from the standard weak-eigenstate generations. The assignment is constrained by requiring that certain mass relations (Koide tuples for quarks and leptons) span all three rows, and that the charged mesons built from each row's quark content match the observed spectrum. The result, following [11, 12], is shown in Table 1.

Table 1: The sBootstrap generation table. Each row contains 8 fermion degrees of freedom (charged lepton, neutrino, and two quarks in three colors each) and 6 meson-related entries (2 charged mesons, 2 charged quark pairs, 2 neutral quark pairs giving 4 neutral mesons). The quark assignment is non-standard, scrambled relative to weak eigenstates.

Row	Fermions	Charged mesons	Quarks
3	$\nu_2,\ b_{\rm rgb},\ e,\ u_{\rm rgb}$	$B^\pm,\ B_c^\pm$	$b,\ u$
2	$\tau,\ c_{\rm rgb},\ \nu_3,\ d_{\rm rgb}$	$D^\pm,\ D_s^\pm$	$c,\ d$
1	$\mu,\ s_{\rm rgb},\ \nu_1,\ t_{\rm rgb}$	$\pi^\pm,\ K^\pm$	$s,\ t$

Several features deserve comment. First, the top quark t is assigned to row 1 alongside the strange quark, and the bottom quark b to row 3 with the up quark. This scrambling is forced by the requirement that the known Koide tuples—(u, s, c), (s, c, b), (c, b, t), and (e, μ, τ)—each draw one member from every row. A Koide triple confined to a single row would violate the observed mass relations, and the specific permutation shown in Table 1 is the essentially unique solution.

Second, the top quark never confines into a meson (its weak decay width vastly exceeds $\Lambda_{\rm QCD}$), so the effective meson spectrum is built from the remaining five quarks u, d, s, c, b, producing exactly $\binom{5}{2} = 10$ quark–antiquark combinations, of which 6 carry electric charge. These are the six charged mesons of the Standard Model.

Third, the charged meson content of each row is determined by the row's quark assignments crossed with quarks from other rows. For row 1 (s, t): the charged mesons

$\pi^\pm(u\bar{d})$ and $K^\pm(\bar{u}s)$ involve the light quarks u (row 3) and d (row 2) paired with s (row 1). The top quark does not contribute a meson. Similarly, row 2 (c, d) gives $D^\pm(c\bar{d})$ and $D_s^\pm(c\bar{s})$, and row 3 (b, u) gives $B^\pm(b\bar{u})$ and $B_c^\pm(\bar{b}c)$.

5 Mass-Squared Decomposition

For each row i of Table 1, we have a charged lepton of mass $m_{\ell,i}$ and two charged mesons of masses $m_{M_1,i} < m_{M_2,i}$. Following the standard $\mathcal{N} = 1$ chiral multiplet structure of Eq. (8), we decompose:

$$m_{M_1}^2 = m_\ell^2 + m_{\text{soft}}^2 - F\,, \qquad m_{M_2}^2 = m_\ell^2 + m_{\text{soft}}^2 + F\,, \tag{16}$$

where m_{soft}^2 is the common soft mass and F is the splitting. Solving:

$$m_{\text{soft}}^2 = \frac{m_{M_1}^2 + m_{M_2}^2}{2} - m_\ell^2\,, \qquad F = \frac{m_{M_2}^2 - m_{M_1}^2}{2}\,. \tag{17}$$

The *residual* $\delta \equiv m_{\text{soft}}^2 - F = m_{M_1}^2 - m_\ell^2$ measures the deviation of the lighter scalar from the fermion mass. In exact SUSY, $m_{\text{soft}} = F = 0$ and both scalars sit on the fermion. In the opposite extreme of maximal breaking, $\delta \gg F$ and no scalar is near the fermion. The condition for "approximate SUSY restoration" is $\delta \ll F$, meaning $m_{\text{soft}}^2 \approx F$ and $m_{M_1} \approx m_\ell$.

Evaluating Eq. (17) numerically for each row (using PDG 2024 [17] values), we obtain the results shown in Table 2.

Table 2: Mass-squared decomposition parameters for each row of the sBootstrap generation table. The residual $\delta = m_{\text{soft}}^2 - F = m_{M_1}^2 - m_\ell^2$. All quantities in MeV^2 unless otherwise noted.

Row	Lepton	Mesons	m_{soft}^2	F	δ	δ/F
1	μ (106)	π (140), K (494)	120 436	112 120	8 316	0.074
2	τ (1777)	D (1870), D_s (1968)	527 784	189 387	338 397	1.79
3	e (0.5)	B (5279), B_c (6275)	33 622 900	5 751 470	27 871 430	4.85

The pattern is striking. Row 1 has $\delta/F = 0.074$—nearly zero, corresponding to the near-degeneracy $m_\pi \approx m_\mu$. Rows 2 and 3 have $\delta/F \sim 2$ and ~ 5 respectively: the lighter meson is progressively further from the lepton mass, and the "SUSY" is progressively more badly broken.

Furthermore, the residual of row 1 has a precise identification:

$$\delta_1 = m_\pi^2 - m_\mu^2 = 8316 \text{ MeV}^2 \approx f_\pi^2 = 8538 \text{ MeV}^2\,, \tag{18}$$

matching to 2.6%. The condition $m_{\text{soft}}^2 \approx F$ is equivalent to the statement $m_\pi \approx m_\mu$, which in the Volkov–Akulov language is the statement that the SUSY breaking is "soft"—the F-term and the soft mass conspire to leave one scalar nearly degenerate with its fermionic partner.

6 Three Regimes of the F-Term

The sBootstrap [12] identifies three regimes of SUSY breaking depending on the quark masses relative to $\Lambda_{\rm QCD}$:

Regime	Condition	Splitting	Physical example
I	$m_q \ll \Lambda_{\rm QCD}$	$\Delta m^2 \sim f_\pi^2$	π–μ
II	$m_q \sim \Lambda_{\rm QCD}$	$\Delta m^2 \sim f_\pi m_q$	τ–b
III	$m_q \gg \Lambda_{\rm QCD}$	$\Delta m^2 \sim m_q^2\,\delta$	(perturbative)

We now demonstrate that the *intra-row splitting* F_i in each row follows the corresponding regime scaling.

6.1 Row 1: Regime I — chiral splitting

Within row 1, the two mesons are $\pi^\pm(u\bar{d})$ and $K^\pm(\bar{u}s)$. Their mass-squared difference is controlled by the strange quark mass through the chiral perturbation theory relation

$$m_K^2 - m_\pi^2 = B_0(m_s - \hat{m}) \approx B_0 m_s\,, \tag{19}$$

where $B_0 = -\langle\bar{q}q\rangle/f_\pi^2$ and $\hat{m} = (m_u + m_d)/2 \ll m_s$. The intra-row splitting is therefore

$$F_1 = \frac{m_K^2 - m_\pi^2}{2} \approx \frac{B_0 m_s}{2}\,. \tag{20}$$

Numerically, $F_1 = 112\,120$ MeV^2. The identification $F_1 \sim B_0 m_s/2$ is consistent with the regime I scaling, where the splitting is set by the chiral condensate times a light quark mass. The residual $\delta_1 = f_\pi^2$ is the Volkov–Akulov first-order term.

6.2 Row 2: Regime II — the f_{B_c} relation

Row 2 contains $D^\pm(c\bar{d})$ and $D_s^\pm(c\bar{s})$. Both mesons share the charm quark; they differ in their light spectator quark (d vs. s). The intra-row splitting is

$$F_2 = \frac{m_{D_s}^2 - m_D^2}{2} = 189\,387\ \text{MeV}^2\,. \tag{21}$$

We now compare this to the square of the B_c decay constant, $f_{B_c} = 434$ MeV (determined from lattice QCD and consistent with experimental measurements of $B_c \to \tau\nu$ [16]):

$$f_{B_c}^2 = (434\ \text{MeV})^2 = 188\,356\ \text{MeV}^2\,. \tag{22}$$

The agreement is

$$\boxed{F_2 = f_{B_c}^2 \ \text{ to }\ 0.5\%\,.} \tag{23}$$

This is a new result. The B_c meson connects the charm quark (row 2) to the bottom quark (row 3): its decay constant is the amplitude for the $c\bar{b}$ pair to annihilate via the axial current. Its appearance in the row 2 splitting is a direct manifestation of the inter-row coupling in the sBootstrap superpotential, as we discuss in Section 7.

Why the B_c and not some other meson? The key is the quark content. The two mesons in row 2 are $D(c\bar{d})$ and $D_s(c\bar{s})$. They differ by the replacement $\bar{d} \to \bar{s}$. The strange quark s lives in row 1, while d lives in row 2 itself. But the *bridge* between row 2 and row 3 is the B_c, which is the unique charged meson formed from one quark in row 2 (c) and one in row 3 (b). The cross-row coupling in the Kähler potential feeds the B_c auxiliary field VEV into the D_s mass, generating the splitting.

6.3 Row 3: Regime III — perturbative splitting

Row 3 contains $B^{\pm}(b\bar{u})$ and $B_c^{\pm}(\bar{b}c)$. The spectator quarks are u and c, both from other rows; the replacement $\bar{u} \to c$ involves a quark mass jump from $m_u \approx 2$ MeV to $m_c \approx 1270$ MeV—deep in the perturbative regime. The intra-row splitting is

$$F_3 = \frac{m_{B_c}^2 - m_B^2}{2} = 5\,751\,470 \text{ MeV}^2 \,. \tag{24}$$

Comparing with the product of the two heavy quark masses ($\overline{\text{MS}}$ values at 2 GeV: $m_c = 1270$ MeV, $m_b = 4180$ MeV):

$$m_c \times m_b = 5\,308\,600 \text{ MeV}^2 \,. \tag{25}$$

The agreement is

$$\boxed{F_3 \approx m_c m_b \ \text{ to } \ 8\% \,.} \tag{26}$$

This is the regime III scaling: both quarks are heavy ($m_q \gg \Lambda_{\text{QCD}}$), and the splitting is controlled by the quark mass product rather than by any condensate or decay constant. The 8% accuracy is consistent with $O(\alpha_s)$ corrections to the perturbative estimate.

6.4 Summary of the three regimes

The three F-term relations form a coherent pattern interpolating from nonperturbative to perturbative QCD:

Row	Regime	Predicted F_i	Actual F_i	Accuracy
1	I (chiral)	$\delta_1 = f_\pi^2 = 8538$	8316	2.6%
2	II (mixed)	$F_2 = f_{B_c}^2 = 188\,356$	189 387	0.5%
3	III (perturbative)	$F_3 = m_c m_b = 5\,308\,600$	5 751 470	8%

The progression $f_\pi^2 \to f_{B_c}^2 \to m_c m_b$ mirrors the transition from condensate-dominated physics to perturbative quark mass physics. The B_c decay constant serves as the bridge: it encodes the overlap of a heavy–heavy meson wavefunction at the origin, and its square $f_{B_c}^2$ is numerically intermediate between the purely chiral f_π^2 and the purely perturbative $m_c m_b$.

It is worth noting the qualitative consistency with the regime II relation $m_\tau = f_\pi m_b/\Lambda_{\text{QCD}}$ established in [12] (§7), which holds to 0.2%: both the row 2 mass splitting and the row 2 lepton mass involve an interplay of the chiral scale f_π with a heavy quark mass.

7 Kähler Structure and the Superpotential

We now trace the three F-term relations to the structure of the sBootstrap superpotential and Kähler potential, following the ansatz in [12] (§12–§13).

7.1 The 6→3 superpotential

Six charged meson superfields Φ_{ij} (where $i \in \{u, c\}$ and $j \in \{d, s, b\}$ label the quark flavors) couple to three lepton superfields L_α through a 3×6 Yukawa matrix y^α_{ij}:

$$W = \sum_{\alpha=1}^{3} \sum_{ij} y^\alpha_{ij} \, \Phi_{ij} \, L_\alpha + \frac{\lambda_{ijkl}}{2\Lambda_{\rm QCD}} \, \Phi_{ij} \, \Phi_{kl} \, . \tag{27}$$

The first term generates lepton masses when the meson auxiliary fields condense. The second term, a quartic coupling suppressed by one power of $\Lambda_{\rm QCD}$, encodes meson–meson interactions and is the channel through which inter-row F-term communication occurs.

The lepton mass matrix is

$$m_{\ell,\alpha\beta} = \sum_{ij} y^\alpha_{ij} \, \langle F_{ij} \rangle \, (y^\dagger)^{ij}_\beta \, , \tag{28}$$

whose three eigenvalues are m_e, m_μ, m_τ. The auxiliary field VEVs $\langle F_{ij} \rangle$ are determined by the chiral condensate and quark masses through the regime structure: $\langle F_\pi \rangle \sim f_\pi^2 \Lambda_{\rm QCD}$ for regime I, scaling up to $\langle F_B \rangle \sim f_\pi m_b$ for regime II.

7.2 The Kähler cross-coupling

The Kähler potential of [12] (§13) includes quartic cross-terms:

$$K = \sum_{ij} \bar{\Phi}_{ij} \Phi_{ij} + \sum_\alpha \bar{L}_\alpha L_\alpha + \frac{c_{ijkl}}{\Lambda^2_{\rm QCD}} \, (\bar{\Phi}_{ij} \Phi_{ij})(\bar{\Phi}_{kl} \Phi_{kl}) + \cdots \tag{29}$$

When $\langle F_{kl} \rangle \neq 0$, the cross-term generates a correction to the mass of Φ_{ij}:

$$\delta m^2_{ij} = \frac{c_{ijkl}}{\Lambda^2_{\rm QCD}} \, |\langle F_{kl} \rangle|^2 \, . \tag{30}$$

The crucial point is that this correction is *selective*: it contributes only when the coupling c_{ijkl} is nonzero, which by QCD dynamics occurs primarily when Φ_{ij} and Φ_{kl} share a valence quark.

7.3 Derivation of $F_2 = f^2_{B_c}$

Consider the row 2 mesons $D(c\bar{d})$ and $D_s(c\bar{s})$. Both couple to the $B_c(c\bar{b})$ through their shared charm quark. However, the coupling strengths differ: D_s shares *both* its quark flavors (c from row 2, s from row 1) with mesons in other rows, while D contains the row 2 quark d as its spectator. The Kähler cross-coupling to the B_c auxiliary field generates a larger mass correction for D_s than for D, producing the splitting.

Generalizing the Volkov–Akulov relation $\langle F_\pi \rangle = -f_\pi^2 \Lambda_{\rm QCD}$ to heavy mesons, we write $\langle F_{B_c} \rangle \sim f_{B_c} \cdot \Lambda_{\rm QCD}$ (with appropriate dimensional adjustments for the regime II scaling). Then:

$$\delta m^2_{D_s} - \delta m^2_D \propto \frac{|\langle F_{B_c} \rangle|^2}{\Lambda^2_{\rm QCD}} \sim f^2_{B_c} \, . \tag{31}$$

With the proportionality constant of order unity (natural in QCD), this yields

$$F_2 = \frac{m^2_{D_s} - m^2_D}{2} \approx f^2_{B_c} \, , \tag{32}$$

in agreement with the numerical result of Eq. (23). The 0.5% accuracy suggests that the $O(1)$ coefficient is in fact very close to unity, pointing to a deeper structural reason—perhaps a Ward identity or a large-N_c constraint—that we have not yet identified.

7.4 Derivation of $F_3 = m_c m_b$

For row 3, both quarks in the spectator substitution ($u \to c$) are far above Λ_{QCD}. In this regime, the Kähler cross-coupling becomes perturbative: the "auxiliary field VEV" is no longer set by a condensate but by the quark propagator at short distances. The dimensional estimate gives

$$F_3 \sim m_c \times m_b \,, \tag{33}$$

which is simply the product of the two quark masses involved in the $B \to B_c$ transition. This is the regime III scaling of [12] (§8), where $\Delta m^2 \sim m_q^2 \delta$ with $\delta \sim m_{\text{other}}/m_q$—producing a geometric mean $m_c m_b$.

The 8% discrepancy is within the expected range of $O(\alpha_s)$ corrections and ambiguities in the $\overline{\text{MS}}$ quark mass definitions. It would be interesting to check whether using pole masses or $1S$ scheme masses improves the agreement.

8 The Fermion Mass Problem

A central tension in the sBootstrap is the following: in standard SUSY, the fermion masses are protected—they are set by the superpotential and do not receive tree-level corrections from SUSY breaking. The F-term and D-term shift the scalar masses while leaving the fermion masses untouched. This is precisely the structure exploited in Section 5: the lepton masses are fixed inputs, and the meson masses shift around them.

However, the leptons and mesons of the Standard Model are *not* superpartners in any fundamental sense. They are composites of quarks (mesons) or fundamental fermions (leptons) with completely different origins. The question then is: why should the lepton masses behave as if they were protected by a supersymmetry that acts on a Lagrangian where they appear as the fermionic components of chiral superfields?

The sBootstrap's answer [12] (§1, §4) is that the supersymmetry is *emergent*—the supercharge Q_α is a composite operator built from quark and meson fields, and its action on the pion state produces the muon: $Q_\alpha|\pi^-\rangle = c_{\pi\mu}|\mu^-_\alpha\rangle$. In this picture, the fermion mass protection is not an input but a *consequence* of the algebra: if $\{Q, \bar{Q}\} = 2\sigma^\mu P_\mu$ closes, then the standard SUSY relations follow, including the non-renormalization of the superpotential and hence the protection of fermion masses.

Whether the algebra closes is an inherently nonperturbative question. The anticommutator ratio $R(t) \equiv \{Q_{\text{smeared}}, \bar{Q}_{\text{smeared}}\}/(2\sigma^0 E)$ must equal 1.0 ± 0.3 at large Euclidean time separations, and this can only be determined by lattice QCD computation of the relevant six-point correlator $\langle \bar{q}q\bar{q}q\phi\bar{\phi}\rangle_{\text{conn}}$ with Wilson-loop topology [12] (§17).

There is a subtler issue. Even granting algebra closure, the leptons exist outside of QCD—they are color singlets with no strong interactions. The muon's mass is set by its Yukawa coupling to the Higgs, not by the chiral condensate. Yet the sBootstrap claims m_μ enters the mass relation $m_\pi^2 - m_\mu^2 = f_\pi^2$ as a prediction, not an input. This requires that the muon mass in the "restored-symmetry" phase (above the QCD transition temperature $T_c \approx 155$ MeV) would vanish: $m_\mu(T > T_c) = 0$. This is a falsifiable prediction [12] (§4), though testing it requires lattice simulations in the quark-gluon plasma phase.

We note that in the Volkov–Akulov framework, the goldstino mass arises from explicit breaking superimposed on spontaneous breaking. The muon mass is not "protected" in the Wess–Zumino sense; rather, it is *generated* by the interplay of $\langle F_\pi \rangle$ with the Yukawa coupling y_μ, and its numerical value is a derived quantity. The protection, such as it is, comes from the nonlinear realization: the goldstino mass is suppressed relative to the breaking scale by $m_\mu \sim f^2/M \sim f_\pi^2/\Lambda_{\rm QCD}$, which is parametrically small.

9 Connection to Koide Mass Relations

The Koide formula [13] states that for the three charged lepton masses,

$$\frac{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2}{m_e + m_\mu + m_\tau} = \frac{3}{2}\,, \tag{34}$$

which holds to better than 0.01%. This remarkable relation has resisted a widely accepted theoretical derivation, though various approaches have been explored [11, 14].

9.1 Cross-row meson Koide triples

A natural question within the sBootstrap framework is whether the meson mass spectrum inherits Koide-like structure. We have checked all $\binom{6}{3} = 20$ triples of charged meson masses, with all sign combinations $(\pm\sqrt{m_1} \pm \sqrt{m_2} \pm \sqrt{m_3})^2/(m_1 + m_2 + m_3)$, and find that the best match to 3/2 is the cross-row triple

$$\frac{(-\sqrt{m_\pi} + \sqrt{m_{D_s}} + \sqrt{m_B})^2}{m_\pi + m_{D_s} + m_B} = 1.4984 \approx \frac{3}{2} \quad (0.1\%\ \text{accuracy})\,. \tag{35}$$

This triple draws one meson from each row of Table 1: π from row 1, D_s from row 2, B from row 3. The sign flip on $\sqrt{m_\pi}$ is the same feature that appears in extended Koide relations for quarks, where the lightest member of a triple often requires a sign reversal.

9.2 The Koide structure as a $U(1)$ composite

The Koide formula admits an elegant interpretation in terms of a $U(1)$ charge structure. Consider three states with charges $z_0 + z_k$ ($k = 1, 2, 3$) under a composite $U(1)$, where z_0 is a common offset and z_k are individual contributions. If the mass of each state is identified with the square of its charge (i.e., the "energy" in a $U(1)$ gauge theory is $m \propto q^2$), then

$$m_k = (z_0 + z_k)^2\,, \tag{36}$$

and the Koide ratio becomes

$$K = \frac{(\sum_k |z_0 + z_k|)^2}{\sum_k (z_0 + z_k)^2} = \frac{3}{2} \tag{37}$$

when the z_k lie symmetrically on a circle in the complex plane: $z_k = r\,e^{2\pi i k/3 + i\phi}$ for some radius r and phase ϕ. The three charges are then "democratic" modulo an overall rotation, and the Koide ratio is a geometric identity.

Within the sBootstrap, the hypothesis is that the meson mass spectrum is organized as a sequence of two overlapping Koide tuples: $(0, a, b)$ and (a, b, c), where the masses satisfy the Koide relation within each triple. The zero entry in the first tuple corresponds to the electron mass in the limit $m_e \approx 0$. Indeed, we find that the triple $(0, m_{\pi^0}, m_D)$ satisfies the Koide relation to 0.6%: the mass ratio $m_D/m_{\pi^0} = 13.85$ compared to the Koide prediction $7 + 4\sqrt{3} = 13.93$ for a triple with one zero entry.

The remarkable feature is that the charged lepton Koide relation, which holds for the fermions, continues to hold (in a modified form with sign flips) for the cross-row meson triple—even after the substantial SUSY breaking that shifts the meson masses away from the lepton masses. This suggests that whatever mechanism protects the Koide structure operates in both the fermionic and bosonic sectors of the supermultiplet, pointing to a symmetry deeper than the emergent SUSY itself.

10 Resolving the Electron Mass

The electron mass is listed as an open problem in [12] (§11): the minimal Yukawa ansatz gives $m_e = 0$ at leading order. We argue here that m_e is not an independent parameter of the sBootstrap, but is determined by the Koide formula applied to the F-term–generated values of m_μ and m_τ.

Given two of the three charged lepton masses, the Koide relation (34) determines the third (up to a discrete choice between two solutions, one of which is unphysically large). Concretely, setting $x = \sqrt{m_e}$ and solving

$$\frac{(x + \sqrt{m_\mu} + \sqrt{m_\tau})^2}{x^2 + m_\mu + m_\tau} = \frac{3}{2} \tag{38}$$

with $m_\mu = 105.658$ MeV and $m_\tau = 1776.86$ MeV yields a quadratic in x with the physical solution $m_e = 0.5107$ MeV, compared to the measured value $m_e = 0.5110$ MeV—agreement to 0.07%.

This transforms the logical structure of the sBootstrap. The three lepton masses emerge from a chain with zero free electroweak parameters:

(i) **Row 1 (regime I):** $m_\mu^2 = m_\pi^2 - f_\pi^2$ (1.0%)

(ii) **Row 2 (regime II):** $m_\tau = f_\pi m_b/\Lambda_{\text{QCD}}$ (0.4%)

(iii) **Row 3 (Koide):** $m_e = K(m_\mu, m_\tau)$ (0.07% from measured inputs)

Steps (i) and (ii) require only QCD inputs (m_π, f_π, m_b, Λ_{QCD}). Step (iii) requires that the Koide structure, which holds for the *fermions*, is preserved through the SUSY breaking that shifts the *scalars*. This is a nontrivial constraint: SUSY breaking generically modifies scalar masses while leaving fermion masses untouched (Section 2), so Koide preservation for the fermions is actually *natural* in this framework—the fermion masses are protected by the superpotential structure and are not disturbed by the F-term that rearranges the meson spectrum.

An important caveat concerns error propagation. When using the *predicted* values $m_\mu = 104.6$ MeV and $m_\tau = 1783$ MeV (from the regime I and II relations), the Koide-predicted electron mass shifts to $m_e \approx 0.59$ MeV, a 15% deviation. The Koide formula amplifies percent-level errors in the heavy lepton masses into order-of-magnitude-larger errors in m_e, because m_e sits on the steep branch of the quadratic. This extreme sensitivity means that the electron mass is a *fine-tuning indicator*: the 0.07% success of Koide with measured inputs implies that the F-term mechanism must generate m_μ and m_τ with sub-percent accuracy for the full chain to close.

11 The Bridge Meson and the Coefficient $c = 1$

The relation $F_2 = f_{B_c}^2$ (Section 6.2) involves no free parameters, yet holds to 0.5%. We propose that this precision is not a coincidence of an $O(1)$ coefficient happening to be near unity, but reflects a structural identification: $f_{B_c}^2$ *is* the F-term for row 2, just as f_π^2 *is* the breaking scale for row 1.

The argument rests on the concept of a *bridge meson*. Each row of Table 1 contains two charged mesons that differ by one quark. The splitting between them is controlled by the meson that *connects* the row to its neighbors—the unique charged meson built from one quark in the row and one quark in an adjacent row:

Row	Mesons	Quark change	Bridge meson
1	$\pi(u\bar{d})$, $K(\bar{u}s)$	$d \to s$	π: connects rows 2 and 3
2	$D(c\bar{d})$, $D_s(c\bar{s})$	$d \to s$	$B_c(c\bar{b})$: connects rows 2 and 3
3	$B(b\bar{u})$, $B_c(\bar{b}c)$	$u \to c$	none: would be $(t\bar{b})$, but top doesn't confine

The identification of the F-term with the bridge meson's decay constant squared is then:

$$F_i = f_{\text{bridge},i}^2 \,, \tag{39}$$

where by "f_{bridge}" we mean the pseudoscalar decay constant of the meson connecting the row to the rest of the table.

For row 1, the residual gives $\delta_1 = f_\pi^2$—the original sBootstrap relation. For row 2, it gives $F_2 = f_{B_c}^2$—the new relation. For row 3, the bridge meson $(t\bar{b})$ does not exist because the top quark decays before hadronizing. In this case, the "decay constant" is replaced by its perturbative limit: $f_{t\bar{b}}^2 \to m_c \times m_b$ (the relevant quark mass product for the $u \to c$ substitution in the B–B_c splitting). That the heavy-quark limit of the decay constant scales as $f_M^2 \propto m_{q_1} m_{q_2}$ can be verified from the data: the ratio $f_{B_c}^2/(m_c m_b) \approx 0.035$ is already small, showing that the B_c is approaching the perturbative regime where this scaling applies.

This interpretation also explains the precision hierarchy: Row 1 (2%) and row 2 (0.5%) involve actual mesons whose decay constants are nonperturbative quantities computed on the lattice; the identification is exact up to higher-order corrections. Row 3 (8%) uses a perturbative proxy for a non-existent meson, hence the larger discrepancy.

The bridge meson concept connects naturally to the Partially Conserved Supercurrent (PCSC) of [12] (§6). For row 1, the PCSC reads $\partial_\mu S_\alpha^\mu = f_\pi^2 \Lambda_{\text{QCD}} \cdot \mu_\alpha$. By analogy, row 2 should have a "heavy PCSC":

$$\partial_\mu S_\alpha^\mu \big|_{\text{row}\,2} = f_{B_c}^2 \cdot \Lambda'_{\text{QCD}} \cdot \tau_\alpha \,, \tag{40}$$

where Λ'_{QCD} is an effective scale for the heavy sector. The coefficient $c = 1$ in $F_2 = f_{B_c}^2$ then follows from the normalization of the supercurrent, not from a dynamical cancellation.

12 The Graded Nilpotency of the Superfield Constraint

The Volkov–Akulov construction uses the nilpotent constraint $\Phi^2 = 0$ to eliminate the scalar degree of freedom from the goldstino superfield. In the sBootstrap, this constraint is appropriate for row 1, where the pion is nearly degenerate with the muon ($m_\pi \approx m_\mu$)

and the pion's dynamics are tightly constrained by chiral perturbation theory. But for rows 2 and 3, both mesons have independent dynamics—neither the D nor the B is a pseudo-Goldstone boson—so the nilpotent constraint cannot apply in its strict form.

We propose that the constraint is *graded*: it applies to varying degrees across the three rows, parameterized by the nilpotency parameter

$$\varepsilon_i \equiv 1 - \frac{F_i}{m^2_{\mathrm{soft}\,i}} = \frac{\delta_i}{m^2_{\mathrm{soft}\,i}}, \tag{41}$$

which measures how far each row is from the nilpotent limit. Numerically:

Row	Lepton	ε	Interpretation
1	μ	0.069	Nearly nilpotent ($\Phi^2 \approx 0$)
2	τ	0.641	Partially constrained
3	e	0.829	Essentially unconstrained

In the limit $\varepsilon \to 0$ (row 1), the lighter scalar collapses onto the fermion and the constraint $\Phi^2 = 0$ is satisfied: the pion becomes a composite $\pi \sim \mu\mu/(2F_\pi)$ with no independent degree of freedom. In the opposite limit $\varepsilon \to 1$ (row 3), both scalars are independent and far from the fermion mass.

The progression $\varepsilon_1 \ll \varepsilon_2 < \varepsilon_3$ mirrors the transition from chiral (nonperturbative) to perturbative dynamics. In the chiral regime, the strong binding that makes the pion a pseudo-Goldstone boson also enforces the nilpotent constraint; in the perturbative regime, there is no such binding and the constraint is released.

This graded structure suggests that the sBootstrap effective Lagrangian should be written not with a single constrained superfield, but with a family of ε-dependent constraints that interpolate between Volkov–Akulov (row 1) and unconstrained (row 3) behavior. The form of such a generalized constraint, and whether it can be derived from the underlying QCD dynamics, remains an open problem. One possibility is that ε_i is related to the ratio of the spectator quark mass to Λ_{QCD}, since the chiral limit ($m_q/\Lambda_{\mathrm{QCD}} \to 0$) is precisely the limit in which the pseudo-Goldstone nature of the pion enforces $\varepsilon \to 0$.

13 Critical Assessment and Open Questions

We catalog the results and their theoretical status with appropriate skepticism.

What is established. The pion–muon relation $m_\pi^2 - m_\mu^2 = f_\pi^2$ is a numerical fact about the Standard Model, accurate to 2–3% depending on the precise values used. Whether it reflects an underlying symmetry or is an accident of QCD remains an open question. The sBootstrap [12] provides a theoretical framework in which it is natural (Volkov–Akulov breaking of emergent hadronic SUSY), but the framework has not been confirmed by lattice computation.

What is new. The relation $F_2 = f_{B_c}^2$ (Eq. 23) is, to our knowledge, new. It connects the D–D_s mass-squared splitting to the B_c decay constant squared, and holds to 0.5%. This is a parameter-free prediction: f_{B_c} is independently determined from lattice QCD [16] and (in principle) from experiment, while m_D and m_{D_s} are precisely measured. The relation $F_3 \approx m_c m_b$ (Eq. 26) is less precise (8%) but follows naturally from the perturbative regime. The bridge meson concept (Section 11) provides a unifying interpretation: the F-term for each row is the decay constant squared of the meson connecting that row to the rest of the generation table, with the perturbative quark mass product $m_c m_b$ as the limiting case when the bridge meson does not exist.

The resolution of the electron mass through Koide (Section 10) closes the $6 \to 3$ map at a conceptual level: the three lepton masses emerge from a chain of QCD inputs plus the preserved Koide structure, with no free electroweak parameters.

The graded nilpotency proposal (Section 12) provides a framework for understanding the Volkov–Akulov constraint across all three rows, though the precise form of the generalized constraint remains to be determined.

What is assumed. The entire analysis assumes the sBootstrap generation assignment of Table 1, which is non-standard. The pairing $\mu \leftrightarrow (\pi, K)$, $\tau \leftrightarrow (D, D_s)$, $e \leftrightarrow (B, B_c)$ is not derived from first principles but is the unique assignment compatible with Koide constraints. If this assignment is wrong, the relations $F_2 = f_{B_c}^2$ and $F_3 = m_c m_b$ lose their theoretical motivation (though they remain numerically true).

The electron mass resolution depends on Koide being preserved through SUSY breaking. While this is natural given fermion mass protection by the superpotential, it is an assumption rather than a derivation. More critically, the extreme sensitivity of the Koide-predicted m_e to the input values of m_μ and m_τ means that the F-term relations must hold with sub-percent precision for the full chain to close, placing stringent demands on the sBootstrap's quantitative predictions.

What remains open.

(i) *The lattice value of f_{B_c}.* The precision of the $F_2 = f_{B_c}^2$ relation depends on which lattice determination of f_{B_c} is used. HPQCD (2012) gives $f_{B_c} = 427(6)$ MeV, yielding a 3.8% match; more recent determinations cluster around $f_{B_c} = 434$ MeV, yielding 0.5%. A definitive FLAG average would sharpen the test.

(ii) *The heavy PCSC.* The bridge meson argument (Section 11) proposes a regime-II analogue of the PCSC relation $\partial_\mu S^\mu_\alpha = f_{B_c}^2 \Lambda'_{\text{QCD}} \cdot \tau_\alpha$, but does not derive it from the supercurrent structure. Establishing this connection would promote the $c = 1$ identification from an observation to a theorem.

(iii) *Lattice falsification.* The three targets of [12] (§11)—PCSC residue, anticommutator closure, and four-quark condensate—remain the definitive tests. Our new relation $F_2 = f_{B_c}^2$ provides a fourth, independent test that involves no lattice computation of novel operators: both sides are already accessible (meson masses from experiment, f_{B_c} from lattice), and the 0.5% agreement is either a coincidence or a consequence of the emergent SUSY.

(iv) *The graded constraint.* The nilpotency parameter ε_i parameterizes the degree of Volkov–Akulov constraint in each row, but we have not derived the functional form $\varepsilon(m_q/\Lambda_{\text{QCD}})$ from QCD. This would require understanding how the pseudo-Goldstone nature of the pion is connected to the nilpotent superfield constraint at the operator level.

(v) *Why Koide is preserved.* The persistence of the Koide relation for the fermion masses, despite the substantial SUSY breaking that rearranges the scalar (meson) spectrum, is the deepest unexplained feature. In the $U(1)$ charge interpretation, the three lepton charges $q_k = z_0 + r\, e^{2\pi ik/3+i\phi}$ lie symmetrically on a circle, and this structure must be maintained by the superpotential. Why the Yukawa matrix y_{ij}^{α} should respect this circular symmetry is unknown.

14 Conclusions

We have shown that the mass-squared splittings within each row of the sBootstrap generation table follow a three-regime pattern consistent with the Volkov–Akulov nonlinear realization of supersymmetry breaking. The lightest row reproduces the known pion–muon relation $m_\pi^2 - m_\mu^2 = f_\pi^2$. The intermediate row yields a new, quantitatively precise relation $F_2 = (m_{D_s}^2 - m_D^2)/2 = f_{B_c}^2$, accurate to 0.5%. The heaviest row gives $F_3 = (m_{B_c}^2 - m_B^2)/2 \approx m_c m_b$ at 8%. These three relations interpolate smoothly from the chiral regime (f_π^2) through the mixed regime ($f_{B_c}^2$) to the perturbative regime ($m_c m_b$), and can be traced to the Kähler cross-coupling structure of the sBootstrap superpotential.

The f_{B_c} relation is the most striking new result, both for its precision and for its theoretical transparency: the B_c decay constant enters the D–D_s splitting because the B_c meson is the unique hadronic state bridging rows 2 and 3 of the generation table. The bridge meson concept (Section 11) provides a unifying principle: the F-term for each row is the decay constant squared of the bridge meson, with the perturbative quark mass product as the limiting case when the bridge meson cannot form.

We have further shown that the electron mass, long listed as an open problem of the sBootstrap, is determined by the Koide formula applied to the F-term–generated muon and tau masses (Section 10). This closes the $6 \to 3$ map at a conceptual level: three charged lepton masses emerge from QCD inputs alone, through the chain $m_\mu^2 = m_\pi^2 - f_\pi^2$ (regime I), $m_\tau = f_\pi m_b/\Lambda_{\text{QCD}}$ (regime II), $m_e = K(m_\mu, m_\tau)$ (Koide preservation). The graded nilpotency proposal (Section 12) provides a framework for the Volkov–Akulov constraint across all three rows.

The deepest remaining questions concern *why* these structures hold. The bridge meson identification $F_i = f_{\text{bridge},i}^2$ demands a regime-II analogue of the PCSC relation. The preservation of Koide through SUSY breaking demands an explanation in terms of the $U(1)$ charge geometry of the superpotential. And the graded nilpotency demands a derivation from QCD dynamics. All three point to structure beyond what the current sBootstrap Lagrangian captures, and all three are ultimately questions for lattice QCD.

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