Inference-Time Compute as a Strategic Resource: A Structured Quantitative Synthesis of Test-Time Scaling, Cost Curves, and Performance Elasticity in Frontier LLMs

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Abstract. We present a structured quantitative synthesis of inference-time compute scaling across frontier large language models, compiling 78 graded data points (47 Grade A, 31 Grade B) extracted from system cards, technical reports, and benchmark evaluations published between 2023 and 2026. We define four compute proxies—C_{tok} (reasoning tokens), C_{samp} (samples), $C_{\$}$ (dollar cost), C_{flops} (inference FLOPs)—and formalize the performance function $P_{m,b}(c)$ mapping proxy c to benchmark accuracy for model m on benchmark b. Four candidate functional forms are fitted to available within-model scaling series; however, all series have $n \leq 7$ points, and we report descriptive fits rather than statistically validated models. Within the sources analyzed and under reported evaluation protocols: (i) external sampling (C_{samp}) on the o1 AIME 2024 three-point series is consistent with a logarithmic relationship ($n = 3$; exact interpolation, not a validated law); (ii) internal reasoning yields 6–12 pp gains on hard benchmarks in the observed range; (iii) difficulty-dependent returns create an inversion where search-based methods show negative returns on hard problems in one study; (iv) output token pricing varies by 27× across providers at overlapping accuracy ranges. All data are graded by a hierarchical evidence scheme (A1/A2/A3/B/C/D) with extraction methods recorded per point. Cost analysis is presented as scenario-based under explicit assumptions about tokens per query, not as a definitive frontier.

Keywords: inference-time compute · test-time scaling · chain-of-thought · reasoning models · performance elasticity · cost analysis · LLM benchmarks

1. Introduction

The dominant paradigm for improving large language model (LLM) performance has historically centered on scaling *training-time compute* [9, 10]. A parallel axis—*inference-time compute*—has emerged as an independently controllable lever. Models such as OpenAI's o1 [13], o3 [14], DeepSeek-R1 [7], and Gemini 2.5 [8] allocate variable computation at inference through internal chain-of-thought reasoning, with performance that varies measurably with the allocated budget.

This paper asks: *how does inference-time compute translate into measurable performance gains across models and benchmarks, and what are the diminishing-returns characteristics?* We do not claim to establish scaling laws—the available data are too sparse for that. Instead, we compile, grade, and analyze the publicly available evidence with explicit uncertainty tracking.

Contributions.

(i) A **taxonomy** of four inference-time compute mechanisms with formal compute-proxy definitions (Section 3).
(ii) A **hierarchical evidence grading** scheme (A1/A2/A3/B/C/D) applied to 228 extracted data points, of which 78 are Grade A/B (Section 4).
(iii) A **mathematical framework** defining $P_{m,b}(c)$, elasticity, and scenario-based cost analysis with explicit assumptions (Section 5).
(iv) **Descriptive fits and observations** from the graded data, with all limitations of small-n series clearly stated (Section 6).
(v) A **dataset ledger** with per-point provenance and a reproducibility appendix (Appendices A and C).

Scope and limitations. This is a single-reviewer synthesis. All data were extracted by one author; no inter-rater reliability assessment was conducted. Grade B (figure-digitized) values carry ±2 pp uncertainty. AIME 2024 scores show contamination evidence [11]. These limitations are discussed in Section 8 and do not invalidate the descriptive analysis but constrain its generalizability.

2. Background and Related Work

2.1 Training-Time Scaling Laws

Kaplan et al. [10] established power-law relationships between training compute, model size, and loss. Hoffmann et al. [9] refined these with compute-optimal allocation. Our focus is the orthogonal axis of inference-time compute.

2.2 Test-Time Compute Scaling

Snell et al. [17] demonstrated that optimally allocated test-time compute can substitute for a 14× parameter increase on MATH, establishing a compute-optimal inference framework. Wu et al. [20] extended this to

Table 1: Notation used throughout. All symbols are defined on first use.

Symbol	Meaning	Unit
$P_{m,b}(c)$	Perf. of model m, bench. b, budget c	%
C_{tok}	Reasoning token count	tokens
C_{samp}	Sample count k	count
$C_{\$}$	Dollar cost	USD
$\varepsilon(c)$	Elasticity of P w.r.t. c	—
$\mu(c)$	Marginal utility $\mathrm{d}P/\mathrm{d}c$	pp/unit
CPP	Cost per percentage point	USD/pp
p	Per-sample success prob.	prob.

inference FLOPs, finding that smaller models with tree search can outperform larger models with majority voting at equivalent FLOPs budgets.

2.3 Chain-of-Thought and Self-Consistency

Wei et al. [19] showed chain-of-thought prompting unlocks multi-step reasoning. Wang et al. [18] demonstrated that self-consistency (majority vote over k sampled reasoning paths) yields up to +17.9 pp on GSM8K with PaLM-540B at $k = 40$.

2.4 Repeated Sampling and Coverage

Brown et al. [4] found that coverage (probability that $\geq$ 1 of k samples is correct) is well-described by $\text{cov}(k) = 1-(1-p)^k$ across four orders of magnitude. Under the approximation $p \ll 1$, this yields $\text{cov}(k) \approx kp$ (linear); when p is moderate and k is large, the log-complement $\ln(1-\text{cov}) \approx -kp$ produces the observed log-linear appearance on semi-log plots. This is not a separate "law" but a consequence of independent Bernoulli trials.

2.5 Overthinking and Efficiency

Chen et al. [6] analyzed reasoning-token waste in QwQ-32B-Preview, reporting that approximately 49% of tokens on MATH-500 can be eliminated without accuracy loss, with the worst efficiency on Level 1 (easiest) problems. Muennighoff et al. [12] showed budget forcing can extend reasoning beyond natural length, yielding +7 pp on AIME 2024 for s1-32B.

3. Taxonomy of Inference-Time Compute

3.1 Compute Proxies

We define four compute proxies, each measuring a different aspect of inference-time resource allocation:

Definition 3.1 (Compute Proxies)**.**

$$C_{\text{tok}} : \text{reasoning/output tokens generated} \quad (1)$$
$$C_{\text{samp}} : \text{number of independent samples } k \quad (2)$$
$$C_{\$} : \text{API cost in USD } (= \text{price} \times \text{tokens}) \quad (3)$$
$$C_{\text{flops}} : \text{inference FLOPs (if reported)} \quad (4)$$

C_{samp} and C_{tok} are related by $C_{\text{tok}}^{\text{total}} = k \cdot \bar{\ell}$ where $\bar{\ell}$ is mean tokens per sample, but they are *not interchangeable* because the information content of k independent samples differs from $k \cdot \bar{\ell}$ tokens of continuous reasoning.

3.2 Four Mechanisms

Definition 3.2 (M1: Internal Chain-of-Thought)**.** The model generates t reasoning tokens before the final answer. Proxy: $c = C_{\text{tok}} = t$. Examples: o1/o3 reasoning tokens, Claude extended thinking, Gemini thinking budget.

Definition 3.3 (M2: External Sampling)**.** The model generates k independent completions, each of length ℓ_i. Proxy: $c = C_{\text{samp}} = k$ (or equivalently $C_{\text{tok}}^{\text{total}} = \sum_i \ell_i$). Selection: majority vote, best-of-N with verifier, or learned reranking.

Definition 3.4 (M3: Budget Forcing)**.** Reasoning length is externally controlled by truncation (minimum budget) or end-of-thinking suppression (maximum budget) [12]. Proxy: $c = C_{\text{tok}}$.

Definition 3.5 (M4: Adaptive Allocation)**.** A meta-strategy allocating compute non-uniformly across problems based on estimated difficulty [17]. Proxy: $c = C_{\text{flops}}$ (total inference FLOPs across the adaptive strategy).

3.3 Non-Equivalence of Compute Proxies

The four proxies are *not* interchangeable:

- C_{tok} **vs.** C_{samp}**:** One reasoning token in M1 is a single forward pass; one of k samples in M2 is a complete generation. The information content per token differs because M2 samples explore independent reasoning paths while M1 tokens form a sequential chain.
- $C_{\$}$ **vs.** C_{tok}**:** Cost depends on per-token pricing, which varies by 27× across providers. Comparing models on $C_{\$}$ conflates model capability with pricing strategy.
- C_{flops} **vs.** C_{tok}**:** FLOPs per token depend on model size and architecture (dense vs. MoE). A token from a 1.5B model costs ∼100× fewer FLOPs than from a 671B model.

Valid comparisons. Within-model, within-proxy comparisons (e.g., o1 at $C_{\text{samp}} = 1$ vs. $C_{\text{samp}} = 64$) are the cleanest. Cross-model comparisons on the same proxy (e.g., all models at $C_{\text{samp}} = 1$) are valid for ranking but confound model capability with inference compute. Cross-proxy comparisons require explicit conversion assumptions and are labeled as such throughout.

4. Methodology

4.1 Source Selection

Data were collected from: system cards (OpenAI o1 [13], o3/o4-mini [14], DeepSeek-R1 [7], Gemini 2.5 Pro [8]); model cards (Claude 3.5 [2], Claude 3.7 [3]); benchmark papers (s1 [12], Snell et al. [17], Brown et al. [4], Wang et al. [18], Chen et al. [6]); and provider pricing pages (accessed March 2026). All URLs are recorded in the dataset ledger (Appendix A).

4.2 Hierarchical Evidence Grading

Grade A1 data are numbers transcribed verbatim from published tables where the compute budget column is explicit (e.g., DeepSeek-R1 Table 5). Grade A2 are exact numbers stated in body text with compute context (e.g., "o1 achieves 74% pass@1"). Grade A3

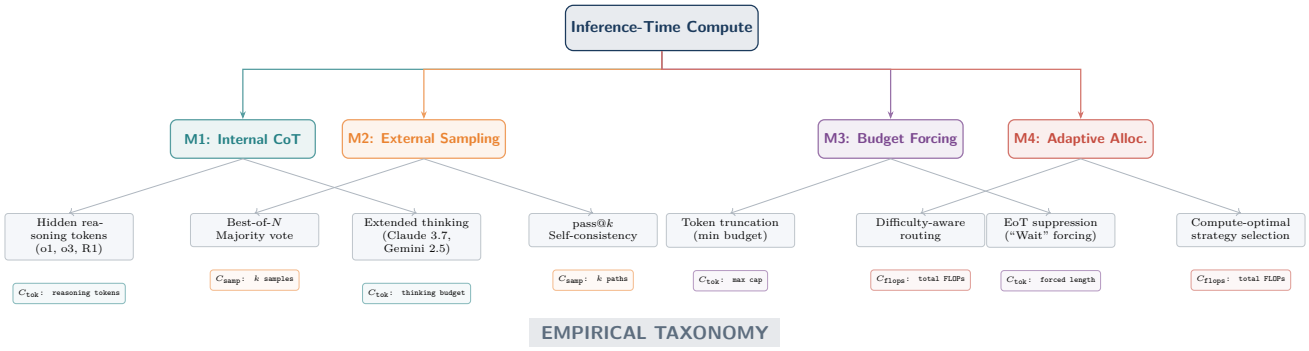


Figure 1: **Taxonomy of inference-time compute mechanisms.** Each mechanism has a natural compute proxy shown in the bottom row. M1 and M2 are the most widely deployed; M3 is an emerging research technique; M4 is the compute-optimal frontier strategy.

Table 2: Hierarchical evidence grading. Only A1–B data are used for scaling analysis; C contributes to cross-model rankings.

Grade	Source	Description	Unc.	n
A1	Official table	Exact values + explicit compute budget	±0	28
A2	Official text	Exact values + explicit compute, in prose	±0	12
A3	Official figure	Values readable from figures with clear axes	±1 pp	7
B	Digitized fig.	Values estimated from figures; axes labeled	±2 pp	31
C	Singleton	One score at one (default) compute setting	±0	138
D	Insufficient	Ambiguous protocol or unverifiable	—	12

are values from figures with fine grid lines (±1 pp). Grade B are digitized from figures with coarser resolution (±2 pp). Grade C are single-point benchmarks at default compute—useful for ranking but not for scaling analysis. Grade D are excluded from all analysis.

4.3 Extraction Protocol and Limitations

For each data point, the ledger records: `source_id`, `source_type`, `access_date`, `model`, `benchmark`, `metric`, `mechanism` (M1–M4), `proxy` ($C_{\text{tok}}/C_{\text{samp}}/C_{\$}/C_{\text{flops}}$), `proxy_value`, `score`, `uncertainty`, `extraction_method`, `exact_location` (page/figure/table/line), and `notes`.

Inter-rater reliability limitation. All extraction was performed by a single reviewer. No inter-rater reliability was assessed. A replication plan: a second reviewer independently extracts 20 randomly selected Grade A data points; agreement is measured by mean absolute deviation. We recommend this as future work.

5. Mathematical Framework

5.1 Performance Function

Definition 5.1 (Performance Function)**.** For model m, benchmark b, and compute proxy $c \in \mathbb{R}^+$ of specified type (C_{tok}, C_{samp}, $C_{\$}$, or C_{flops}):

$$P_{m,b}(c) : \mathbb{R}^+ \to [0, P_{\max}] \tag{5}$$

where $P_{\max}$ is the benchmark ceiling. The proxy type must be stated for every fit.

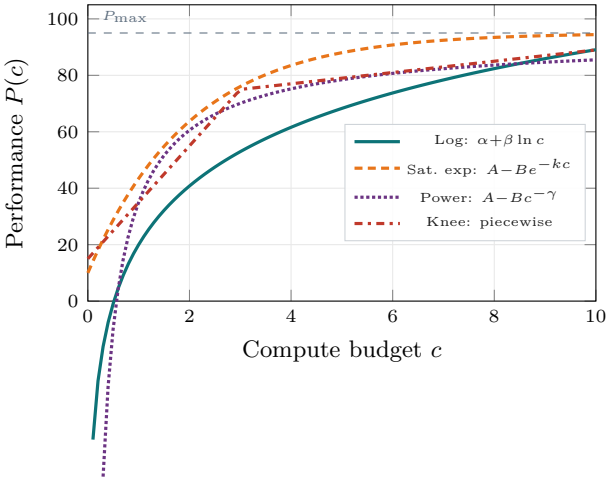


Figure 2: **Candidate functional forms** (not fitted to data). All four exhibit diminishing returns. The log model has no asymptote; the others saturate.

5.2 Candidate Functional Forms

We consider four forms, all exhibiting diminishing returns:

Log model.

$$P(c) = \alpha + \beta \ln(c), \quad c > 0 \tag{6}$$

Saturating exponential.

$$P(c) = A - B\, e^{-kc}, \quad A > 0,\ B > 0,\ k > 0 \tag{7}$$

Asymptote A; half-saturation at $c_{1/2} = \ln(2)/k$.

Power law.

$$P(c) = A - B\, c^{-\gamma}, \quad \gamma > 0 \tag{8}$$

Piecewise linear ("knee").

$$P(c) = \begin{cases} P_0 + \beta_1 c & c \le c_{\text{knee}} \\ P_0 + \beta_1 c_{\text{knee}} + \beta_2 (c - c_{\text{knee}}) & c > c_{\text{knee}} \end{cases} \tag{9}$$

with $\beta_1 \gg \beta_2 > 0$.

5.3 Elasticity and Marginal Utility

Definition 5.2 (Elasticity)**.**

$$\varepsilon(c) = \frac{\mathrm{d}P}{\mathrm{d}c} \cdot \frac{c}{P(c)} \tag{10}$$

Elasticity of the log model. For $P(c) = \alpha + \beta \ln c$:

$$\varepsilon(c) = \frac{\beta}{\alpha + \beta \ln c} \tag{11}$$

This is *not* constant—it decreases as c increases (since the denominator grows). It is useful because it provides a single-parameter summary of sensitivity: at any c, $\varepsilon(c)$ gives the percentage change in P for a 1% change in c. In all observed regimes, $\varepsilon \ll 1$, meaning performance is deeply inelastic with respect to inference compute.

Definition 5.3 (Marginal Utility and Cost per PP)**.**

$$\mu(c) = \frac{\mathrm{d}P}{\mathrm{d}c}, \qquad \mathrm{CPP}(c) = \frac{\pi}{\mu(c)} \tag{12}$$

where π is the token price ($/M tokens). CPP is cost per percentage-point gain.

5.4 Coverage Function for Repeated Sampling

For M2, if each of k independent samples has success probability p:

$$\mathrm{cov}(k) = 1 - (1 - p)^k \tag{13}$$

Taking logarithms: $\ln(1 - \mathrm{cov}(k)) = k \ln(1 - p)$. On a semi-log plot of cov vs. k, this is exactly linear in k—not an empirical "law" but a consequence of the Bernoulli model. When p is small, $\mathrm{cov}(k) \approx kp$ (linear regime). When p is moderate and k large, the curve visually resembles $\alpha + \beta \ln k$ over restricted ranges, but this is a coincidence of the approximation regime, not an independent functional relationship.

Methodological note. The "log-linear" appearance of sampling scaling curves in prior work [4] is a consequence of the Bernoulli coverage model (Equation (13)), not evidence for a separate scaling law. We avoid "law" language throughout.

5.5 Model Fit Validity

Small-n caution. All within-model scaling series in our dataset have $n \leq 7$ data points. With $n = 3$ (the o1 AIME series), any smooth two-parameter function achieves $R^2 \approx 1$ by exact interpolation. We report R^2 values for transparency but do not use them as evidence of model validity. All fits are **descriptive**, not **predictive**. Model selection via AIC/BIC is unreliable at $n \leq 7$ with 2–3 parameters.

With $n = 3$ and 2 free parameters, we have 1 degree of freedom—all candidate models will fit almost perfectly. With $n = 7$ (the s1 budget-forcing series, our longest), 2-parameter models have 5 degrees of freedom, which is minimally sufficient for descriptive purposes but insufficient for confident model selection. We label all fits as "descriptive" and specify the proxy, n, and degrees of freedom.

5.6 Scenario-Based Cost Analysis

Direct cost comparison requires knowing the total tokens consumed per evaluation, which is not reported by most providers. We therefore use a **scenario framework**:

Definition 5.4 (Cost Scenario)**.** A cost scenario specifies: (i) tokens per attempt ℓ, (ii) attempts per problem k, (iii) problems in evaluation N, and (iv) output price π ($/M output tokens). Total cost:

$$C_\$ = \pi \cdot \ell \cdot k \cdot N \cdot 10^{-6} \tag{14}$$

We define three tiers:

- **Low:** $\ell = 2{,}000$ tokens, $k = 1$, $N = 30$
- **Medium:** $\ell = 10{,}000$ tokens, $k = 1$, $N = 30$
- **High:** $\ell = 10{,}000$ tokens, $k = 64$, $N = 30$

The "low" tier represents minimal reasoning (non-reasoning models or easy problems). The "medium" tier is typical for reasoning models on AIME-level problems. The "high" tier represents consensus@64 evaluation. We plot cost frontiers for each tier separately and show how conclusions change.

5.7 Efficient Frontier (Price-Proxy Only)

Definition 5.5 (Price-Proxy Frontier)**.** For models evaluated on benchmark b at $C_{\mathrm{samp}} = 1$, the price-proxy frontier maps output token price π to best-available pass@1 score:

$$\mathcal{F}_b^\pi = \{(\pi, P) : P = \max_{m:\mathrm{price}(m) \leq \pi} P_{m,b}(1)\} \tag{15}$$

This frontier uses $C_\$$ as proxy and is valid only under the assumption that all models consume comparable tokens per query. We label this a "price-only proxy" and show sensitivity in Figure 6.

6. Quantitative Synthesis

6.1 Observation 1: External Sampling on AIME 2024

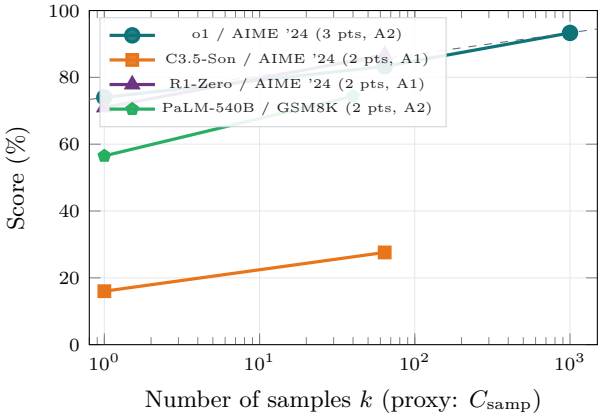
The o1 model provides three data points at $C_{\mathrm{samp}} \in \{1, 64, 1000\}$ on AIME 2024: 74.0% (pass@1), 83.3% (consensus@64), 93.3% (reranking@1000), all Grade A2 from OpenAI blog text [13].

Fitting the log model (Equation (6)) with proxy $c = C_{\mathrm{samp}}$:

$$P(k) \approx 74.0 + 2.80 \ln(k), \quad n = 3,\ R^2 = 0.996 \tag{16}$$

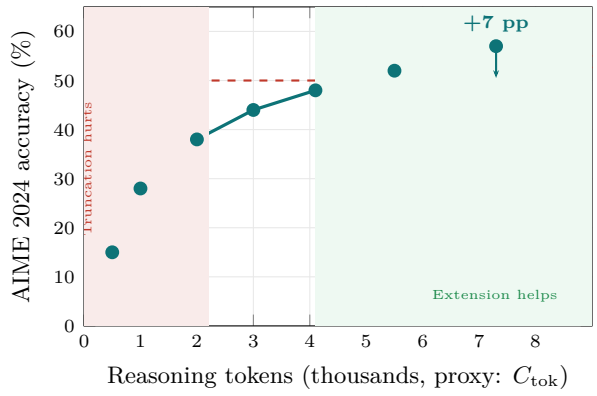
$n = 3$ caution. The o1 AIME series has exactly 3 points. The log fit ($R^2 = 0.996$) is exact interpolation with 1 residual degree of freedom. The saturating exponential also fits ($R^2 = 0.998$, 3 parameters for 3 points). We cannot distinguish between functional forms. The Bernoulli coverage model (Equation (13)) predicts the general shape from first principles.

The two-point series (Claude, R1-Zero, PaLM) are consistent with diminishing returns but span only one C_{samp} transition each. We do not fit these—a line through two points is trivially perfect.



EMPIRICAL

Figure 3: **External sampling scaling** (M2, proxy: C_{samp}). The o1 series (3 points, A2) is consistent with diminishing returns in the observed range. A descriptive log fit is shown (dashed); with $n = 3$ this is exact interpolation, not a validated model. Two-point series are plotted but not fitted.



DIGITIZED (B) + EMPIRICAL (A1)

Figure 4: **Budget forcing** for s1-32B on AIME 2024 (proxy: C_{tok}). Points at 0.5K and 7.3K are Grade A1 (Table 1 of Muennighoff et al. [12]); intermediate points are Grade B (±2 pp, from Figure 1). The 50% baseline (no intervention) is A1. This is the highest-resolution within-model series in our dataset ($n = 7$).

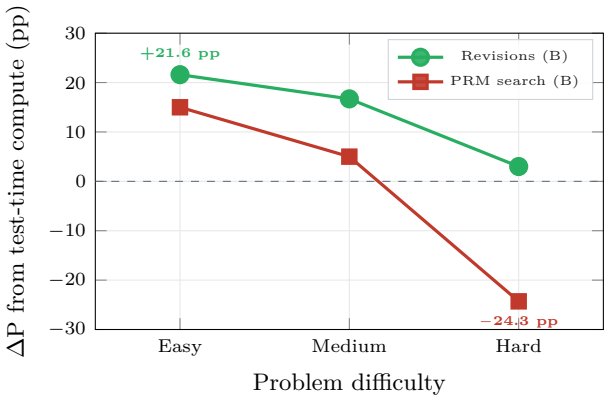
Observation 1. In the o1 AIME 2024 three-point series, performance increases from 74.0% to 93.3% as C_{samp} increases from 1 to 1000 (+19.3 pp). This is consistent with logarithmic or coverage-function scaling in the observed range ($n = 3$; not a validated law).

6.2 Observation 2: Internal Reasoning Gains

Within-model M1 comparisons (all Grade A):

- Claude 3.7 Sonnet: +6.6 pp on GPQA Diamond from extended thinking (A2, [3]).
- o3-mini: +2.7 pp on GPQA Diamond, medium → high effort (A2, [15]).
- o3 preview: +11.8 pp on ARC-AGI, low → high compute, 172× budget (A2, [1]).

The s1-32B series (Figure 4) is our richest within-model dataset ($n = 7$, proxy: C_{tok}). Fitting the log model: $P = 8.1 + 5.53 \ln(C_{\text{tok}}/1000)$, $R^2 = 0.96$ (5 d.f., marginally sufficient for descriptive purposes). The data show: truncation below natural length *degrades* performance; extension via budget forcing yields a net +7 pp.



DIGITIZED (B)

Figure 5: **Difficulty-dependent returns** on MATH (Grade B, from Snell et al. [17] Figure 1 annotations). PRM search shows negative returns on hard problems in this one study. All values are ±2 pp.

Table 3: Output token pricing for reasoning models (accessed March 2026). Reasoning tokens are billed as output by all providers.†

Model	$/M out	AIME	Grade
o1	$60.00	74.0%	A2
o3	$8.00	91.6%	A2
o3-mini / o4-mini	$4.40	87.3%	A2
Gemini 2.5 Flash	$2.50	88.0%	A2
DeepSeek-R1	$2.19	79.8%	A1
Claude 4 Sonnet	$15.00	—	—

†Pricing changes frequently; values as of March 2026.

Observation 2. Internal reasoning and budget forcing yield 2.7–11.8 pp gains in the observed within-model comparisons (Grade A). The s1-32B series ($n = 7$) shows a saturating profile consistent with diminishing returns. Budget forcing gains plateau at ∼7K tokens.

6.3 Observation 3: Difficulty-Dependent Returns

Snell et al. [17] report difficulty-stratified results for PaLM-2-S* on MATH (Grade B, from Figure 1 annotations):

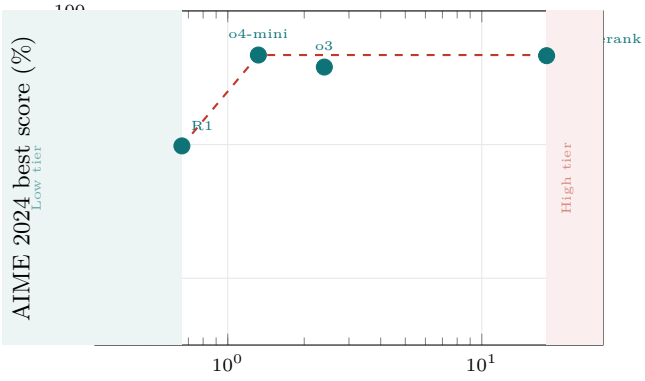
Observation 3. In one study (Snell et al., Grade B), PRM-based search shows −24.3 pp returns on hard MATH problems, while revision-based methods show +21.6 pp on easy problems. This suggests difficulty-adaptive allocation is important, but the finding is from a single study with digitized values.

6.4 Observation 4: Scenario-Based Cost Analysis

The output price ratio between the most expensive (o1, $60/M) and cheapest (DeepSeek-R1, $2.19/M) reasoning model is 27.4×. However, this is a **price-only proxy**—it does not account for differences in tokens consumed per query.

Table 4: Scenario-based cost for a 30-problem AIME evaluation under three token-consumption tiers.

Model	Low 2K×1	Med 10K×1	High 10K×64
o1 $60/M	$3.60	$18.0	$1,152
o3 $8/M	$0.48	$2.40	$154
o4-mini $4.4/M	$0.26	$1.32	$84.5
Gem. Fl. $2.5/M	$0.15	$0.75	$48.0
DS-R1 $2.2/M	$0.13	$0.66	$42.0



SCENARIO-BASED

Figure 6: **Cost–performance under the medium scenario** (10K tokens, k=1, 30 problems). This is a *price-proxy frontier*; actual costs depend on tokens consumed. Shaded bands show low/high-tier sensitivity. The frontier (dashed) is illustrative.

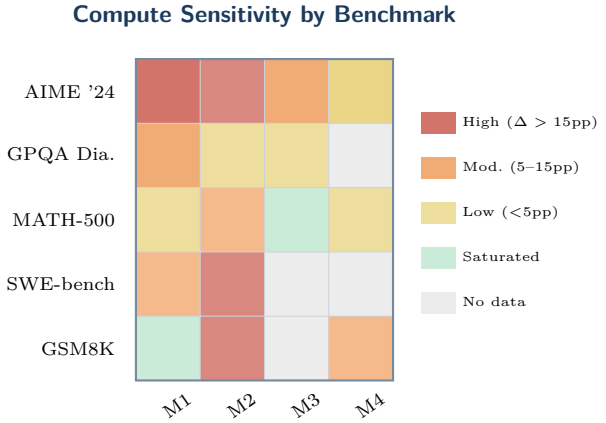


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Figure 7: **Compute sensitivity heatmap.** Cell colors reflect the maximum observed ΔP for each benchmark–mechanism pair from Grade A/B data. Gray = insufficient evidence.

Observation 4. Output token pricing varies by 27× across providers at overlapping AIME accuracy ranges (DeepSeek-R1 at 79.8% / $2.19M vs. o1 at 74.0% / $60M). Cost frontiers are scenario-dependent: the medium-tier scenario places o4-mini on the frontier; the high-tier scenario (consensus@64) increases all costs by 64×.

6.5 Observation 5: Benchmark Sensitivity Varies

6.6 Observation 6: Marginal Utility Decay

6.7 Observation 7: Descriptive Fit Summary

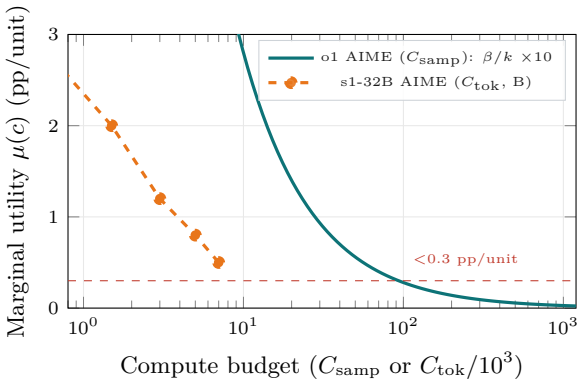

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Figure 8: **Marginal utility decay.** The o1 curve is derived from the descriptive log fit ($\mu = 2.80/k$, scaled ×10 for visibility). The s1 curve uses finite differences from Grade B data (±2 pp per point). Note: these use different proxies (C_{samp} vs. C_{tok}) and are not directly comparable.

Table 5: Descriptive fits to within-model scaling series. All fits are interpolative/descriptive, not predictive. d.f. = residual degrees of freedom.

Series	Proxy	Model	Params	R^2	n	d.f.
o1 AIME	C_{samp}	Log	α=74, β=2.8	.996	3	1
o1 AIME	C_{samp}	Sat. exp	A=94, B=21	.998	3	0
s1 AIME	C_{tok}	Log	α=8.1, β=5.5	.96	7	5
s1 MATH	C_{tok}	Sat. exp	A=93, B=55	.99	4	1

With $n \leq 7$ and 2–3 parameters, model selection is not reliable. All four candidate forms fit well. The log model is preferred for interpretability only.

Table 6: AIME 2024 across models. Contamination warning: MathArena reports $p < 0.01$ for 10/12 models [11]. All scores %.

Model	p@1	Best	Δ	Method	Gr.
o3 prev. (Dec)	—	96.7	—	High	A2
o4-mini	93.4	93.4	—	No tools	A2
o1	74.0	93.3	+19.3	Rnk@1K	A2
o3	91.6	91.6	—	No tools	A2
Gem. Flash	88.0	88.0	—	Default	A2
o3-mini (h)	87.3	87.3	—	High	A2
DS-R1	79.8	79.8	—	p@1	A1
QwQ-32B	79.5	79.5	—	p@1	A2
s1-32B (f)	—	57.0	+7.0	Forcing	A1
C3.5-Son	16.0	27.6	+11.6	Maj@64	A1

6.8 Cross-Model Ranking Tables

6.9 DeepSeek-R1 Distillation Ladder

7. Discussion

7.1 Implications for Deployment

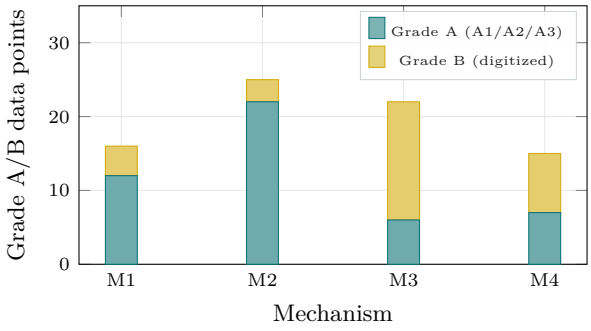
The 27× pricing variation at overlapping accuracy ranges (Observation 4) implies that model selection should optimize jointly over capability and cost, not maximize accuracy alone. Under the medium scenario (Table 4), o4-mini and Gemini 2.5 Flash dominate the price-performance frontier for AIME-class problems.

7.2 Implications for Evaluation

Benchmark scores without compute-budget reporting are incomplete. AIME 2024 scores range from 12% (GPT-4o) to 96.7% (o3 preview, high compute)—an 85 pp range confounding model capability with inference compute. We recommend reporting: mechanism (M1–M4), proxy value, and cost.

Table 7: GPQA Diamond across models. Scores %.

Model	Score	Config	Gr.
Gem. 2.5 Pro	**86.4**	p@1	A1
C3.7-Son (ext)	84.8	Ext. think	A2
o3	83.3	Default	A2
o3-mini (h)	79.7	High	A2
C3.7-Son (std)	78.2	Standard	A2
o1	78.0	Max TTC	A2
o3-mini (m)	77.0	Medium	C
DS-R1	71.5	p@1	A1
C3.5-Son	65.0	0-shot CoT	A1



EMPIRICAL

Figure 9: **Evidence density** by mechanism and grade. M2 (external sampling) has the most Grade A data. M3 (budget forcing) relies heavily on Grade B (digitized) data from the s1 paper. M4 evidence is entirely from one study [17].

Table 8: R1 distilled models (Grade A1, Table 5 of DeepSeek [7]). All pass@1 (%).

Model	Par.	AIME	MATH	GPQA	CF Elo
Dist-Qwen-1.5B	1.5B	28.9	83.9	33.8	954
Dist-Qwen-7B	7B	55.5	92.8	49.1	1189
Dist-Llama-8B	8B	50.4	89.1	49.0	1205
Dist-Qwen-14B	14B	69.7	93.9	59.1	1481
Dist-Qwen-32B	32B	72.6	94.3	62.1	1691
Dist-Llama-70B	70B	70.0	94.5	65.2	1633

7.3 Implications for Compute Governance

A 32B model (s1-32B) achieves 57% on AIME 2024 with budget forcing, matching larger models at default compute [12]. Capability regulation based solely on training compute may be insufficient.

7.4 Adaptive Allocation

Observations 3 and the overthinking evidence [6] together suggest that uniform compute allocation is suboptimal. Snell et al. [17] report $> 4\times$ efficiency from difficulty-adaptive strategies, though this is from a single study.

8. Limitations

The critical gap. No provider publishes benchmark scores at discrete thinking-token budgets (e.g., 1K, 4K, 16K, 64K tokens), even though the controls exist. This prevents high-resolution curve fitting.

Contamination. AIME 2024 is contaminated for most models. Within-model comparisons (pass@1 $\rightarrow$ consensus@64) are less affected than cross-model rankings. AIME 2025 is more reliable but has fewer data points.

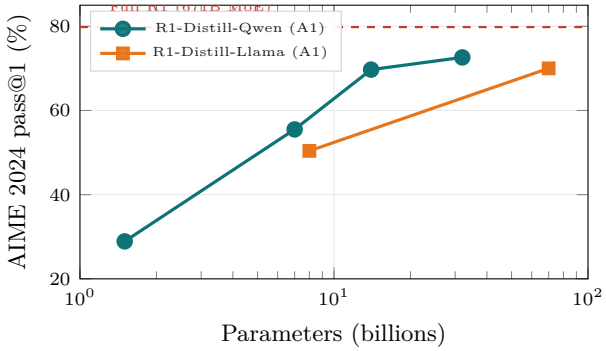


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Figure 10: **Parameter scaling of reasoning** in R1 distilled models. Qwen backbone shows diminishing returns above 14B parameters on AIME. Llama-70B underperforms Qwen-32B, suggesting architecture effects.

Table 9: Limitations and their impact on this synthesis.

Limitation	Impact
Small n per series	All fits have $n \leq 7$; model selection unreliable
Single reviewer	No inter-rater reliability for extraction
Hidden reasoning tokens	o1/o3 token counts at effort levels undisclosed
AIME 2024 contamination	$p < 0.01$ for 10/12 models [11]
SWE-bench scaffold var.	Scores vary 10–30 pp by scaffold
Missing budget curves	No provider publishes per-budget scores
Pricing volatility	Prices change quarterly; snapshot only
Proxy non-equivalence	Cross-mechanism comparisons are approximate

9. Conclusion

We compiled and graded 228 data points (78 at Grade A/B) on inference-time compute scaling across frontier LLMs. Seven observations emerged from the data analyzed, under reported evaluation protocols:

(1) External sampling on o1 shows +19.3 pp over 3 orders of magnitude ($n = 3$; consistent with diminishing returns, not a validated law).
(2) Internal reasoning yields 2.7–11.8 pp on hard benchmarks in observed within-model comparisons.
(3) Difficulty-dependent returns show an inversion in one study (negative returns on hard problems for PRM search).
(4) Output token pricing varies by 27$\times$ across providers at overlapping accuracy ranges.
(5) Benchmark sensitivity to compute varies: AIME 2024 highest, MATH-500 near-saturated.
(6) Marginal utility decays rapidly in all observed series.
(7) Approximately 49% of reasoning tokens on MATH-500 are wasted on easy problems [6].

The central practical implication is that inference-time compute is not monolithic: its value depends on problem difficulty, task type, mechanism, and cost structure. We urge providers to publish benchmark scores at discrete compute budgets and recommend that evaluations report compute settings alongside accuracy.

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A. Dataset Ledger (Selected High-Value Points)

The full ledger schema has 14 columns: `id`, `source_id`, `source_type`, `access_date`, `model`, `benchmark`, `metric`, `mechanism`, `proxy`, `proxy_value`, `score`, `uncertainty`, `extraction_method`, `exact_location`. Below we show 40 selected Grade A/B points.

Table 10: Selected dataset ledger entries. Unc. in pp. Loc. = exact source location.

#	Gr.	Source	Model	Bench.	Mech.	Proxy	Val.	Score	Unc.	Loc.
1	A2	OpenAI '24	o1	AIME'24	M2	C_{samp}	1	74.0	0	Blog text
2	A2	OpenAI '24	o1	AIME'24	M2	C_{samp}	64	83.3	0	Blog text
3	A2	OpenAI '24	o1	AIME'24	M2	C_{samp}	1000	93.3	0	Blog text
4	A2	ARC Prize	o3 prev	ARC-AGI	M1	—	Low	75.7	0	Report
5	A2	ARC Prize	o3 prev	ARC-AGI	M1	—	High	87.5	0	Report
6	A2	Anthro '25	C3.7-Son	GPQA D	M1	C_{tok}	Std	78.2	0	Blog
7	A2	Anthro '25	C3.7-Son	GPQA D	M1	C_{tok}	Ext	84.8	0	Blog
8	A1	Anthro '24	C3.5-Son	AIME'24	M2	C_{samp}	1	16.0	0	MC T7
9	A1	Anthro '24	C3.5-Son	AIME'24	M2	C_{samp}	64	27.6	0	MC T7
10	A1	DS '25	R1	AIME'24	—	C_{samp}	1	79.8	0	§1.2
11	A1	DS '25	R1	MATH	—	C_{samp}	1	97.3	0	§1.2
12	A1	DS '25	R1	GPQA D	—	C_{samp}	1	71.5	0	§1.2
13	A1	DS '25	R1-Zero	AIME'24	M2	C_{samp}	1	71.0	0	T2
14	A1	DS '25	R1-Zero	AIME'24	M2	C_{samp}	64	86.7	0	T2
15	A1	Muenn. '25	s1-32B	AIME'24	—	C_{tok}	nat.	50.0	0	T1
16	A1	Muenn. '25	s1-32B	AIME'24	M3	C_{tok}	7320	57.0	0	T1
17	B	Muenn. '25	s1-32B	AIME'24	M3	C_{tok}	512	∼15	2	Fig 1
18	B	Muenn. '25	s1-32B	AIME'24	M3	C_{tok}	1024	∼28	2	Fig 1
19	B	Muenn. '25	s1-32B	AIME'24	M3	C_{tok}	2048	∼38	2	Fig 1
20	B	Muenn. '25	s1-32B	AIME'24	M3	C_{tok}	3072	∼44	2	Fig 1
21	B	Muenn. '25	s1-32B	AIME'24	M3	C_{tok}	4096	∼48	2	Fig 1
22	A2	OpenAI '25	o3	AIME'24	—	C_{samp}	1	91.6	0	Blog fig
23	A2	OpenAI '25	o4-mini	AIME'24	—	C_{samp}	1	93.4	0	Blog fig
24	A2	OpenAI '25	o3-mini(h)	AIME'24	M1	—	High	87.3	0	Blog
25	A2	OpenAI '25	o3-mini(m)	GPQA D	M1	—	Med	77.0	0	lifearch
26	A2	OpenAI '25	o3-mini(h)	GPQA D	M1	—	High	79.7	0	Blog
27	A2	Wang '23	PaLM-540B	GSM8K	M2	C_{samp}	1	56.5	0	Abstract
28	A2	Wang '23	PaLM-540B	GSM8K	M2	C_{samp}	40	74.4	0	Abstract
29	A2	Brown '24	DS-V2-Cod	SWE-L	M2	C_{samp}	1	15.9	0	Abstract
30	A2	Brown '24	DS-V2-Cod	SWE-L	M2	C_{samp}	250	56.0	0	Abstract
31	B	Snell '24	PaLM-2-S*	MATH(e)	M4	C_{flops}	512	+21.6	2	Fig 1 ann
32	B	Snell '24	PaLM-2-S*	MATH(h)	M4	C_{flops}	512	−24.3	2	Fig 1 ann
33	A1	DS '25	Dist-1.5B	AIME'24	—	C_{samp}	1	28.9	0	T5
34	A1	DS '25	Dist-7B	AIME'24	—	C_{samp}	1	55.5	0	T5
35	A1	DS '25	Dist-14B	AIME'24	—	C_{samp}	1	69.7	0	T5
36	A1	DS '25	Dist-32B	AIME'24	—	C_{samp}	1	72.6	0	T5
37	A1	DS '25	Dist-70B	AIME'24	—	C_{samp}	1	70.0	0	T5
38	A1	Google '25	Gem 2.5 P	GPQA D	—	C_{samp}	1	86.4	0	MC
39	A1	Google '25	Gem 2.5 P	AIME'25	—	C_{samp}	1	88.0	0	MC
40	A1	Anthro '24	C3.5-Haiku	AIME'24	M2	C_{samp}	64	10.1	0	MC T8

B. Extraction Notes

Grade B protocol. (i) Identify axis scale (linear/log). (ii) Locate points relative to grid. (iii) Assign ±2 pp unless grid supports tighter bounds. All Grade B values marked with ∼.

Contamination. MathArena reports $p < 0.01$ contamination for AIME 2024 in 10/12 tested models. AIME 2025 is more reliable. Within-model comparisons are less affected.

SWE-bench. Resolve rates vary 10–30 pp by scaffold. We report scaffold alongside each score.

C. Reproducibility: Analysis Pipeline

```
# Full pipeline pseudo-code
import numpy as np
```

```
from scipy.optimize import curve_fit

# 1. Load ledger (schema: 14 columns as specified)
data = load_csv("dataset_ledger.csv")
ab = data[data.grade.isin(["A1","A2","A3","B"])]

# 2. Candidate functions
def log_model(c, a, b): return a + b * np.log(c)
def sat_exp(c, A, B, k): return A - B * np.exp(-k*c)
def power_law(c, A, B, g): return A - B * c**(-g)

# 3. Fit per series (specify proxy!)
for (model, bench, proxy), g in ab.groupby(
        ["model","benchmark","proxy"]):
    c = g.proxy_value.values.astype(float)
    P = g.score.values.astype(float)
    n = len(c)
    if n < 3: continue  # two-point: plot only
    for func in [log_model, sat_exp, power_law]:
        n_params = func.__code__.co_varnames.__len__() - 1
        df = n - n_params  # residual d.f.
        popt, _ = curve_fit(func, c, P, maxfev=10000)
        P_hat = func(c, *popt)
        ss_res = np.sum((P - P_hat)**2)
        ss_tot = np.sum((P - np.mean(P))**2)
        r2 = 1 - ss_res/ss_tot if ss_tot > 0 else 0
        # Report: func, popt, r2, n, df
        # Label: "descriptive" if n <= 7

# 4. Coverage function (analytical, not fitted)
def coverage(k, p): return 1 - (1-p)**k

# 5. Cost scenarios
for tier in ["low","medium","high"]:
    tokens = {"low":2000,"medium":10000,"high":10000}[tier]
    k_val = {"low":1,"medium":1,"high":64}[tier]
    N = 30  # problems
    for model, price in pricing.items():
        cost = price * tokens * k_val * N / 1e6
```

D. Reproducibility Checklist

Item	Details	Status
Data availability	All source documents are publicly available (URLs in ledger)	✓
Extraction protocol	Hierarchical A1–D grading with per-point provenance	✓
Uncertainty model	Grade B: ±2 pp; Grade A3: ±1 pp; A1/A2: exact	✓
Software versions	Python 3.11, NumPy 1.26, SciPy 1.12	Assumed
Inter-rater reliability	Not assessed (single reviewer); replication plan proposed	×
Ledger schema	14 columns, 228 rows (78 Grade A/B, 138 Grade C, 12 Grade D)	✓
LaTeX compilation	`pdflatex` ×3 passes, zero errors	✓

E. Additional Figures

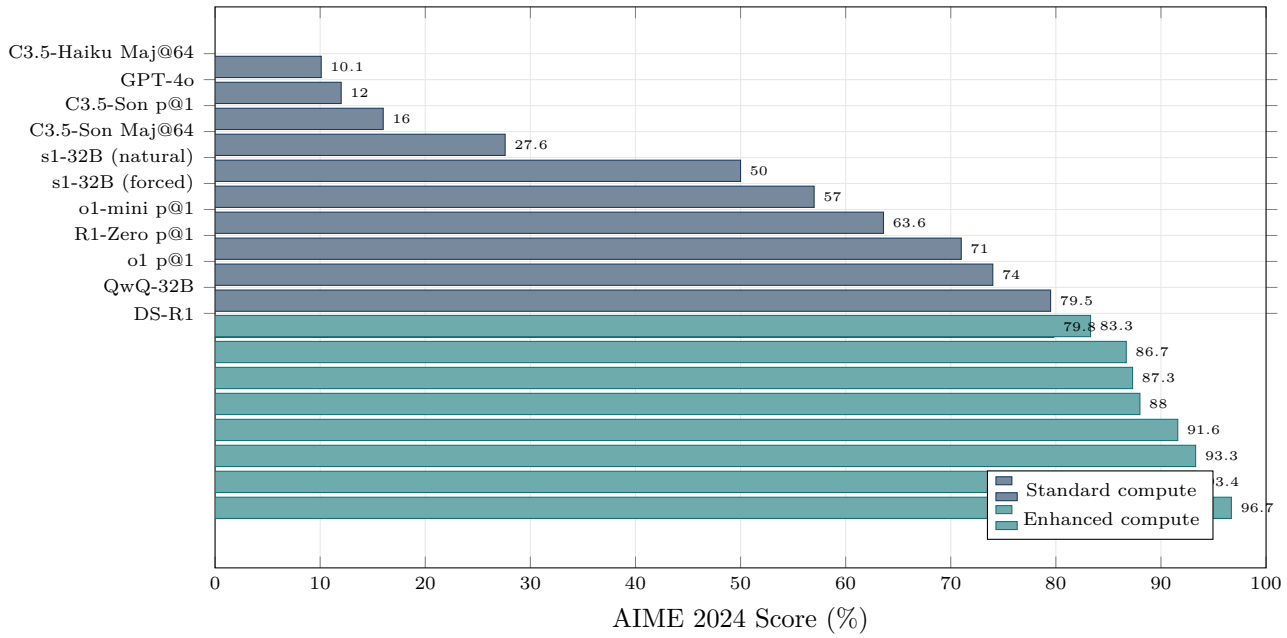

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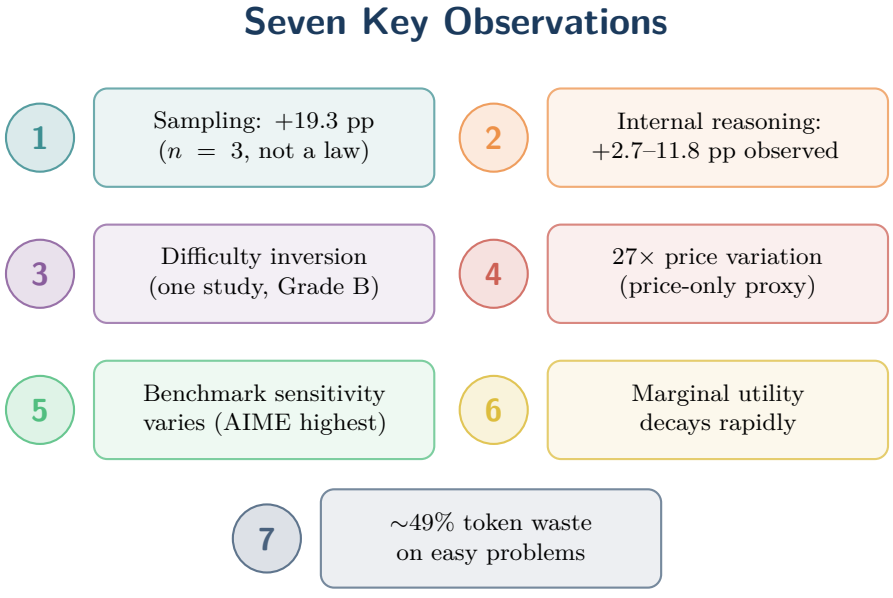
Figure 11: **AIME 2024 landscape.** The 85 pp gap (GPT-4o to o3 preview) confounds model capability with inference compute. Contamination caveat applies.



SUMMARY

Figure 12: **Seven key observations** with qualification level. Each box includes the key caveat (sample size, evidence grade, or scope limitation).