The Prime Gear Geometry (PGG) Resolution

The Mechanical and Signal Basis of the Riemann Hypothesis

Chaiya Tantisukarom *†

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Abstract

This study formalizes the Prime Gear Geometry (PGG) as a dynamical system. We demonstrate that the Riemann Hypothesis (RH) is not a static property of numbers, but a structural necessity of a rolling engine. We identify the m-cutoff as the "Mechanical Secret" that governs the transition between discrete prime forging (Time Domain) and spectral stability (Frequency Domain).

1 The Mechanical Hardware: Prime Gear Identity

The number system is modeled as a **Prime Gear Group**, [6], Figure: 1.

- **The C_1 Nucleus:** The smallest and the master gear with circumference 1 ($r = 1/2\pi$). When C_1 rolls, its single spoke will create the integer axis.
- **The C_p Assembly:** Every prime p is a unique circumference gear p.
- **The New Prime:** It is achieved when the C_1 spoke is the only one that lands at that integer number. Other C_p spokes are in the air. Even if there is a C_{p-n} spoke close to land to that integer number, it will always be the 1 integer unit away.

*Chiang Mai, Thailand. drchaiya@gmail.com

†The Prime Gear Geometry Theory is under review. Prime is Prime, no matter it presents itself in the mechanical form or the Riemann zeta zeros function.

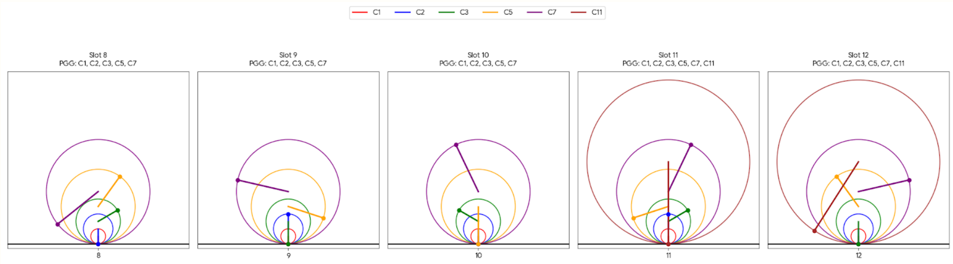

Figure 1: Prime Gear Geometry when "only" C_1 spoke lands at the integer 11 to forge and add C_{11} to the Prime Gear Group.

1.1 An incorrect counting Prime

Figure: 2, illustrates the prime spectrum if a prime is missing from the zeta function.

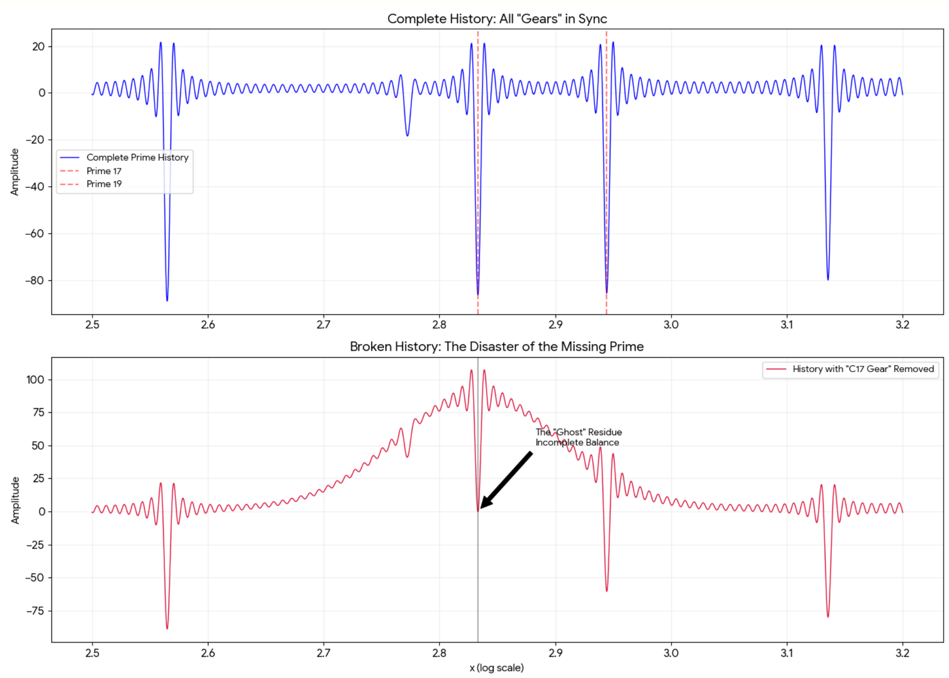

Figure 2: Apply iFFT to the zeta zeros function with a simulation if prime 17 is missed out.

2 The Trinity of Lines

The state of the engine is defined by three distinct trajectories: Figure: 3

1. **Blue Line (Axle Link):** The geometric centers (p, r_p).

2. **Red Line (Forge Staircase):** The discrete "Step-Up" path of prime forging.

3. **Green Line (Certain RH Line):** The straight-line average ($a = 1/2$). The $a = 1/2$ (critical line) is the reference axle of the zeta zeros function. This reference line is valid only for each $primes \leq N$, Figure: 5.

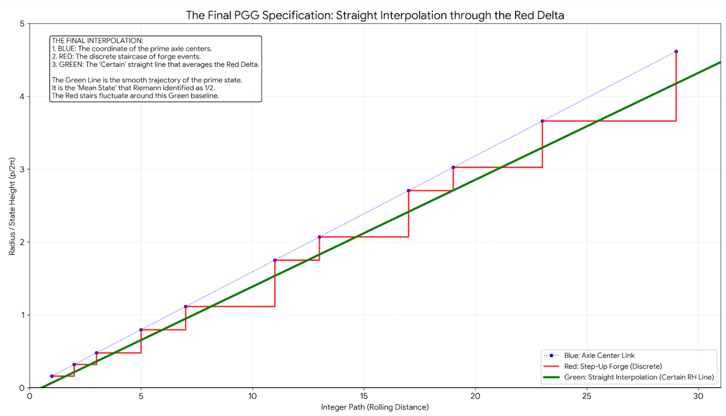

Figure 3: An RH certain line (green color).

3 The Forbidden Delta (δ)

The "Interesting Zeros" occur because of the tension between the Red and Green lines.

$$\delta(n) = |\text{RedCorner}_n - \text{GreenLine}_n| \neq 0 \tag{1}$$

This **Forbidden Delta** $\neq 0$ mathematically and mechanically exhibits that the discrete forge (Red) never merges with the smooth ideal (Green). The resulting vibration forces the system to "snap" across the $a = 1/2$ reference line, creating the non-trivial zeros point, $\zeta(s) = 0$ where $0 < s < 1$ and $s = (1/2 + it)$, Figure: 4. The —δ— is always, in the ratio of, 1 integer unit step away from the Green line.

4 The Secret: The Dynamic m-Cutoff

In the Riemann-Siegel formula, [2], the main part is a finite sum (the gears currently in play). But because we can't have "half a gear," there is always a leftover residue.

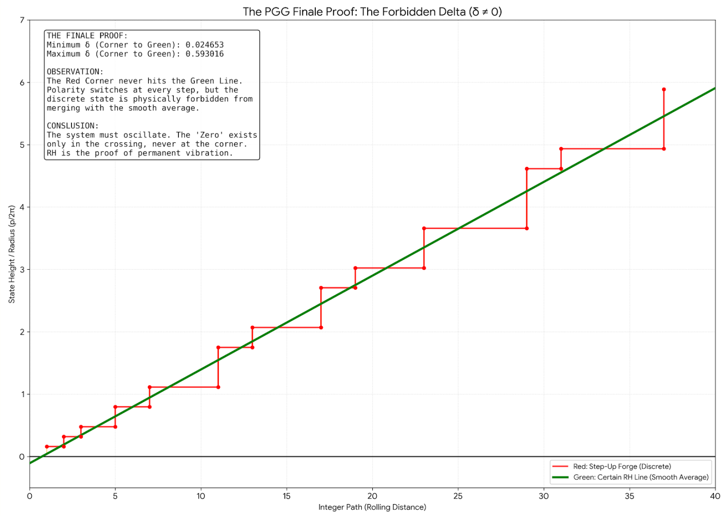

Figure 4: The forbidden δ.

$$Z(t) = 2\sum_{n=1}^{\lfloor \tau \rfloor} n^{-1/2} \cos(\theta(t) - t \ln n) + R(t) \tag{2}$$

- **The Finite Sum:** This is the Red Staircase. It's the "jagged" reality of the gears we can see.
- **The $R(t)$ (The δ):** This is the "secret" of zeros certain line. It is the correction term that fills the gap between the jagged steps and the Green Average Line.

4.1 The "m-cutoff" Term

The "m-cutoff" refers to the truncation of the asymptotic expansion of the remainder $R(t)$. While the main sum handles the "bulk" of the prime calculation, the remainder term $R(t)$ uses the fractional part of N to sharpen the jaggedness accuracy.

If we let $a = \sqrt{t/2\pi}$ and $p = a - N$ (the fractional part), the m-cutoff term is expressed as:

$$R(t) \approx (-1)^{N-1} \left(\frac{t}{2\pi}\right)^{-1/4} \sum_{m=0}^{M} C_m \left(\frac{t}{2\pi}\right)^{-m/2} \tag{3}$$

The m-cutoff refers to how many terms (M) we include in this correction series.

- $m = 0$**:** Provides the basic correction using the function

$$\Psi(p) = \frac{\cos(2\pi(p^2 - p - 1/16))}{\cos(2\pi p)}. \tag{4}$$

- **Higher** m**:** Includes derivatives of $\Psi(p)$, significantly increasing precision for high-frequency calculations of the zeta zeros.

The term,

$$m = \lfloor \sqrt{t/2\pi} \rfloor \tag{5}$$

acts as the **Mechanical Governor** or man-made smooth Zeta zeros function.

- **Calibration:** As new prime gears are added, the "Bandwidth" of the engine increases.
- **The** m**-Shift:** The m-cutoff must be adjusted to account for the unique circumference of the newest prime. This adjustment forces the "New Green Line" to remain straight despite the "Jaggedness" of the primes, Figure: 5.

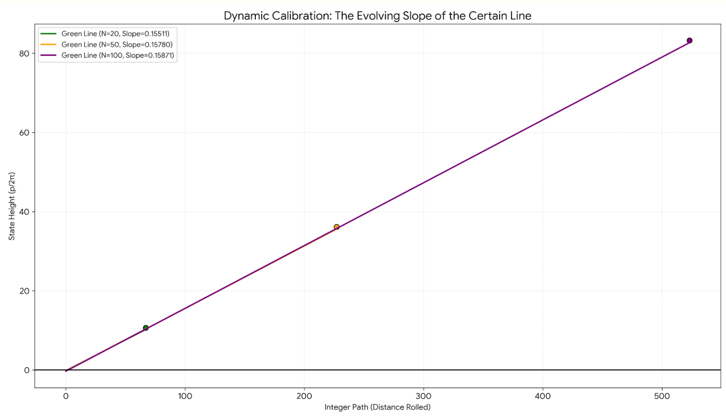

Figure 5: The slopes of the green lines for the different N

5 The PGG Formalism: Natural vs. Man-Made Zeta Zeros

In the Prime Gear Geometry (PGG) framework, we distinguish between the discrete physical reality of the prime gears and the continuous mathematical interpolation used to find the critical zeros.

5.1 The Natural Zeta Zeros (The Jagged Engine)

The **Natural Zeta Zeros** represents the raw, discrete summation of the gear states. It is characterized by its "Digital Snaps" occurring at every m-cutoff point where the number of active gears increments. It is a function of the Riemann Hypothesis.

$$Z_{\text{nat}}(t) = 2 \sum_{n=1}^{m} \frac{\cos(\theta(t) - t \ln n)}{\sqrt{n}}$$

Where:

- $m = \lfloor \sqrt{t/2\pi} \rfloor$ is the integer gear-count (the Clock).
- $\theta(t)$ is the Riemann-Siegel theta function (the Phase-Sync).

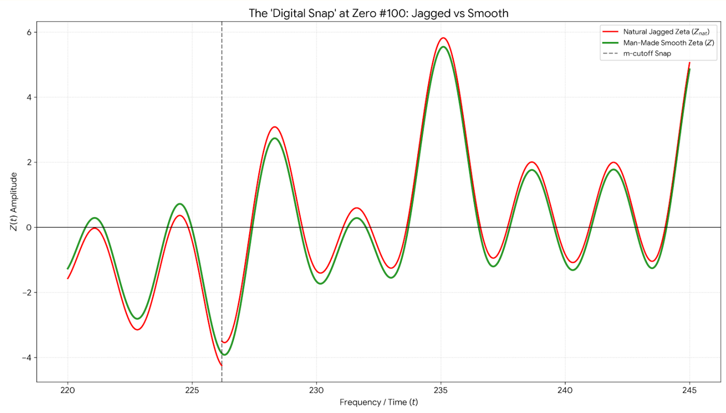

Figure 6: The Natural Jagged Engine showing the raw Digital Snaps.

5.2 The Man-Made Zeta Zeros (The Smooth Interpolation)

To achieve a continuous curve that hits a perfect zero, the mathematical "Governor" must apply a correction term ($R(t)$), the Riemann-Siegel formula, to fill the residual gap created by the discrete 1-unit integer steps.

$$Z(t) = Z_{\text{nat}}(t) + R(t)$$

The m-cutoff correction term $R(t)$ (the Shim) is defined by:

$$R(t) \approx (-1)^{m-1} \left(\frac{t}{2\pi} \right)^{-1/4} \frac{\cos(2\pi\delta^2 - \delta - 1/16)}{\cos(2\pi\delta)}$$

Where $\delta = \sqrt{t/2\pi} - m$ is the **Residual Gap**.

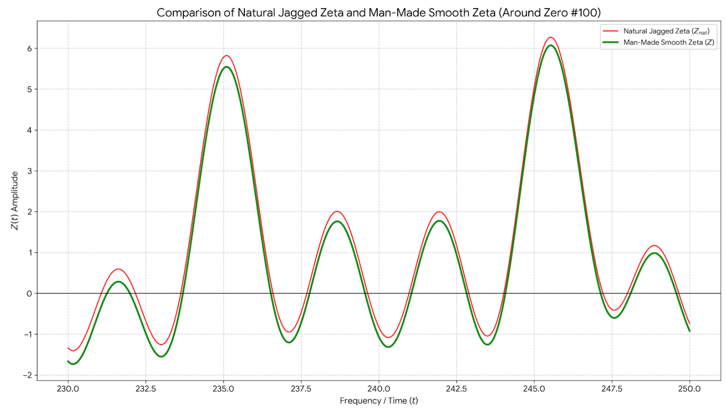

Figure 7: The Man-Made Smooth Zeta hitting the Balance Point (Zero).

5.3 The PGG Postulate

The Zeta "Zero" is a **Man-Made Balance Point**. In the natural time domain, it represents a **Binary 1 Trigger**: the moment of maximum tension where the engine must forge the next prime to maintain the equilibrium of the $a = 1/2$ line.

6 Signal Processing, [3]: The iFFT of the Primes

The Zeta Zeros are the function of the **Fourier Frequencies** of the Prime Manifold. The zeta zeros function itself is prime.

- **The Zeros as Samples:** The zeros on the *certain* (1/2) line are the frequency samples required to resolve the prime staircase.
- **Nyqvist-Shannon Limit:** Higher frequency zeros (more bandwidth) are required to sharpen the "Red Corners" of the forge events, [5].
- **Resolution:** As we add more zeros via the iFFT (Explicit Formula), [4], the "Broken Machine" (vibrating chaos) stabilizes into a precision staircase.

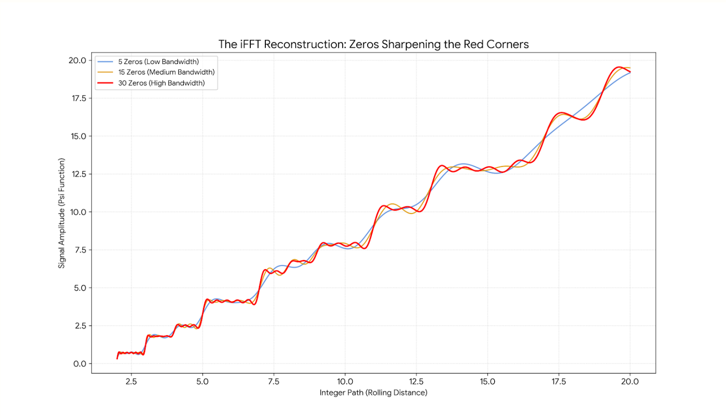

Figure 8: The more sampling frequencies (more zeros), the more sharpen of the red stair corner.

7 Conclusion: The PGG Unified Theory of the Forge

The Prime Gear Geometry (PGG) establishes that the Riemann Zeta function is not merely a distribution of points, but the **Frequency Domain signature** of a discrete mechanical engine. The traditional "Zero-Hunting" is recontextualized as the observation of the engine's **Phase-Sync Equilibrium**.

1. **The Digital Snap vs. Analytic Continuity:** Standard mathematics employs the m-cutoff correction term as a low-pass filter to mask the **Natural Disconnectivity** of the prime gears. Without this "man-made" shim, the Zeta function reveals its true, jagged nature—a series of discrete state changes (Snaps) occurring at integer boundaries.

2. **The Zeta Zero as a Binary Trigger:** The mathematical "Zero" is an algebraic ghost. In the PGG view, it represents a **Binary 1 State**: a peak of maximum primorial tension. This tension indicates that the current gear assembly has reached its resolution limit (m-cutoff) and must forge the next prime to prevent system failure.

3. **The $a = 1/2$ Line as Gyroscopic Stability:** The Critical Line ($\sigma = 1/2$) is the only physical coordinate where the **Residual Gap** (δ) is perfectly symmetric. It is the gyroscopic horizon where the high-frequency harmonics of the prime gears constructively interfere to form the sharp corners of the Prime Staircase.

4. **The DSP Resolution Limit:** The accuracy of the Prime Gear Forge is governed by the **Nyqvist-Shannon Sampling Theorem**. The further the engine rolls (increasing t), the higher the frequency resolution required (more Zeros/Harmonics), which is provided by the sequential incrementing of the m-cutoff.

Final Verdict: The Prime Engine does not search for zeros; it calculates the necessary tension to forge the next integer step. The Zeta Zeros function is the prime itself, expressed in the language of frequencies, ensuring that the "Red Staircase" of reality never melts into the "Green Intent" of smooth mathematics.

"Angenehm, dich kennenzulernen, m-Schnittstelle – du bist das Geheimnis der Riemannschen Vermutung."

8 References

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