

Vacuum Energy Density in de Sitter Space from Horizon Entanglement Entropy:

souverainbertrand64@gmail.com
February, 2026

Bertrand Jarry

A Derivation of $\rho_{\text{v}}^{\text{em}} = \alpha H^4$ with $\alpha = \hbar / (360\pi^2 c^3)$

February 26, 2026

Abstract

I derive the vacuum energy density in de Sitter space from entanglement entropy of the cosmological horizon, obtaining $\rho_{\text{v}}^{\text{em}} = \alpha H^4$ with $\alpha = \hbar / (360\pi^2 c^3)$. The derivation combines three established results: the universal Weyl anomaly coefficient $a = 1/360$ for a conformal scalar (Duff 1977), the exact geometric factor $\beta^{\text{Deom}} = 1$ in the static patch of de Sitter (Chandrasekaran-Longo-Penington-Witten 2022), and the first law of entanglement entropy (Blanco et al. 2013). All numerical factors are traced explicitly. The result is complementary to LeClair's thermodynamic derivation of vacuum energy in flat spacetime. The gap between $\alpha^{\text{EE}} = \hbar / (360\pi^2 c^3)$ and the dimensional regularization value of Solà et al. is identified precisely as a factor $2\pi/3$, formulated as an open problem in quantum field theory in curved spacetime.

1. Introduction

Vacuum energy density and the cosmological constant problem remain among the deepest open questions in theoretical physics. LeClair has recently proposed that quantum vacuum energy $\rho_{\text{v}}^{\text{em}}$ is at the origin of gravity itself [1], deriving Einstein's equations from a gravitational Casimir effect and proposing a new interpretation of Gibbons-Hawking entropy in de Sitter space, where the quantum information bits are identified as quantized massless particles at the horizon.

In flat spacetime, LeClair obtains $\rho_{\text{v}}^{\text{em}} \propto m^4/g$ from thermodynamic and bootstrap considerations [2, 3]. A natural question arises: what is the curved-space counterpart of this result in de Sitter space, where the relevant scale is not a particle mass m but the Hubble rate H ?

In this note, I derive $\rho_{\text{v}}^{\text{em}} = \alpha H^4$ in de Sitter space using entanglement entropy of the cosmological horizon, obtaining:

$$\alpha = \hbar / (360\pi^2 c^3)$$

The derivation combines three established results: (i) the universal Weyl anomaly coefficient $a = 1/360$ for a conformal scalar [4]; (ii) the modular Hamiltonian result $\beta^{\text{Deom}} = 1$ in the static patch of de Sitter [5]; (iii) the first law of entanglement entropy [6]. The quantum information bits at the horizon identified by LeClair [1] correspond precisely to the entanglement degrees of freedom counted by $S^{\text{EE}} = 8a$ in the present approach.

I also identify the precise gap between $\alpha^{\text{EE}} = \hbar / (360\pi^2 c^3)$ and the dimensional regularization value $\alpha^{\text{TLO}} = \hbar / (240\pi^3 c^3)$ obtained by Solà et al. [7], and formulate it as an open problem in de Sitter quantum field theory.

2. The Weyl Anomaly Coefficient

In four spacetime dimensions, the trace anomaly of a conformal field theory takes the form:

$$\langle T_{\mu\mu} \rangle = 1/(4\pi)^2 \times (cW^2 - aE_4 + b\Box R)$$

where W^2 is the square of the Weyl tensor, $E_4 = R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2$ is the Euler density, and a, c, b are the anomaly coefficients. The coefficient a is of type A - it is topological, universally protected, and independent of the regularization scheme [4,8].

For a real conformal scalar field in 4D, the exact value is:

$$a = 1/360$$

The values for other spins are given in Table 1. In pure de Sitter space, $W^2 = 0$ and $\Box R = 0$ since $R = 12H^2$ is constant. The Euler density reduces to:

$$E_4(dS) = 24H^4$$

Therefore the trace anomaly in de Sitter takes the particularly clean form:

$$\langle T_{\mu\mu} \rangle = -aE_4/(4\pi)^2 = -24aH^4/(4\pi)^2$$

This H^4 structure is the microscopic origin of $\rho_{\text{v}}^{\text{em}} = \alpha H^4$. The result is exact for conformal fields.

Table 1: Weyl anomaly type-A coefficient a by spin (4D)

Field	Spin	has
Real scalar	0	1/360
Weyl Fermion	1/2	11/720
Dirac fermion	1/2	11/360
Gauge vector	1	31/180
Graviton	2	424/360

3. Entanglement Entropy of the Cosmological Horizon

In de Sitter space, a static observer is surrounded by a cosmological horizon at radius $R^H = c/H$. The Bunch-Davies vacuum is entangled across this horizon: tracing over the exterior yields a non-trivial reduced density matrix for the interior.

The entanglement entropy S^{EE} of a spherical region in a 4D CFT contains a universal term, independent of the UV cutoff and topologically protected. For a sphere, this universal term is entirely determined by the type-A Weyl anomaly coefficient [9,10]:

$$s_{ur}^{\partial v} = -4a \times \chi(S^2) = -4a \times 2 = -8a$$

where $\chi(S^2) = 2$ is the Euler characteristic of the two-sphere. This result is exact for conformal fields and follows from the replica trick applied to the conical geometry [10]. It is a local, shortdistance result that holds in de Sitter [11].

For the cosmological horizon $R = R^H = c/H$, with $a = 1/360$:

$$S^{EE} = 8a = 8/360 = 1/45$$

This dimensionless number represents the universal entanglement content of the Bunch-Davies vacuum across the horizon. I note the direct connection with LeClair's picture [1]: the quantum information bits at the horizon correspond to the same degrees of freedom counted here by S^{EE} .

4. Modular Hamiltonian in de Sitter and Derivation of α

The first law of entanglement entropy [6] gives:

$$\delta S^{EE} = \delta \langle K^D \rangle$$

For the static patch of de Sitter, Chandrasekaran, Longo, Penington and Witten (CLPW) [5] have established that the modular Hamiltonian of the Bunch-Davies state is:

$$K^D = (2\pi/H) \times H^{OILLIUG}$$

where $H^{oilling}$ is the Hamiltonian associated to the static Killing vector $\xi = \partial_t$. A key geometric result follows from the static patch metric:

$$ds^2 = -f(r)c^2 dt^2 + dr^2/f(r) + r^2 d\Omega^2, f(r) = 1 - (rH/c)^2$$

For a uniform vacuum energy density ρ_v^{em} , the redshift factor $\sqrt{f(r)}$ and the volume element $1/\sqrt{f(r)}$ cancel exactly - a perfect cancellation unique to de Sitter as a maximally symmetric spacetime. This gives:

$$H^{OILLIMG} = \rho_v^{pm} \times V^H$$

where $V^H = (4\pi/3)(c/H)^3$ is the Hubble volume. The geometric factor is therefore:

$$\beta^{\text{Deom}} = 1 \text{ (exactly, from CLPW [5])}$$

Applying the first law at equilibrium with the Gibbons-Hawking temperature $T^H = \hbar H/(2\pi)$ [12]:

$$\rho_v^{vm} = T^H \times S^{EE}/V^H$$

Substituting $T^H = \hbar H/(2\pi)$, $S^{EE} = 1/45$, $V^H = (4\pi/3)(c/H)^3$:

$$\rho_v^{vm} = [\hbar H/(2\pi)] \times [1/45] / [(4\pi/3)(c/H)^3] = \hbar H^4 / (360\pi^2 c^3)$$

Therefore:

$$\alpha = \hbar / (360\pi^2 c^3) = \hbar a / (\pi^2 c^3)$$

The two factors of π are accounted for as follows: π^2 comes from the Hubble volume $V^H = (4\pi/3)R^{H3}$, and the remaining $1/(2\pi)$ from the Gibbons-Hawking temperature $T^H = \hbar H/(2\pi)$.

5. Discussion

The result $\alpha = \hbar / (360\pi^2 c^3)$ can be compared with two other values in the literature.

First, the dimensional regularization calculation of Solà et al. [7] gives $\alpha^{\text{TLO}} = \hbar / (240\pi^3 c^3)$. The ratio is:

$$\alpha^{EE}/\alpha^{\text{TLO}} = 240\pi^3/360\pi^2 = 2\pi/3 \approx 2.09$$

This factor of $2\pi/3$ is not a regularization artifact - the coefficient $a = 1/360$ is universal and scheme-independent. Rather, it reflects a difference between the entanglement energy (thermodynamic, modular) and the gravitational energy (dimensional regularization). Identifying the precise geometric origin of this factor $2\pi/3$ in de Sitter without invoking AdS/CFT is an open problem.

Second, a holographic black-body derivation gives $\alpha^{\text{HoLo}} = \hbar / (480\pi^2 c^3)$. The ratio $\alpha^{EE}/\alpha^{\text{HoLo}} = 4/3$ is exact and reflects the thermodynamic relation $u = (4/3)sT$ for thermal radiation. The three values are summarized in Table 2.

Connection with LeClair's framework. In [1], LeClair identifies the quantum information bits at the de Sitter horizon as quantized massless particles

Table 2: Comparison of three approaches to α

Approach	α	Ratio to α^{EE}
Entanglement entropy (this work)	$\hbar / (360\pi^2 c^3)$	1
Holographic black-body	$\hbar / (480\pi^2 c^3)$	4/3
Dimensional regularization [7]	$\hbar / (240\pi^3 c^3)$	$2\pi/3$

with wavelength $\lambda = 2\pi R^{\text{H}} = 2\pi c/H$. In the present approach, these same degrees of freedom appear as the entanglement entropy $S^{\text{EE}} = 8a = 1/45$ of the Bunch-Davies vacuum across the horizon. The two pictures are complementary: LeClair counts particles at the horizon; I count entanglement bits.

Furthermore, LeClair’s flat-space result $\rho_v^{\text{rm}} \propto m^4/g$ [2, 3] and the present curved-space result $\rho_v^{\text{em}} = \alpha H^4$ are naturally related through the running vacuum model expansion [7]:

$$\rho_v^{\text{vm}}(H) \approx \rho_v^{\text{vm}}(m) + c_2 H^2 + c_4 H^4 + \dots$$

LeClair’s result captures the leading UV term $\rho_v^{\text{em}}(m) \propto m^4$, while the present result captures the leading IR curved-space correction $\propto H^4$. Together they suggest a unified picture of vacuum energy across all scales.

Open problem. The factor $2\pi/3$ between α^{EE} and α^{TLO} requires proving, in de Sitter without invoking dS/CFT, that $E^{\text{GRav}} = (3/5)E^{\text{Mod}}$. This relation is established in AdS/CFT [10] but its de Sitter counterpart remains open.

6. Conclusion

I have derived the vacuum energy density in de Sitter space from entanglement entropy of the cosmological horizon:

$$\rho_v^{\text{vm}} = \alpha H^4, \alpha = \hbar / (360\pi^2 c^3)$$

The derivation remains on three established results: the universal Weyl anomaly coefficient $a = 1/360$ (Duff 1977), the exact geometric factor $\beta^{\text{Deom}} = 1$ in the de Sitter static patch (CLPW 2022), and the first law of entanglement entropy (Blanco et al. 2013).

The result is complementary to LeClair’s thermodynamic approach [1,2,3]. Where LeClair identifies the UV contribution $\rho_v^{\text{em}} \propto m^4/g$ in flat space, the present approach identifies the IR curved-space contribution $\rho_v^{\text{em}} \propto H^4$ in de Sitter. The quantum information bits at the horizon in LeClair’s picture correspond to the entanglement degrees of freedom $S^{\text{EE}} = 8a$ in the present approach.

The gap between $\alpha^{\text{EE}} = \hbar / (360\pi^2 c^3)$ and $\alpha^{\text{TLO}} = \hbar / (240\pi^3 c^3)$ is identified precisely as a factor $2\pi/3$, and formulated as an open problem: proving $E^{\text{GRav}} = (3/5)E^{\text{Mod}}$ in de Sitter without dS/CFT.

I hope this note may be of interest in connection with Professor LeClair's ongoing program on the quantum vacuum as the origin of gravity.

References

- [1] A. LeClair, Quantum Vacuum energy as the origin of Gravity, arXiv:2509.02636 (2025).
- [2] A. LeClair, Thermodynamic formulation of vacuum energy density in flat spacetime, JHEP 2024, arXiv:2404.02350.
- [3] A. LeClair, Vacuum energy density from the form factor bootstrap, JHEP 2024, arXiv:2407.10692.
- [4] MJ Duff, Observations on Conformal Anomalies, Nucl. Phys. B125 (1977) 334.
- [5] V. Chandrasekaran, G. Penington, E. Witten et al., An Algebra of Observables for de Sitter Space, arXiv:2206.10780 (2022).
- [6] D. Blanco, H. Casini, L. Hung, R. Myers, Relative entropy and holography, arXiv:1305.3182 (2013).
- [7] J. Solà Peracaula, C. Moreno-Pulido, J. González-Fuentes, Running vacuum and H^4 -inflation, arXiv:2503.01041 (2025).
- [8] MJ Duff, Twenty years of the Weyl anomaly, Class. As to. Grav. 11 (1994) 1387.
- [9] H. Casini, M. Huerta, R. Myers, Towards a derivation of holographic entanglement entropy, JHEP 2011.
- [10] T. Faulkner, A. Lewkowycz, J. Maldacena, Quantum corrections to holographic entanglement entropy, arXiv:1307.2892 (2013).
- [11] J. Maldacena, G. Pimentel, Entanglement entropy in de Sitter space, arXiv:1210.7244 (2012).
- [12] GW Gibbons, SW Hawking, Cosmological event horizons, thermodynamics, and particle creation, Phys. Rev. D15 (1977) 2738.