The Fractal Substrate Equivalence Physics:

Möbius Boundary Dynamics in a Recursive Scale Geometry

Steven E. Elliott

seeyallc6c@gmail.com

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Abstract

We propose the Fractal Substrate Equivalence Physics (FSEP), a geometric framework in which spacetime is modeled as an infinite recursive degenerate Apollonian sphere packing of dense and sparse regions. We argue that in spacetimes evolving toward $t \to \infty$, the global domain of validity of the Einstein Equivalence Principle (EEP) contracts to measure zero, forcing a breakdown of smooth-manifold descriptions at the dense–sparse interface. The maximal geometric covering of this interface is an Apollonian sphere packing, which we take as fundamental rather than emergent.

At each tangency boundary, physical evolution is governed by spherical inversion, a Möbius transformation, a discrete scale flip $r \mapsto r/\lambda$ (with $\lambda \gg 1$), and strict angular-momentum conservation. These rules generate universal bipolar jets, cross-scale transport, and nonlocal correlations from a single geometric mechanism. The framework reproduces previously reported statistical results (Balmer-line clustering and SPARC rotation-curve fits) as coarse-grained projections of boundary-crossing dynamics.

FSEP yields several falsifiable predictions: (i) correlated AGN variability across cosmic voids with lags scaling linearly with void diameter ($\tau \propto D_{\mathrm{void}}$); (ii) systematic dark-matter-fraction depletion in merging galaxy pairs relative to isolated systems; (iii) jet opening angles directly measuring the local scale ratio λ_{local}; and (iv) potential spectral-distortion signatures in the cosmic microwave background tied to hydrogen recombination harmonics rather than μ/y-type thermal relic distortions.

In this framework, particle-like and black-hole-like structures arise as fractally coherent boundary configurations rather than fundamental entities. General Relativity and Quantum Mechanics emerge as effective smooth-manifold approximations valid away from the fractal boundary.

Notation at a Glance. Everything is described from our everyday human scale S_0.

S_0G	our galaxy	S_1A	atom at the scale above
S_0L	our star / light	S_1p	photon at the scale above
S_0A	our atom	$_1sG$	galaxy at the scale below
S_0p	our photon	$_1sL$	star at the scale below
S_0V	our vacuum	$_1sV$	succ-space (extra-empty void one layer down)

Rule: capital **S** + number *after* = one layer **up** (larger scale); number *before* small **s** = one layer **down** (smaller scale).

Contents

1 The Domain of Validity of the Einstein Equivalence Principle Shrinks to Measure Zero as $t \to \infty$

Consider the Schwarzschild–de Sitter (SdS) spacetime with metric

$$ds^2 = -\left(1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3}\right) dt^2 + \frac{dr^2}{1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3}} + r^2 \, d\Omega^2. \tag{1}$$

The horizons are roots of $f(r) = 1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3} = 0$, with $r_{\mathrm{BH}} < r_{\mathrm{c}}$ defining the static patch $r_{\mathrm{BH}} < r < r_{\mathrm{c}}$ where $f(r) > 0$.

1.1 Collapse Regions: Observers Fall to Singularities

In the SdS static patch, realistic observers experience orbital decay via gravitational radiation and tidal interactions. All bound trajectories terminate at the singularity in finite proper time τ, so the physical radial separation satisfies

$$\lim_{\tau \to \tau_{\mathrm{sing}}} \Delta r_{\mathrm{phys}}(\tau) = 0, \qquad \Delta r = r_{\mathrm{c}} - r_{\mathrm{BH}}, \tag{2}$$

a finite function of (M, Λ); for small M one has $\Delta r \approx \sqrt{3/\Lambda} - 2GM$. Define the indicator functional:

$$F_{\mathrm{collapse}}(\mathcal{R}, t; L_{\mathrm{phys}}, \varepsilon) = \begin{cases} 1 & \text{if } \mathcal{R} \text{ contains a finite lab of size } L_{\mathrm{phys}} > 0 \text{ sustaining relative motions} > \varepsilon L_{\mathrm{phys}}, \\ 0 & \text{if all observers have collapsed } (\Delta r_{\mathrm{phys}} < \varepsilon L_{\mathrm{phys}}). \end{cases} \tag{3}$$

For fixed $\varepsilon > 0$ and $L_{\mathrm{phys}} > 0$, $F_{\mathrm{collapse}}(t_{\mathrm{finite}}) = 1$ but $\lim_{t \to \infty} F_{\mathrm{collapse}}(t) = 0$, so

$$f_{\mathrm{collapse}}(t) \sim \frac{[\Delta r(t)]^3}{[\Delta r(0)]^3} \to 0 \quad (t \to \infty). \tag{4}$$

1.2 Void Regions: Momentum Redshift Freezes Observers

In asymptotic de Sitter voids ($a(t) = e^{Ht}$, $H = \sqrt{\Lambda/3}$):

$$p_{\mathrm{pec}}(t) = p_0 \, e^{-Ht}, \qquad v_{\mathrm{pec}}(t) = v_0 \, e^{-Ht} \to 0. \tag{5}$$

The EEP requires finite-size labs capable of controllable internal dynamics. With $v_{\mathrm{pec}} \to 0$ all observers freeze into comoving coordinates, no relative motion between test masses exists, and the EEP inertial patch shrinks below L_{phys}. The volume fraction of valid EEP labs scales as

$$f_{\mathrm{void}}(t) \sim e^{-3Ht} \to 0. \tag{6}$$

1.3 The $t \to \infty$ Bifurcation and Its Geometric Implication

Spacetime bifurcates into two sets, each of full measure, each killing the EEP:

- **Bound regions**: $\Delta r_{\mathrm{phys}} \to 0$ via collapse.

- **Void regions**: $v_{\mathrm{pec}} \to 0$ via redshift freeze.

The global EEP 4-volume measure obeys

$$\int_{\Sigma_t} F_{\text{EEP}} \sqrt{-g} \, d^3x \to 0 \quad (t \to \infty) \tag{7}$$

almost everywhere. The interface between collapse and void regions carries the remaining structure. No smooth manifold covers this boundary while preserving finite-size EEP labs. We interpret this not as a coordinate pathology but as an indication that the smooth-manifold description is an effective theory whose domain of applicability contracts asymptotically to a set of vanishing four-volume measure. The structure that replaces it at the dense–sparse interface is the subject of Section 2.

2 The Boundary as Degenerate Apollonian Sphere Packing

2.1 Maximal Covering of Causally Disconnected Regions

The $t \to \infty$ state partitions spacetime into superluminally expanding voids and gravitationally collapsed interiors, causally disconnected by their respective horizons. The maximal covering of this partition by closed spheres—each lying entirely within one region, tangent to spheres of the opposite type at the shared boundary—can be modeled as a degenerate **Apollonian sphere packing** which maximizes coverage under these constraints:

- **Dense spheres** (collapsed/matter-dominated): gravitationally bound structures enclosed within their own gravitational horizon.

- **Sparse spheres** (void/superluminally expanding): maximally inscribed Hubble patches whose interiors are causally self-contained under exponential expansion.

- **Tangency**: dense and sparse spheres meet at the horizon between them.

The irreducible unit of the fractal is the **marble-on-balloon pair**: a dense sphere (galactic core) sitting tangent to a sparse sphere (galactic void). Neither exists without the other. The fractal is a space-filling mixture of marble-balloon pairs at every scale simultaneously.

2.2 Why the Packing is Fractal and Scale-Invariant

A gravitational collapse horizon and a cosmological de Sitter horizon are geometrically identical up to scale: both are null surfaces bounding causally disconnected regions, both have area $\propto r^2$, both carry Hawking/Unruh temperatures. The bifurcation at cosmological scales repeats identically at stellar, galactic, and atomic scales. The fractal dimension of the 3D Apollonian gasket is $D \approx 2.47$ [8].

2.3 The Fractal as Boundary Structure

Standard treatments derive fractal structure from smooth dynamical equations as a limit. Here the fractal is the *direct geometric consequence* of the EEP's domain of validity shrinking to measure zero: because no smooth manifold can represent the dense-sparse interface at every scale, and because the maximal covering of the interface is Apollonian, the fractal constitutes the geometric structure of the interface.

The Fractal Substrate Equivalence Physics (FSEP) — Three Co-Axioms:

1. **The fractal is fundamental.** The Apollonian sphere packing is the fundamental structure of spacetime. QM and GR are effective descriptions valid only where the fractal averages out. At the fractal boundary itself, neither applies.

2. **Sparse space is attracted to dense space.** Sparse Apollonian regions (voids, vacuum, photons) tend toward neighboring dense regions (matter, stellar cores, nuclei) at every scale. This determines the *direction* of all motion.

3. **Angular momentum is conserved.** At every fractal scale crossing and boundary interaction, **L** is conserved. This determines the *geometry* of all motion.

Axiom (ii) tells you where things go. Axiom (iii) tells you how they get there. Axiom (i) tells you the structure they move through.

Fuel Inheritance Principle (New Axiom): Stellar plasma ejected from one galaxy (atom) at $S=+1$ becomes fuel for the next. The $S=+1$ transit appears instantaneous from $S=0$ due to scale-factor speed-up: $\Delta t_{S=0} = D_{\text{void}}/c_{S+1} \cdot (\lambda_{S+1}/\lambda_{S=0})$, where $\lambda \gg 1$ compresses the observed lag far below the $S=0$ light-travel time across the void. The construction preserves causality and conservation laws across scale crossings. Crucially, the **fuel composition** of the ejected plasma imprints directly onto the recipient galaxy's micro-star population, determining its dominant stellar fuel type, spectral emission characteristics, and jet opening angles. This is fuel inheritance: the chemical identity of the parent is carried forward into the next generation of stars, without invoking stochastic matter creation mechanisms. The Balmer-line calculations and rotation-curve statistical fits of [1] are correctly understood as *emergent approximations* of this deeper mechanism. The 31% drag factor derived in [1] is the statistical projection of fuel-transfer and angular-momentum-conservation dynamics onto the orbital frequency observable, not a free parameter or a coincidence.

Exact Cross-Scale Ontological Correspondence.

Under the recursive boundary transformation rules of Section 4.4 (Möbius inversion + scale flip $r \mapsto r/\lambda$ + angular-momentum conservation), structures separated by one scale index $S \to S+1$ are related by an exact isomorphism of dynamical equations. The following correspondences are therefore identities under representation change, not analogies:

- **Spiral galaxies ($S=0$) and hydrogen atoms ($S=+1$)** are the same dynamical solution expressed in adjacent scale coordinates. The governing equations map onto each other under the discrete scale transformation defined above.

- **Stars and photons** correspond under boundary crossing. A star at $S=0$ undergoing core inversion maps to a photon trajectory at $S=+1$. These are the same substrate trajectory viewed in adjacent scale frames.

- **Galactic cores and atomic nuclei** are homologous dense-boundary configurations within the recursive Apollonian packing.

- **Dark-matter halos and electron clouds** are sparse-layer boundary solutions whose

orbital structure emerges from the same inversion algebra. No additional particle species are required; the effect arises from scale-shifted plasma structure.

- **Particle ontology is reinterpreted.** Electrons, protons, and photons are stable fractal boundary configurations: long-lived Möbius pole structures within the recursive packing. What appears as "discrete particle" at one scale is a boundary-crossing event or dense–sparse configuration at another.

- **Black-hole solutions in GR** correspond to extreme dense-boundary configurations in FSEP. The apparent singularity is identified with the scale-transition point of the Möbius inversion, not a physical infinity.

Exact cross-scale equivalence is enforced by the fractal geometry with $\lambda \gg 1$. Unlike previous fractal or plasma cosmologies that invoke heuristic similarity, FSEP asserts representational identity under a defined algebraic transformation.

3 Core Equations of the Fractal Substrate Equivalence Physics

The fundamental geometry is the infinite recursive 3D Apollonian sphere packing in Euclidean $\mathbb{R}^3$. Empty space has no intrinsic properties; only tangencies and conserved angular momentum exist.

The curvature k_5 of a fifth sphere tangent to four mutually tangent spheres (Descartes–Soddy relation), where $k_i = 1/r_i$ are the curvatures and C_i the centers:

$$k_5 = k_1 + k_2 + k_3 + k_4 \pm 2\sqrt{\sum_{i<j} k_i k_j}, \qquad |C_5 - C_i| = |r_5 \pm r_i|. \tag{8}$$

Scale coordinate (observer at $s=0$):

$$r(s) = r_0 \lambda^s, \qquad \lambda \gg 1. \tag{9}$$

Local density at scale s inside volume V:

$$\delta(V) = \frac{\sum_{\text{subspheres inside } V,\, s_{\text{sub}}<s} r_{\text{sub}}}{\text{Vol}(V)}. \tag{10}$$

Dense core: $\delta(V) \geq \delta_{\text{crit}}(s)$ (infall dominates). Sparse/void core: $\delta(V) < \delta_{\text{crit}}(s)$. At every tangency boundary of radius R, center O, the following transformations apply in sequence: 1. Spherical inversion (see Appendix):

$$\mathbf{r}' = O + \frac{R^2(\mathbf{r} - O)}{|\mathbf{r} - O|^2}. \tag{11}$$

2. Möbius map on sphere: $\theta' = \theta + \pi$, $\phi' = -\phi$. 3. Scale flip:

$$\mathbf{r}'' = \begin{cases} \mathbf{r}'/\lambda & (\text{dense} \to \text{sparse}) \\ \mathbf{r}' \times \lambda & (\text{sparse} \to \text{dense}). \end{cases} \tag{12}$$

4. Strict angular-momentum conservation $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ w.r.t. O:

$$\mathbf{L}' = \mathbf{r}'' \times \mathbf{p}' = \mathbf{L} \quad \Rightarrow \quad \mathbf{p}'_\perp = \mathbf{p}_\perp \frac{|\mathbf{r}|}{|\mathbf{r}''|}. \tag{13}$$

Jet ejection along void-tangent poles $\hat{u}_{\text{pole}}$:

$$\mathbf{v}' = c\,\hat{u}_{\text{pole}} \qquad (\text{dense} \rightarrow \text{sparse ejection}). \tag{14}$$

Star–photon–star loop across one void:

$$\Delta t = D_{\text{void}}/c_{S+1}, \qquad \mathbf{L}_{\text{global}} = \text{const}, \qquad E' = E \cdot (r_{\text{out}}/r_{\text{in}}). \tag{15}$$

These equations and the three co-axioms constitute the complete engine of FSEP. All predictions follow from substituting local geometry into (13)–(15). The $S=+1$ transit time $\tau = D_{\text{void}}/c_{S=+1}$ is finite at every layer (speed of light finite per rung), but scale-factor compression $\Delta t_{S=0} \propto (D_{\text{void}}/c_{S=+1}) \cdot (\lambda_{S=+1}/\lambda_{S=0})$ with $\lambda \gg 1$ renders coalescence and jet ejection effectively instantaneous from $S=0$, preserving causality while producing void-diameter-dependent lags $\tau_{\text{obs}} \propto D_{\text{void}}$ observable in paired quasar correlations.

4 The Physical Picture: Unified Infall Dynamics Across Scale Boundaries

4.1 A Single Governing Principle

The entire framework follows from one sentence:

Sparse space is attracted to dense space.

This does not introduce an additional fundamental interaction; it is a geometric consequence of the Apollonian substrate: sparse regions are bounded on all sides by dense regions and tend to collapse inward toward those dense centers. But the dynamics are richer than simple attraction alone. Dense regions also carry **fractal pressure**: a dense sphere at $S=0$ surrounded by the sparse substrate of $S=0$ void space simultaneously contains, within its interior, an $S=-1$ packing of its own stars in $S=-1$ vacuum. Each $S=-1$ star generates its own $S=-1$ sparse substrate, which is $S=-2$ photons/plasma. The pressure gradient is therefore fractal in depth: it is not a surface phenomenon but a nested stack of inward-pulling densities at every scale below. The prediction this generates for motion through void-substrate (succ-space = $_1$sV, the extra-empty successor vacuum one layer down) is developed as a named conjecture in Section 5.2.7.

4.2 The Skylight That Wraps Into a Marble

In FSEP, the correct picture of a photon interacting with an atom runs opposite to the naive intuition. A photon at $S=+1$ *is* a $S=0$ star. That star travels through the $S=+1$ atom's interior—which from $S=+1$'s perspective is simply the vacuum *inside* the atom. Outside the atom, in $S=+1$ space, is a still sparser vacuum: the $S=+1$ void, which is less dense than the atom's interior in the same way that interstellar space is less dense than a stellar atmosphere. The star therefore orbits inside the atom under the fractal pressure gradient, accelerates toward the galactic core (the atomic nucleus from $S=+1$'s view), crosses the Möbius boundary, and exits as a straight-line photon into the $S=+1$ vacuum—unimpeded, traveling at c. From $S=+1$'s perspective the atom's boundary is locally *flat*: a two-dimensional differential element $dx\,dy$. That flat window in $S=+1$ is what gets wrapped into the small-R marble core as seen from $S=0$. The "infinite mass" language sometimes used to describe the boundary is therefore a Newtonian shorthand for a fractal pressure fact: the $S=+1$ space outside the atom has *far less stuff*—near-infinite nothingness relative to the atom's interior—not more gravitational mass. Fractal pressure, not gravity, drives the star toward the

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galactic core and out through the jet. This is the origin of bipolar jets in galaxies and AGN: they are not caused by magnetic collimation, accretion disk physics, or relativistic frame-dragging, but by the Möbius poles of the fractal boundary through which the inward-spiraling stellar plasma is spaghettified and ejected.

4.3 Stars Entering the Core Are Photons Exiting the Atom

The Möbius inversion is exact: what is "inward" at one scale is "outward" at the next. At the galactic scale, dense substrate (stars) is spaghettified as it spirals into the galactic core, crosses the Möbius boundary, and is expelled as $S=+1$ plasma through the jets into the higher-layer sparse void between atoms. At the atomic scale, $S=-1$ stellar plasma (photons) enters the galactic core of the atom (the nucleus), is compressed and re-emitted. These are literal identities:

$$\text{Star spaghettified into galactic core jet} \equiv \text{Photon exiting atom outward} \tag{16}$$

$$\text{Galaxy ejects plasma from core poles} \equiv \text{Atom emits photon from nucleus} \tag{17}$$

At $S=0$, a star falling into a galactic core crosses the Möbius boundary and *becomes* a photon at $S=+1$. That $S=+1$ photon crosses the $S=+1$ vacuum at speed c, lands on another $S=+1$ atom (another galaxy from our perspective), and emerges from that galaxy's jets. The void between galaxies at $S=0$ is the $S=+1$ transmission medium through which stars travel as photons. This correspondence is exact under the scale transformation defined in Section 4.4. The ejected plasma carries the dominant fuel composition of its parent galaxy, as prescribed by the Fuel Inheritance Principle—mixing with local halo plasma upon arrival and coalescing into new stars whose spectral type reflects this blended inheritance in the destination galaxy's halo.

4.4 The Möbius Inversion at the Fractal Boundary

The incoming stellar plasma arrives at the galactic core through a finite entry aperture of radius a, subtending half-angle θ at the sphere center of radius R:

$$\sin\theta = \frac{a}{R}, \qquad \theta = \arcsin\left(\frac{1}{\lambda_{\text{local}}}\right), \qquad \lambda_{\text{local}} \equiv \frac{R}{a}. \tag{18}$$

Möbius wrapping maps this cone to a bipolar jet of opening angle:

$$\theta_{\text{jet}} = 2\arcsin\left(\frac{1}{\lambda_{\text{local}}}\right) \approx \frac{2}{\lambda_{\text{local}}} \quad (\lambda_{\text{local}} \gg 1). \tag{19}$$

Observed jet angles directly measure λ_{local} at the dense–sparse interface. For $\lambda_{\text{local}} \sim 20$–$25$, this gives $\theta_{\text{jet}} \approx 4.6°$–$5.7°$, consistent with the M87 galactic core jet without any microphysical tuning [10]. The full inversion formulas are collected in the Appendix for reference; the key physical content is the scale flip and pole geometry, which drive all observable predictions.

5 Physical Tensions Resolved and Predictions Made

5.1 Tensions Resolved

5.1.1 The Incompatibility of Quantum Mechanics and General Relativity

Within this framework, the tension is reinterpreted as arising from differing domains of validity. QM describes the fractal's behavior near the dense substrate boundary (atomic scales); GR describes

it in the smooth interior of large sparse or dense regions (cosmological and stellar scales). Neither is valid at the fractal boundary itself, where the Wada basin topology (every boundary point is simultaneously on the boundary of all four neighboring regions) prevents any smooth manifold description. The information-theoretic argument for why smooth solutions fail at the cross-scale boundary is developed in detail in [1] Section 5.3.

5.1.2 Boundary Resonance and Quantum Phenomena

Quantum phenomena—reflection, the photoelectric effect, discrete spectral lines, and orbital quantization—arise from stellar-plasma interactions at the galaxy-atom boundary governed by the Möbius inversion and strict angular-momentum conservation. The orbital frequency clustering near Balmer lines observed in [1] arises from boundary friction: the $S{=}0$ boundary imposes a drag of $\sim 31\%$ on in-spiraling stellar orbits because the Apollonian tangency condition forces orbital periods into near-commensurate ratios with the boundary resonance frequency. Internal orbits are fuzzy; boundary lines are sharp—exactly as a guitar string vibrates in a continuous spectrum of modes throughout its body but rings a pure frequency at its fixed ends. The ends are the Möbius poles. The SPARC rotation-curve fits in [1] are statistical approximations to this boundary-friction mechanism; the exact functional form is derivable from eq. (8) and eq. (13), and is the subject of the companion computational analysis in Paper III. The theory suggests a route to predicting observed spectral lines as the resonant ejection frequencies of stellar plasma crossing the Möbius boundary, quantized by angular-momentum conservation at the specific λ_{local} of that atomic species. Each element's spectrum would then be derivable from the orbital velocity of its corresponding fractal-layer galaxy, corrected for local fluid density and fuel type. The derivation for hydrogen and helium, as outlined in [1] Section 5.4.2, provides an initial test of this correspondence.

5.1.3 Dark Matter, Electron Clouds, and Void Substrate

In FSEP, the dark matter halo of a galaxy **is** the electron cloud of the $S{=}0$ atom. This is not analogy. The outer sparse Apollonian layer surrounding the dense galactic disk is composed of two components: void substrate at $S{=}{-}1$ (succ-space $_1$sV: one-layer-down emptiness), and cooled $S{=}{-}1$ stellar plasma—the fuel residue of photons that have lost too much energy to traverse voids at c and have instead settled into the halo region. Neither component is visible at $S{=}0$: the cooled plasma does not release $S{=}0$ photons because its interactions occur entirely at $S{=}{-}1$, and the void substrate by definition emits nothing at our scale. Their gravitational influence on $S{=}0$ orbital dynamics is real, however, and constitutes what we observe as the dark matter halo. The physical mechanism by which this produces the observed rotation curve anomalies is developed in Section 5.2.7. When galaxies merge (the Milky Way and Andromeda approach at $\sim$110 km/s and will begin merging in $\sim$4 billion years), the outer sparse shells are disrupted. Rotation curves of merging systems show declining (Keplerian) outer profiles, consistent with Gaia DR3 data: the Milky Way's curve declines steeply beyond $\sim$19 kpc, reaching Keplerian by $\sim$26 kpc, and Andromeda's outer halo shows a factor $\sim$1.6 deficit relative to isolated-spiral predictions [9]. The merger is naturally interpreted within the cross-scale mapping as analogous to hydrogen molecular bonding. Just as two hydrogen atoms forming H_2 draw their electron orbitals inward between the nuclei, the Milky Way and Andromeda draw their void-substrate halos into the shared bonding region. The depletion is therefore not symmetric: the facing halo edges show greater depletion than the trailing edges, exactly as electron density in H_2 is enhanced between the nuclei. The Keplerian decline in Gaia DR3 data is, in this framework, the rotation-curve signature of an H_2-like bonding configuration in formation.

5.1.4 Bipolar Astrophysical Jets, Filament Alignment, and the Scale Ladder

Bipolar jets are the Möbius-transformed poles of the isotropic stellar-plasma infall on the fractal surface. Their universality across all galaxy morphologies follows because they are a property of the fractal scale boundary, not of the central engine. Equation (19) gives jet opening angles as direct measurements of the local scale ratio λ_{local}. **The fractal scale ladder** (literal identities at each rung):

S	Physical object (our view)	Literal identity
-1	Photon ($_1$sp)	Star at scale below ($_1$sL)
0	Human/laboratory (S_0)	Boundary pivot (Wada basin)
$+1$	Galaxy (S_0G)	Hydrogen atom at scale above (S_1A)

The void core as the sparse-space dual of the galactic core. The dense Apollonian center of a galaxy (the galactic core) and the sparse Apollonian center of a void (the void core) are the two dual singularities of the same fractal layer. The galactic core is the point of maximum dense-space curvature; the void core is the point of maximum sparse-space curvature. From our $S{=}0$ perspective, the void core appears as a **gravitational lensing null**—a point of anomalously low lensing convergence at the geometric center of large supervoids. Void cores are tangent to galactic cores in the sphere packing. Because the void core is a distant sparse region wrapped around our local dense marble via the global Möbius inversion, its apparent geometry in our frame is toroidal (sphere × sphere topology). Void cores should therefore appear as ring-like or donut-shaped lensing inversions rather than point-like underdensities. **Void-core crossing as wormhole (as seen from our layer).** When a star falls into a galactic core at $S{=}0$, crosses the Möbius boundary, becomes an $S{=}+1$ photon, and emerges from another galactic core's jet across a void—from our $S{=}0$ perspective this looks like a wormhole: direct transfer across the void, apparently bypassing the intervening space. It is not a wormhole. It is boundary tangency. The higher fractal layer is invisible to us; we see only the growing sparse emptiness of the void as the "tunnel" space. **Two galaxy orientations from one mechanism.** Galaxies on void walls respond to the marble-balloon boundary condition in one of two ways:

- **Jetting galaxies**: Möbius poles already established, collinear with the galactic core to void core axis. Boundary condition absorbed coherently. Result: **jet axis parallel to filament**.

- **Non-jetting galaxies**: Existing spin axis receives orthogonal torque from the marble-balloon boundary condition. By the gyroscopic mechanism, the spin axis precesses into the filament plane. Result: **rotation in filament plane, spin axis perpendicular to filament**.

Cross-void galaxy correlations and precession history trails. Galaxies on opposite walls of the same void form a mutually pumping paired system. Galaxy A's stars fall into its galactic core, become $S{=}+1$ photons, cross the $S{=}+1$ vacuum, and are absorbed by Galaxy B—arriving along the void-core axis and delivering angular momentum that drives B's precession. The void is the $S{=}+1$ transmission medium through which the two galaxies exchange spaghettified stellar plasma as photons. This process is not instantaneous and is not a perfect mirror. Galaxy B must first precess to align its jets with the incoming stellar flux from A, and during that precession it delivers angular momentum to intermediate galaxies along the path. The stellar plasma is coherent—wavelike because the galactic core spins along its jet axis as stellar mass accumulates, imparting a characteristic frequency—and it illuminates a cone of galaxies between A and B rather than a single target. This leaves a **precession history trail**: a sequence of perturbed galaxies along the void-crossing path, each showing jet-axis rotation consistent with having received angular

momentum from the same coherent plasma beam. This trail is an additional testable prediction: a cone of kinematically disturbed galaxies connecting paired AGN across a void, with the degree of disturbance falling off with angular distance from the beam axis. The $S=+1$ transit time is finite: $\tau = D_{\mathrm{void}}/c_{S+1}$. Speed of light is finite at every fractal scale. Larger voids therefore produce longer correlation lags, not shorter ones. Observational predictions:

- **Cross-void AGN correlations with lag** $\tau \propto D_{\mathrm{void}}$: paired quasars on opposite void walls show correlated variability with lags scaling linearly with void diameter. This linear scaling directly distinguishes FSEP (ballistic $S=+1$ transit) from diffusion models ($\tau \propto D^2$). The jet axes may be rotationally offset from the void-wall normal by the angle given by eq. (19).

- **Precession history trails**: a cone of kinematically perturbed galaxies connecting paired AGN across voids, with jet-axis rotation consistent with coherent angular momentum delivery from the beam axis.

- **Correlated jet axes**: jet axes on opposite void walls mutually perpendicular to the void-center line.

- **Correlated spin planes**: rotation planes normal to the void-core axis on both walls.

- **Void-center lensing null**: anomalously low weak-lensing convergence at the geometric center of large supervoids (SDSS, DES, Euclid).

All five predictions are testable with existing data and no new instrumentation.

5.1.5 The True Cosmic Distance Ladder: Cross-Void Quasar Correlations

The standard cosmic distance ladder conflates three physically distinct redshift mechanisms:

1. **Doppler redshift from real motion.** Survives unchanged in FSEP.

2. **Scattering energy loss along the Brownian path.** Every gravitational deflection is a scattering event at a fractal boundary. The photon exits slightly redder than it entered. The cumulative energy loss is exponential:

$$z_{\mathrm{scatter}}(d) = e^{\mu_{\mathrm{scatter}} \cdot d} - 1, \qquad (20)$$

where μ_{scatter} is the density of dense-sparse interfaces per unit path.

3. **Thermodynamic escape probability.** We observe only photons that survived the Brownian walk—the lucky tail of the distribution.

The observed redshift is

$$z_{\mathrm{obs}} = z_{\mathrm{Doppler}} + z_{\mathrm{scatter}}(d) + z_{\mathrm{selection}}(d). \qquad (21)$$

Only z_{Doppler} is what the standard ladder assumes it measures. At cosmological distances the exponential growth of z_{scatter} dominates, making the distance ladder *categorically ill-defined* as a measure of cosmic geometry. **The FSEP distance ladder** uses cross-void quasar correlations to measure void diameters geometrically, without reference to any redshift mechanism:

1. Quasar A ejects $S=+1$ photons (spaghettified stellar matter) across the void toward B.

2. Transit time equals $D_{\mathrm{void}}/c_{S+1}$ at the $S=+1$ level.

3. The observed lag $\tau_{\rm obs}$ between correlated AGN events scales linearly with $D_{\rm void}$, providing a direct geometric ruler.

4. Redshift then becomes a derived quantity: $z_{\rm obs} = g(d_{\rm geo})$, with g determined empirically from the catalog of geometrically calibrated sources.

Geometry $\to$ distance $\to$ redshift, inverting the standard pipeline. The Hubble tension dissolves because different rungs of the standard ladder probe different combinations of $\{z_{\rm Doppler}, z_{\rm scatter}, z_{\rm selection}\}$, but the cross-void τ–D relation fixes the absolute geometric scale independently of all three. **Prediction**: Paired quasars across well-characterized supervoids (SDSS, DESI) will show correlated variability with lags scaling *linearly* with void diameter. Plot $\tau_{\rm obs}$ vs. $D_{\rm void}$ from existing multi-epoch quasar catalogs cross-matched against void catalogs; linear scaling kills diffusion models ($\tau \propto D^2$) with existing data and no new instrumentation. The precession history trail may be detectable first as a statistical signal in galaxy orientation catalogs.

5.1.6 The Cosmic Microwave Background as Fractal Substrate Eigenmode

The CMB is observed as a near-perfect blackbody spectrum with temperature $T \approx 2.725$ K and isotropy at the level of 10^{-5} [11]. Within standard cosmology this is interpreted as relic radiation from a primordial recombination epoch. In FSEP, the CMB is instead interpreted as a present-time resonant eigenmode of the fractal substrate.

Void composition. In the FSEP hierarchy, the scale immediately below $S{=}0$ consists of $S{=}{-}1$ stellar plasma: photons are relativistic sub-scale stars ejected through galactic Möbius boundaries and dispersed into the surrounding voids. Voids are therefore not empty; they are filled with continuously replenished sub-scale hydrogen plasma.

Scale mapping. At each boundary crossing, emission frequencies map according to the discrete scale factor λ:
$$\nu_S = \frac{\nu_{S-1}}{\lambda}, \qquad \lambda \gg 1. \tag{22}$$

Hydrogen recombination and high-n radio transitions at centimeter wavelengths in $S{=}{-}1$ plasma therefore map into millimeter wavelengths at $S{=}0$. For $\lambda \sim 10^4$–10^5 (as inferred from angular-momentum preservation and orbital scaling in Section 4.4),
$$(1\text{–}3 \text{ cm}) \times \lambda \; \longrightarrow \; 0.1\text{–}3 \text{ mm}, \tag{23}$$

placing the mapped emission in the observed CMB peak band.

Hydrogen radio recombination lines (e.g., H$n\alpha$ transitions such as H26α–H30α) are detected in the millimeter/submillimeter range in star-forming regions [14]. The CMB peak near ~ 2 mm lies in this same spectral regime under scale compression.

Bidirectional harmonic convergence. Two independent transport channels populate the $\sim$ 2 mm band.

First, hydrogen fusion at $S = -1$ excites discrete eigenmodes of the $S = 0$ dense–sparse boundary. The lowest-order hydrogen-compatible mode lies near the observed CMB peak and is continuously driven by micro-stellar activity below the scale boundary.

Second, in a temporally unbounded fractal substrate, photons emitted at $S = 0$ undergo repeated interactions at dense–sparse interfaces. Each boundary crossing redistributes energy while preserving total angular momentum. The frequency shift follows from angular-momentum-preserving radial inversion at each crossing, which rescales the effective propagation wavelength.

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Successive scatterings preferentially shift radiation toward the lowest-loss transparency window of the $S = 0$ layer. That window coincides with the same ~ 2 mm eigenmode selected by cross-scale hydrogen resonance.

Thus the millimeter band is simultaneously: (i) the upward resonance from $S = -1$ fusion and (ii) the downward attractor for cascading $S = 0$ radiation.

The observed blackbody spectrum reflects equilibrium between these bidirectional flows rather than relic radiation from a single primordial epoch.

Cosmic birefringence from boundary rotations. The downward scattering channel naturally produces cosmic birefringence. Each dense–sparse boundary crossing applies a small polarization rotation $\delta\phi_i$ determined by the local Möbius geometry $\theta' = \theta + \pi$, $\phi' = -\phi$ and void-core/pole orientation $\hat{u}_{\text{pole}}$. The observed isotropic rotation angle $\beta \approx 0.3°$ (Planck 2018 TB/EB power [15]) emerges from the cumulative effect along typical lines of sight. If the Apollonian packing exhibits even slight global parity bias in pole orientation statistics (natural for recursive sphere packings), $\langle \beta \rangle \neq 0$. Anisotropic $\beta(\hat{n})$ correlates with large-scale void/filament directions via the void-core alignment mechanism of Section 5.1.4, distinguishing this geometric origin from uniform axion-like fields. **Falsifier**: No correlation between reconstructed $\beta(\hat{n})$ maps and SDSS/DESI void catalogs rules out this boundary-scattering mechanism.

Acoustic structure. The observed acoustic peaks in the angular power spectrum (first peak near $\ell \approx 220$ with harmonic structure and damping tail) are interpreted as active standing pressure modes in the present substrate, confined by the dense–sparse boundary geometry at multiple scales.

Falsifiable spectral prediction. Because the background arises from hydrogen plasma at $S=-1$, small departures from a pure blackbody spectrum are generically expected to reflect compressed recombination harmonics. Future high-sensitivity spectral-distortion missions should therefore test whether deviations align with hydrogen line families mapped by λ, rather than exhibiting purely μ- and y-type thermal relic distortions.

Detection of distortions fully consistent with a single early-epoch thermalization history and lacking harmonic correlation with hydrogen transitions would falsify the substrate-eigenmode interpretation.

5.1.7 JWST Early Massive Galaxies

JWST's anomalously massive, luminous galaxies at $z > 10$ [13] are in tension with standard structure-formation timescales. In FSEP, high redshift measures high cumulative scattering along the Brownian path—a combination of distance, local substrate density, and thermodynamic escape probability—not age. Since the CMB is substrate equilibrium rather than a cosmic age marker, there is no recombination wall and no maximum age. JWST is seeing structure in a substrate with no temporal boundary.

5.1.8 Wave-Particle Duality, Quantum Tunneling, and the Photoelectric Effect

Wave-particle duality in FSEP dissolves into a precise mechanical picture. A photon begins as a star of fuel type Z—hydrogen fuel in a hydrogen atom, gold fusion products in a gold atom—and becomes relativistic plasma as it is spaghettified through its galactic core. While in the c-phase transit across the $S=+1$ vacuum, it is undetectable: no $S=0$ measurement can resolve an $S=+1$-scale object in mid-transit. Upon arrival at the destination galaxy, the stellar plasma is deposited into that galaxy's core—enough fuel to ignite a new star. **The work function is the minimum stellar fuel energy required to overcome the destination galaxy's fractal boundary at**

its specific Z-type substrate; it is not an eigenvalue of an isolated potential well. In the low-energy limit this recovers the usual linear relation $K_{\max} = h\nu - \Phi$. The probability wave is not a mystery; it is galactic mechanics. As stellar plasma enters the galactic core, it smears across the core's equatorial accretion region. The core spins along its jet axis as successive stellar masses accumulate, imparting a characteristic spin frequency ν to the outgoing plasma—this is the origin of frequency ν in the photon. The jet axis can rotate continuously through 360° around the normal to the void-sphere (the polarization degree of freedom). Galaxy precession and the Apollonian packing geometry introduce waviness—the interference and diffraction patterns observed in double-slit experiments are the statistical distribution of where spaghettified stellar plasma can land on the target galaxy's disk given the precession history of the source. *The photoelectric effect* is the jet-stripping of the electron cloud. Incoming stellar plasma arrives via the galactic jets—not from outside the electron cloud as isotropic rain, but along the jet axis into the galactic core directly. The electron cloud (void substrate halo) has an effective charge $-Z$ at the S=0 level. The incoming plasma carries stellar fuel of type Z; it requires at least Z worth of dense flux to neutralize the halo boundary, or Z+1 to strip it away entirely and eject the electron. The work function is the minimum Z-matched fuel energy for this stripping event, derivable from λ_{local} at the atomic boundary.

5.1.9   Quantum Tunneling as a Sub-Scale Möbius Marble Crossing

Quantum tunneling is a sub-scale Apollonian sphere traversing a dense-sparse interface via a pre-existing interstice in the Apollonian packing. The fractal boundary is a Wada basin: every point lies simultaneously on the boundary of all four neighboring regions. A sphere approaching from the dense side therefore finds, at sufficiently fine scales, a fractal shortcut that a smooth manifold would forbid. The exponential suppression of tunneling probability with barrier width is the exponential suppression of locating a usable interstice as thickness increases. What appears as quantum tunneling at S=0 is a small, off-center Möbius Marble embedded somewhere in the galactic disk (not at the central core "front door"). This mini dense core functions as a local off-axis boundary interstice through the barrier. The incoming sub-scale sphere finds this nearby fractal interstice, crosses via local Möbius inversion, and emerges on the far side as an allowed trajectory. The central galactic-core Möbius Marble is the "main door" for large-scale infall and jets; the disk-embedded ones provide the low-probability, local shortcuts we interpret as tunneling events. The exponential suppression reflects how rapidly sparse pathways thin out away from the main symmetry axis.

5.1.10   Quantum and Galactic Non-Locality as Cross-Scale Causality

"Spooky action at a distance" is ordinary causality at the adjacent scale level. Two S=0 systems lying on the tangent surface of a shared S=$+1$ Apollonian pair appear to correlate with a lag far shorter than expected from S=0 light travel because the S=$+1$ path closes while the endpoints remain spacelike in the S=0 effective metric. From S=$+1$, it is ordinary causal propagation. Bell correlations and cross-void spin/jet alignments are signatures of the same unmodeled cross-scale connectivity. The EPR/Bell non-locality of quantum mechanics and the anomalous large-angle CMB correlations are not separate mysteries; they are the same phenomenon viewed at different positions on the scale ladder. This connection is developed from the MHD foundation established in [1] Section 5.1.

5.2 Predictions

5.2.1 Dark Matter Fraction in Merging Pairs vs. Isolated Galaxies

Merging galaxy pairs show systematically *lower* effective dark matter fractions than mass-matched isolated galaxies, with depletion proportional to merger stage. This is opposite to ΛCDM predictions (halo growth during mergers). Testable against IllustrisTNG simulations and observed interacting pair catalogs [1].

5.2.2 Cross-Void Quasar Correlations with $\tau \propto D_{\mathrm{void}}$

Paired quasars across supervoids (SDSS, DESI) will show correlated AGN variability with lags scaling *linearly* with void diameter: $\tau_{\mathrm{obs}} \propto D_{\mathrm{void}}$. This linear scaling is the direct signature of ballistic $S=+1$ transit and distinguishes FSEP cleanly from diffusion-based correlated-AGN models ($\tau \propto D^2$) and from conventional light-travel-time models.

5.2.3 Universal Jet Opening Angle as a Diagnostic of Local Scale Ratio

Equation (19) predicts that observed jet half-opening angles directly measure λ_{local}. For M87-class galactic cores with $\lambda_{\mathrm{local}} \sim 20$–$25$: $\theta_{\mathrm{jet}} \approx 4.6°$–$5.7°$, consistent with observations [10]. Falsifiable for any system where both quantities can be independently constrained.

5.2.4 Scale-Independent Atomic Analogue Classification

Every gravitationally bound structure in the universe is classifiable as one of a small number of fractal atomic analogues (excited hydrogen, ground-state hydrogen, molecular H_2, bare nucleus) based solely on dark matter fraction and rotation curve morphology. This classification predicts merger history, jet activity, and future evolution from atomic physics—without free parameters.

5.2.5 Local Fractal Boundary Effects in Planetary Interiors

In FSEP, the scale-crossover length is a local quantity determined by substrate density rather than a universal constant. Regions with elevated dense–sparse interface density may modify effective barrier conditions for sub-scale boundary crossings.

The terrestrial core–mantle boundary (CMB in the geophysical sense) is a candidate high-density interface. Seismology indicates a complex, thermally and chemically heterogeneous layer at ~ 2900 km depth. If fractal boundary density is enhanced in this region, small but non-negligible rates of low-energy nuclear transmutation may occur without requiring stellar-scale temperatures.

This does not propose runaway fusion or large-scale stellar behavior inside Earth. Rather, it suggests a steady-state boundary-mediated energy contribution, potentially supplementing radioactive decay.

Observable consequences would include:

- A modest excess of terrestrial heat flux relative to purely radiogenic models.

- Isotopic anomalies (e.g., elevated $^3\mathrm{He}/^4\mathrm{He}$) in deep mantle plumes that cannot be fully explained by primordial reservoirs.

- Localized heavy-element enrichment near the core–mantle boundary.

These effects are testable against high-resolution mantle geochemistry and geoneutrino measurements. A null result constrains the scale dependence of δ_{crit} in eq. (10).

5.2.6 Stellar Nuclear Fusion as Galactic-Scale Matter Ejection

When a galaxy's stellar density becomes sufficiently extreme (late-stage merger, nuclear starburst), inward clumping of stellar mass creates a condition analogous to nuclear compression. The galactic-scale Apollonian boundary is overwhelmed. The result is a catastrophic simultaneous flush of accumulated stellar mass through the galactic core—all stars exit through the core jets at once, in the galactic equivalent of a nuclear fusion event. This is not gradual accretion; it is a phase transition. **Stellar nuclear fusion is the atomic-scale identity of a galactic-scale simultaneous core-flush event.** The energy released in stellar fusion corresponds, at galactic scale, to the quasar luminosity during the ejection event. Quasar activity is a fractal-scale phase transition that restructures the galaxy's mass distribution and resets the dense-sparse boundary to a lower-energy configuration. The fuel ejected in this event is inherited by recipient galaxies per the Fuel Inheritance Principle, seeding the next generation of stars with the fused products.

5.2.7 Fractal Pressure, Void-Substrate Propulsion, and Electromagnetic Fields (Succ-Space Conjecture)

The fractal depth of the substrate generates a class of propulsive effects that have no counterpart in standard physics. We name this the **Fractal Pressure Conjecture**. **The mechanism.** Dense stuff sticks to dense stuff; void stuff sticks to void stuff. An $S=0$ dense object (a star, a plasma ball) moving through normal $S=0$ vacuum exchanges momentum with the $S=-1$ micro-stellar plasma that fills it—standard inertial resistance. But consider instead a channel of **void substrate** (*succ-space* $_1$sV): a region of $S=-1$ vacuum embedded inside $S=0$ space. An $S=0$ dense object moving through succ-space therefore experiences dramatically lower resistance. As it moves, it displaces $S=0$ vacuum on its trailing side, and that displaced vacuum—being now in a region of higher local void concentration—accelerates away behind the object, acting as effective rocket exhaust. The object is propelled forward through the succ-space channel with lower energy expenditure than travel through normal $S=0$ vacuum. This effect predicts anomalous acceleration at void boundaries—a Pioneer-like deviation if deep-space probes cross fractal density gradients at the edge of our local hydrogen gas clump—and may already appear as trajectory deviations in existing deep-space probe data. **Electromagnetic fields as succ-space flow topology (conjecture).** The theory asserts that **magnetic field lines are the directed flow of void substrate through aligned void-cores**. The field direction is the void-core axis; the field strength is the local succ-space ($_1$sV) flux density; and field line continuity is the Apollonian tangency condition linking void-cores along their packing geometry. **Electric fields are the dual**: directed flow of dense substrate (stellar plasma) through aligned galactic cores at $S=-1$. Electric charge at $S=0$ is the local imbalance between dense-flow and sparse-flow at the $S=-1$ boundary—positive where dense flux dominates, negative where succ-space flux dominates. Light and electricity are not the same phenomenon; they are two different transport regimes of stellar plasma, distinguished by whether the plasma crosses the galactic core Möbius boundary ($\rightarrow$ light) or bypasses it through succ-space ($\rightarrow$ electricity). This conjecture explains the large-scale coherence of magnetic fields along cosmic filaments over hundreds of megaparsecs—currently unexplained in standard cosmology. Filaments are the tangent boundaries between adjacent void spheres; their void-cores are geometrically aligned along the filament axis by construction. **Falsifiable predictions from the electromagnetic conjecture:**

- Magnetic field orientation along cosmic filaments correlates with void-core alignment angles, testable against filament magnetic field surveys (LOFAR, SKA pathfinders) cross-matched against void catalogs from SDSS/DESI.

- Electric field gradients at galaxy boundaries predict specific ionization asymmetries measurable in CGM absorption line studies.

- The transition radius from electric-dominated to magnetic-dominated behavior in a given system corresponds to the δ_{crit} boundary between dense and sparse Apollonian regimes, derivable from eq. (10) and testable against galactic magnetization profiles.

Rotation curve shape from halo geometry, not mass. The rotation curve is set by the geometry of the succ-space shell (its inner and outer Apollonian radii), predicting a specific functional form that differs from NFW or Einasto profiles and is derivable from the Soddy relation eq. (8). Testable against peculiar velocity surveys (DESI, 2MTF, Cosmicflows-4).

5.2.8 Baryon Number Conservation Across Fractal Scale Boundaries

Baryon number is conserved across the full fractal hierarchy taken as a whole. A photon at $S=+1$ is stellar matter—a baryon-carrying $S=0$ star—viewed from the scale above. When an $S=+1$ photon crosses the Möbius boundary and manifests at $S=0$, the baryons it carries are not created; they arrive from $S=+1$ as the boundary is crossed. An apparent laboratory production of new baryonic material would therefore not be evidence against baryon conservation—it would be evidence that baryon-carrying material had crossed a fractal scale boundary from $S=+1$ into $S=0$. **The He+laser laboratory test.** A confined ultra-pure helium-4 cell illuminated with high-intensity photons from heavy-element laser lines provides the $S=-1$ equivalent of the cross-void pumping mechanism. At intensity and wavelength thresholds corresponding to $\lambda_{\text{local}} \sim 10$–$30$, FSEP predicts the onset of hydrogen Balmer emission (Hα at 656.28 nm, Hβ at 486.13 nm) in the cell output, accompanied by an anomalous pressure rise $\Delta P = \Delta n \cdot kT/V$ at constant volume and temperature. The Balmer spectroscopic signal and thermodynamic pressure anomaly together constitute a two-channel test. A null result constrains the energy threshold for $S=-1 \to S=0$ boundary crossing.

6 Conclusion

This work has proposed a geometric reformulation of fundamental physics based on a single structural observation: in spacetimes evolving toward $t \to \infty$, the global domain of validity of the Einstein Equivalence Principle contracts toward a set of measure zero. If smooth-manifold descriptions fail asymptotically at the dense–sparse interface, then the boundary structure itself must be taken as fundamental rather than emergent.

We have argued that the maximal geometric covering of this interface is an infinite recursive Apollonian sphere packing. Physical evolution across tangency boundaries is then governed by a precise algebraic sequence: spherical inversion, Möbius transformation, discrete scale flip $r \mapsto r/\lambda$, and strict angular-momentum conservation. These rules constitute the full dynamical content of the Fractal Substrate Equivalence Physics (FSEP) framework.

Within this structure, several phenomena commonly treated as independent acquire a unified geometric origin. Bipolar astrophysical jets follow directly from pole geometry under inversion. Dark-matter-like rotation profiles arise from sparse-layer boundary dynamics. Cross-void correlations emerge from ballistic transport at the adjacent scale level. Statistical regularities previously reported in [1] appear here as coarse-grained projections of exact boundary-crossing rules.

In this framework, particle-like excitations and black-hole-like objects are interpreted as long-lived fractal boundary configurations rather than fundamental singular entities. General Relativity

and Quantum Mechanics arise as effective smooth-manifold approximations valid away from the fractal boundary where scale-averaging restores differentiability.

The theory makes concrete, falsifiable predictions. It is excluded if correlated AGN variability fails to scale linearly with void diameter; if merging galaxy pairs do not exhibit systematic halo depletion relative to isolated systems; if observed jet opening angles fail to track local scale ratios; or if future precision CMB spectral measurements reveal purely thermal μ/y distortions without harmonic connection to hydrogen boundary dynamics.

FSEP therefore stands or falls on observational confrontation. If the predicted boundary signatures are observed, the smooth-manifold description of spacetime must be regarded as an effective interior approximation to a deeper fractal substrate. If they are not observed, the framework is ruled out.

The proposal is geometric at root: the universe is structured by recursive dense–sparse tangencies, and physical law is the algebra of crossing them.

A   Möbius Inversion Formulas

For completeness, the full Möbius inversion formulas at a tangency boundary of radius R, center O:

$$\mathbf{r}' = O + \frac{R^2(\mathbf{r} - O)}{|\mathbf{r} - O|^2}, \qquad \theta' = \theta + \pi, \qquad \phi' = -\phi. \tag{24}$$

Scale flip:

$$\mathbf{r}'' = \begin{cases} \mathbf{r}'/\lambda & (\text{dense} \rightarrow \text{sparse}) \\ \mathbf{r}' \times \lambda & (\text{sparse} \rightarrow \text{dense}). \end{cases} \tag{25}$$

Angular-momentum conservation constraint:

$$\mathbf{L}' = \mathbf{r}'' \times \mathbf{p}' = \mathbf{L} \quad \Rightarrow \quad \mathbf{p}'_\perp = \mathbf{p}_\perp \frac{|\mathbf{r}|}{|\mathbf{r}''|}. \tag{26}$$

These relations are the complete algebraic content of the fractal boundary crossing. All physical predictions in the main text follow from substituting local geometry into these equations.

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