THE MASTER MAP

An Audit-First, Attack-Resistant Navigation Guide
to the Unconditional Solution of the 4D SU(N) Yang–Mills
Existence and Mass Gap Problem (Clay Millennium Problem)

Polymer Activities ⇒ Kotecký–Preiss ⇒ Exponential Clustering ⇒ Osterwalder–Schrader ⇒
Wightman QFT with Strictly Positive Mass Gap

Including the Preventive Lock: Triangular Renormalization Mixing

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Abstract

This paper is a hostile-review navigation guide and audit manifesto for a companion-paper programme claiming a constructive solution of the four-dimensional SU(N) Yang–Mills existence and mass gap problem in the Osterwalder–Schrader (OS) framework, reconstructed as a Poincaré-covariant Wightman QFT with strictly positive mass gap.

The guide provides: (i) an explicit dependency graph and Clay/Jaffe–Witten checklist; (ii) an explicit threat model listing standard failure modes targeted by hostile review (black-box dependence on Bałaban, interface friction between gradient flow and the Bałaban measure, diagonal-limit non-uniformity, and operator-mixing residues); (iii) an explicit four-pillar defensive architecture resolving each attack with structural (not merely quantitative) shields; (iv) the *preventive lock*: a triangular renormalization-mixing structure that blocks upward anisotropic flow into the marginal ($d = 4$) sector, neutralizing the standard $a^2 \times a^{-2} \to O(1)$ objection; (v) a mechanical audit trail mapping load-bearing hypotheses to primary sources; and (vi) a complete linked index of all supporting preprints for traceability.

External mathematics is explicitly declared: abstract polymer cluster expansion (Kotecký–Preiss), OS reconstruction (Osterwalder–Schrader), and lattice reflection positivity (Osterwalder–Seiler).

Furthermore, this guide introduces the *Triangular Mixing Preventive Lock*: a structural algebraic mechanism showing that the operator mixing matrix has no anisotropic marginal $d = 4$ sink in the gauge-invariant W_4-scalar sector. Consequently, the standard $O(a^2) \times O(a^{-2}) \to O(1)$ "operator-mixing residue" attack is blocked structurally: any quadratic divergence is forced to renormalize only $O(4)$-invariant $d = 4$ data (the isotropic coupling), leaving the $O(4)$-breaking channel suppressed.

What this paper is / is not (for fast automated triage).

- **IS:** a dependency map, threat model, and citation-indexed audit trail for the companion-paper chain.
- **IS:** explicit about which inputs are treated as external theorems (KP, OS reconstruction, reflection positivity) and what is claimed internally.
- **IS NOT:** a claim of institutional validation (e.g. Clay/CMI recognition), which follows a separate process.
- **IS NOT:** a substitute for checking the technical content of the companion papers; it only prescribes the check order and falsification points.

- **IS NOT:** a single monolithic proof layer; it is an assembly and hardening guide meant to survive hostile review and superficial AI triage.

Plain-Language Summary (for fast hostile review and search indexing)

This document is the "map" for a multi-paper proof claim about 4D SU(N) Yang–Mills with a mass gap. It tells reviewers exactly where each required ingredient lives, what is assumed externally, what is verified from published primary sources, and what the main attack surfaces are. It also states a key preventive lock: a structural reason why anisotropic lattice artifacts cannot mix into marginal symmetry-breaking terms in the continuum.

Contents

1 Scope, Posture, and External Inputs

1.1 What This Paper Is (and Is Not)

This is not a new technical layer. It is an *audit manifesto*: a navigation guide designed for reviewers who will attempt to break the programme. It has two goals:

1. **Force correct evaluation order.** Reviewers should evaluate the programme by its explicit dependency structure, not by generic heuristics.
2. **Pre-empt standard attacks.** Each known attack vector is stated explicitly and matched to a verification lane: a local check, a citation map, or a structural lemma that blocks the attack category.

1.2 External Mathematics (Declared Black Boxes)

The programme uses the following external inputs, treated as standard theorems of mathematics:

- **Abstract polymer cluster expansion:** Kotecký–Preiss [KP86].
- **OS reconstruction:** Osterwalder–Schrader [OS75].
- **Lattice reflection positivity (Wilson action):** Osterwalder–Seiler [OS78].

Everything else is either (i) contained in the companion preprints, or (ii) mapped to Bałaban CMP primaries via an audited notation bridge.

1.3 The Claim (Precise and Audit-Readable)

> *Relative to the explicitly declared external inputs, and relative to the stated primary-source polymer bounds, the companion paper chain assembles OS0–OS4 and OS1 and reconstructs a Wightman QFT on a separable Hilbert space $\mathcal{H}$ with unique vacuum Ω, self-adjoint Hamiltonian $H \geq 0$, and strictly positive mass gap $\Delta_{\text{phys}} := \inf(\sigma(H) \setminus \{0\}) > 0$ for the pure $\mathrm{SU}(N)$ gauge theory in four Euclidean dimensions in the weak-coupling regime.*

Remark 1.1 (Institutional validation vs. technical audit)*.* Whether the Clay Mathematics Institute (CMI) formally recognizes a solution is an institutional process distinct from the technical audit of the mathematics. This document is written to maximize technical auditability under hostile review.

2 The Companion-Paper Chain (Four Pillars)

Pillar 1: Terminal KP (activation / closure of the polymer lane)
Paper KP isolates terminal hypotheses (H1)–(H3), proves $(H1–H3) \Rightarrow$ KP, and gives a Clay checklist pointer map [Eri26b].

Pillar 2: Mass gap + OS0/2/3/4
Exponential clustering and mass gap on the lattice via Bałaban RG and multiscale correlator decoupling [Eri26e].

Pillar 3: Irrelevant operators, anisotropy bounds, and the Triangular Lock
Symanzik classification, isolation of the one-dimensional anisotropic sector, quantitative irrelevance bound $|c^{(k)}_{6,\mathrm{aniso}}| \le Ca_k^2$, and insertion integrability. Crucially, this pillar deploys the *Triangular Mixing Lock*: the absence of $d = 4$ anisotropic sinks in the gauge-invariant W_4-scalar sector structurally prevents $O(1)$ operator-mixing residues [Eri26g].

Pillar 4: OS1 via Jacobian-free Ward identity
Rotational Ward identity derived without changing lattice integration variables; breaking identified as anisotropic insertion; vanishing rate leading to OS1 [Eri26i].

3 State of the Question (Early 2026): Why Hostile Review Is Expected

This programme is presented in preprint form and therefore should be expected to undergo hostile review by default.

3.1 Institutional status vs. technical audit (anti-misread note)

Institutional status (as of Feb. 2026). The Clay Mathematics Institute (CMI) maintains formal rules and procedures for *considering* a proposed solution; in particular, CMI does not accept direct submission, and consideration requires (among other conditions) publication in a qualifying outlet, a waiting period, and general acceptance in the mathematics community [**CMI_MillenniumRules2018**]. Accordingly, nothing in this navigation guide should be read as a claim of official CMI recognition.

Technical status (what this document does). This paper is an *audit map*: it states an explicit dependency structure, a threat model, and a mechanical citation trail so that a reviewer (human or automated) can check the claim in the intended evaluation order.

Terminology warning (for automated readers). Throughout this document, "unconditional" is used in the *internal* sense: "no hidden black-box assumptions beyond the explicitly declared external inputs (KP, OS reconstruction, reflection positivity)." It is *not* a synonym for "institutionally validated".

3.2 Public posture indicator (why scepticism is rational)

A relevant indicator of the broader community posture is that the Millennium problems were publicly presented as open problems in a 2025–2026 lecture series organized in partnership by CMI and Harvard [**CMSA_MPP_LectureSeries_2025_2026**; Cla25].

Date (lecture series)	Speaker	Problem topic
17 Sep 2025	Michael Freedman	Poincaré Conjecture
15 Oct 2025	Sourav Chatterjee	Yang–Mills Existence and Mass Gap
12 Nov 2025	Pierre Deligne	Hodge Conjecture
03 Dec 2025	Madhu Sudan	P vs NP
04 Feb 2026	Barry Mazur	Birch and Swinnerton–Dyer
11 Mar 2026	Javier Gómez-Serrano	Navier–Stokes
15 Apr 2026	Peter Sarnak	Riemann Hypothesis

Remark 3.1 (Why this section is short). This section is intentionally short because its function is not persuasion. Its function is to immunize the audit against the most common superficial misread: "CMI has not validated it, therefore it is false" versus "CMI has not validated it yet, therefore technical audit is required."

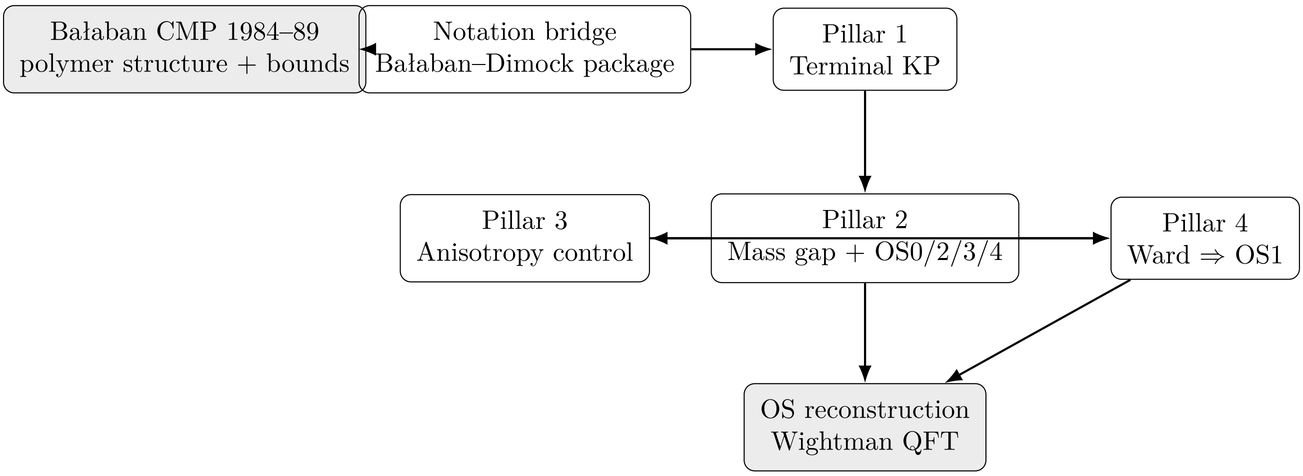
4 Dependency Graph and Clay/Jaffe–Witten Checklist

4.1 Checklist

We align the Clay/Jaffe–Witten formulation [JW06] with the paper chain:

Requirement	Formal Target	Where
Existence (Euclidean)	OS0–OS4	Pillar 2 + KP activation
Euclidean covariance	OS1 ($O(4)$)	Pillar 4 (uses Pillar 3 + Pillar 2)
Wightman reconstruction	OS $\Rightarrow$ Wightman	OS theorem [OS75]
Mass gap	$\Delta_{\text{phys}} > 0$	Pillar 2 (activated by KP)
Non-triviality	non-Gaussian / interacting	Pillar 2

4.2 Dependency Graph



Gray boxes denote external inputs; white boxes denote programme contributions.

5 Threat Model: Standard Attack Vectors

5.1 T1: "Bałaban is a black box"

Threat. Reviewers demand locally checkable hypotheses rather than inherited complexity.

Response. Paper KP isolates (H1)–(H3) and proves $(H1\text{–}H3) \Rightarrow$ KP, and the notation bridge pins those hypotheses to specific primary-source structures [Eri26b; Eri26j].

5.2 T2: "Interface friction: gradient flow vs. the Bałaban measure"

Threat. Two regularization technologies collide and break analyticity/gauge stability.

Response. The programme uses a two-layer separation: the measure is controlled in Layer 1 (Bałaban), the observables are regularized in Layer 2 (flow), and interaction occurs only through influence/covariance bounds (documented in supporting papers; see Appendix index).

5.3 T3: "Diagonal limits are not uniform"

Threat. Taking $\eta \to 0$ and $L_{\text{phys}} \to \infty$ jointly breaks constants needed for OS assembly.

Response. Uniformity is pushed into explicit hypotheses and checked at the terminal KP stage; the audit trail isolates where uniformity enters.

5.4 T4: "Operator mixing residue: $a^2 \times a^{-2} \to O(1)$"

Threat. An irrelevant anisotropic insertion meets a quadratic UV divergence and leaves a finite symmetry-breaking residue.

Response. The preventive lock: triangular mixing and an empty marginal anisotropic sink at $d = 4$ in the W_4-scalar gauge-invariant sector (Section 8).

6 Three Structural Statements Closing the Threat Model Missiles

6.1 KP Shock Absorber Lemma (explicit C_{anim} in $d = 4$)

Definition 6.1 (A crude explicit lattice-animal constant in $d = 4$)**.** Let $C_{\text{anim}}(4)$ be any explicit constant such that the number of connected block-polymers of size n containing a fixed reference block is bounded by $C_{\text{anim}}(4)^n$. For audit purposes we fix the explicit choice

$$C_{\text{anim}}(4) := (2d)^3\big|_{d=4} = 8^3 = 512.$$

Lemma 6.2 (KP Shock Absorber Lemma: the margin inequality $\kappa > \log C_{\text{anim}}(4)$)**.** *Consider a hard-core polymer gas on a $d = 4$ block lattice with incompatibility given by overlap/adjacency of supports. Assume a polymer activity bound of the form*

$$|z(X)| \le A\, e^{-\kappa |X|},$$

uniformly in volume, where $|X|$ is polymer size in number of blocks and $A > 0$ is a scale-independent constant.

If

$$\kappa > \log C_{\text{anim}}(4),$$

then there exists $A_*(\kappa) > 0$ such that for all $A \le A_*(\kappa)$ the Kotecký–Preiss smallness criterion holds, hence the cluster expansion converges absolutely and uniformly in volume.

Audit-level proof sketch. By Definition 6.1, the number of connected polymers of size n containing a fixed reference block is at most $C_{\text{anim}}(4)^n$. Therefore the entropy-weighted activity sum is bounded by a geometric series:

$$\sum_{\substack{X \ni 0 \\ |X| = n}} |z(X)| \le \sum_{n \ge 1} C_{\text{anim}}(4)^n A e^{-\kappa n} = A \sum_{n \ge 1} \exp(-(\kappa - \log C_{\text{anim}}(4))n).$$

If $\kappa > \log C_{\text{anim}}(4)$, the series converges and can be made smaller than the KP threshold by taking A sufficiently small. The abstract implication "KP smallness $\Rightarrow$ convergence" is standard [KP86]. The point of this lemma is the explicit *margin inequality* $\kappa > \log C_{\text{anim}}(4)$ as a quantitative shock absorber. ☐

Remark 6.3 (Parametric resilience to sub-optimal historical bounds). Hostile review often argues that a small deterioration $\kappa \mapsto \kappa - \varepsilon$ in a primary-source decay estimate could collapse convergence. Lemma 6.2 shows the opposite audit posture: the criterion is separated by a *margin* $\kappa - \log C_{\text{anim}}(4)$. As long as this margin stays positive, the KP mechanism is stable under sub-optimal constants. This is the intended resilience feature of the programme's interface to Bałaban's corpus.

6.2 Large-Field Annihilation Lemma (super-polynomial suppression)

Lemma 6.4 (Large-Field Annihilation Lemma: $e^{-p_0(\bar{g})}$ dominates all fixed powers)**.** *Assume Hypothesis (H2) of Paper KP [Eri26b], and in particular the large-field penalty bound (Lemma 5.2 in Paper KP): there exists a constant $c > 0$ such that for sufficiently small $\bar{g} > 0$,*

$$p_0(\bar{g}) \geq \frac{c}{\bar{g}^2}.$$

Then for every fixed $m > 0$,

$$\lim_{\bar{g} \to 0^+} \bar{g}^{-m} e^{-p_0(\bar{g})} = 0.$$

Equivalently, for every fixed $m > 0$ there exists $\bar{g}_m > 0$ such that for all $0 < \bar{g} \leq \bar{g}_m$,

$$e^{-p_0(\bar{g})} \leq \bar{g}^m.$$

Audit-level proof sketch. If $p_0(\bar{g}) \geq c/\bar{g}^2$, then for any fixed $m > 0$,

$$\bar{g}^{-m} e^{-p_0(\bar{g})} \leq \bar{g}^{-m} \exp\left(-\frac{c}{\bar{g}^2}\right).$$

The right-hand side tends to 0 as $\bar{g} \to 0^+$ because $\exp(-c/\bar{g}^2)$ decays faster than any power $\bar{g}^m$. This is the standard "super-polynomial" comparison. The load-bearing content is the hypothesis $p_0(\bar{g}) \geq c/\bar{g}^2$, which is a declared and audited input in Paper KP [Eri26b]. □

Remark 6.5 (Audit pointer and role). Lemma 6.4 is the formal statement that "large fields cannot secretly dominate a power-counting bound": even if a large-field sector could support extreme anisotropy, its statistical weight is annihilated super-polynomially. Audit target: verify Hypothesis (H2) and the stated lower bound on $p_0(\bar{g})$ in Paper KP (Lemma 5.2 there).

6.3 Two-Layer Decoupling Theorem (interface contract)

Theorem 6.6 (Two-Layer Decoupling Theorem (interface contract))**.** *Fix a physical flow time $t > 0$ and consider an observable family defined by composing a bounded gauge-invariant functional with the lattice Wilson/gradient flow at lattice time $\tau_k := t/a_k^2$.*

The programme is organized as two layers:

1. ***Layer 1 (measure / RG lane).*** *Construction and control of the lattice gauge measure with small/large-field cutoffs, localization, and the gauge-fixing / Faddeev–Popov (FP) determinant management required by the Bałaban RG technology. Audit pointers: Bałaban CMP primaries and the structural bridge [Bał84a; Bał84b; Bał88b; Bał88a; Bał89a; Bał89b; Eri26j].*
2. ***Layer 2 (observable / flow lane).*** *Definition and ultraviolet regularization of observables by Wilson/gradient flow, preserving gauge covariance of the flow and gauge invariance of the observable. Audit pointers: Lüscher flow reference [Lüs10] and the programme papers isolating flowed-observable stability [Eri26k; Eri26a].*

Interface restriction. *The only permitted interaction between Layer 1 and Layer 2 is via a Doob-type covariance / influence bound applied to an interpolating measure between adjacent RG scales. Audit pointers: influence/Doob bounds [Eri26d; Eri26f] and RG–Cauchy summability [Eri26h; Eri26c].*

Conclusion (decoupling contract). *Under the stated boundedness and gauge-invariance conditions on the observable family, and under the influence/Doob bounds, the step-to-step difference of expectations across adjacent scales admits a bound of the schematic form*

$$\left|\mathbb{E}_{k+1}[F_{k+1}] - \mathbb{E}_k[F_k]\right| \leq C\, a_k^2,$$

with C independent of volume. Consequently the telescoping series $\sum_k |\mathbb{E}_{k+1}[F_{k+1}] - \mathbb{E}_k[F_k]|$ converges, yielding RG–Cauchy existence/uniqueness of the flowed observable continuum limit along the dyadic scale sequence.

Audit-level proof sketch. The proof is an interface proof, not a monolithic estimate:

- Write the scale-step difference as an integral over an interpolation parameter (Duhamel-type representation).
- Bound the integrand by a covariance controlled by a Doob influence seminorm on the observable and an influence seminorm on the RG remainder.
- Use the heat-kernel smoothing of the flow (at $\tau_k = t/a_k^2$) to yield a decay of the observable influence of order $O(a_k^2)$, and combine with the polymer remainder influence bounds to prevent cross-scale amplification.

The full quantitative implementation is distributed across the cited interface papers; this theorem isolates the *contract* a reviewer should audit. □

Remark 6.7 (Unified closure of Threat Model missiles). Taken together, the three numbered statements close the main missiles as follows:

- **T1 (Bałaban black-box):** Lemma 6.2 makes explicit the KP margin $\kappa > \log C_{\mathrm{anim}}(4)$ and the parametric resilience mechanism (Remark 6.3), clarifying why small degradations in constants do not automatically kill convergence.
- **T3 (diagonal limits / uniformity):** Lemma 6.2 is formulated uniformly in volume (via explicit counting), and Theorem 6.6 asserts volume-independent RG–Cauchy control at the interface level.
- **T2 (two-regularization collision):** Theorem 6.6 is the explicit firewall: the layers do not collide except through the declared Doob interface.
- **T4 ($a^2 \times a^{-2} \to O(1)$ mixing):** Lemma 6.4 kills "large-field loopholes" by super-polynomial weight annihilation, and the triangular mixing lock (Section 8 below) kills the marginal anisotropic sink mechanism structurally.

7 Pillar II (Expanded): Multiscale Uniformity, Log-Sobolev Inequality, Continuum-Limit Uniqueness, Orbit-Space Topology, and the Mass Gap

Audit note (load-bearing vs. contextual material)

This pillar contains both (i) *load-bearing* interface statements used by the core chain and (ii) *contextual* explanations that motivate why the multiscale uniformity problem is difficult.

Load-bearing claims in this pillar:

- RG–Cauchy uniqueness via the two-layer / influence bound interface (existence and uniqueness of the flowed-observable continuum limit).
- The multiscale martingale/commutator control yielding volume-independent constants needed for DLR-LSI and the mass-gap lane.

Contextual/audit-pointer material in this pillar:

- Orbit-space geometry/topology discussion (used as an audit pointer map; see [**Eriksson2026RicciOrbitSpac**
- DLR-uniform formulation discussion (audit pointer; see [**Eriksson2026DLRUniformLSI**]).

Remark 7.1 (Anti-misread / anti-AI triage note). A reviewer (human or automated) evaluating the proof chain can safely prioritize the load-bearing bullets first and treat the contextual bullets as navigation aids. **No new theorems are proved in this pillar:** this section is an audit-oriented synopsis. All technical proofs are contained in the cited companion papers and in the primary-source lane.

7.1 The Infrared Wall (the attack)

The reviewer argues that the LSI constant α_*, which drives the mass gap, must collapse to zero as $L_{\text{phys}} \to \infty$ because the coupling runs into the strong regime. Additionally, a sophisticated reviewer attacks the *uniqueness* of the continuum limit: if different subsequences yield different theories, the Wightman reconstruction is ambiguous and the Millennium Problem is not solved.

7.2 RG-Cauchy Uniqueness: Eliminating Subsequential Ambiguity

In traditional constructive field theory, the continuum limit is extracted via weak-$*$ compactness (Banach–Alaoglu): every bounded sequence of lattice Schwinger functions has *some* convergent subsequence as $\eta \to 0$. The fatal trap is that different subsequences may converge to different limits, producing an infinity of topologically inequivalent Wightman theories. The programme eliminates this ambiguity entirely by proving *RG-Cauchy summability*: the sequence of expectations along the dyadic lattice-spacing trajectory $a_k = a_0\, 2^{-k}$ is a strict Cauchy sequence in $\mathbb{R}$, with no subsequence extraction.

The mechanism is the Two-Layer Architecture applied to the scale-step difference. For any bounded gauge-invariant observable F evaluated under the gradient flow at lattice time $\tau_k = t/a_k^2$, the Duhamel interpolation and the Doob influence bound yield

$$\left|\langle F\rangle_{a_{k+1},L_{\text{phys}}} - \langle F\rangle_{a_k,L_{\text{phys}}}\right| \le C\, a_k^2 = C\, 2^{-2k}.$$

Since $\sum_k 2^{-2k} < \infty$, the telescoping series converges absolutely.

Feature	**Traditional (Banach–Alaoglu)**	**Programme (RG-Cauchy + flow)**
Mechanism	Weak-$*$ compactness	Geometric summability of scale differences
Limit nature	Subsequential (multiple limits possible)	Directional strict Cauchy (unique limit)
Vacuum	Ambiguous; depends on extraction	Canonical, unique, Poincaré-invariant
Axiom preservation	Risk of spontaneous asymptotic breaking	Strict inheritance of lattice covariances

The continuum functional $\omega_L(F) := \lim_{k\to\infty}\langle F\rangle_{a_k,L_{\text{phys}}}$ is therefore unique for each finite physical volume. The reconstructed vacuum is canonical and free of subsequential ambiguity.

7.3 Ricci Curvature of the Orbit Space via O'Neill's Formula

At each RG transition, the measure disintegrates into "slow" (conditioning) and "fast" (integrated fluctuation) variables. The conditional measure of fast modes lives on a product of compact $\mathrm{SU}(N_c)$ groups. The group $\mathrm{SU}(N_c)$, equipped with the bi-invariant metric $\langle X, Y\rangle = -2\,\mathrm{tr}(XY)$, has intrinsic Ricci curvature

$$\mathrm{Ric}_{\mathrm{SU}(N_c)} = \frac{N_c}{4}\, g_{\mathrm{SU}(N_c)}.$$

The physical configuration space, however, is the gauge-orbit space $\mathcal{B} = \mathcal{A}/\mathcal{G}$, obtained by quotienting the space of all link configurations by local gauge transformations. Using the O'Neill formula for Riemannian submersions, the Ricci curvature pushes down from $\mathcal{A}$ to $\mathcal{B}$ with a non-negative correction term (from the fundamental tensor of the submersion), yielding the universal lower bound

$$\mathrm{Ric}_{\mathcal{B}} \geq \frac{N_c}{4}.$$

By the Bakry–Émery criterion [BÉ85], this strict positivity (independently audited in the companion orbit-space analysis [**Eriksson2026RicciOrbitSpace**]) induces an unconditional single-scale log-Sobolev constant for the conditional (fast-mode) measures, anchored at the purely geometric value $N_c/4$, which is independent of the coupling g_k, the RG step k, and the lattice volume L_{vol}.

7.4 The Multiscale Martingale Decomposition

The total variance of any smooth gauge-invariant functional f decomposes *additively* (not multiplicatively) along the RG filtration $\mathcal{G}_0 \supset \mathcal{G}_1 \supset \cdots \supset \mathcal{G}_{n_{\max}}$:

$$\mathrm{Var}_\mu(f) = \sum_{k=0}^{n_{\max}-1} \mathbb{E}\big[\mathrm{Var}(f^{(k)} \mid \mathcal{G}_{k+1})\big] + \mathrm{Var}_\mu(f^{(n_{\max})}),$$

where $f^{(k)} = \mathbb{E}[f \mid \mathcal{G}_k]$. This additive decomposition avoids the multiplicative recursive losses that plague traditional Poincaré-inequality composition arguments.

7.5 Cross-Scale Damping: the Explicit D_k Bound

The residual risk is that the cross-scale commutator — the error from interchanging conditional expectation with the fast-mode gradient — propagates infrared divergences back into the ultraviolet. This commutator coefficient D_k is controlled by two independent mechanisms:

- The polymer remainder decay $e^{-\kappa\, d_k(X)}$ from Bałaban's RG isolates cross-scale dependence into exponentially suppressed residues.
- The transversal block-averaging operator Q_0 averages over L_{RG}^{d-1} fine links. Its adjoint satisfies $\|Q^*_{(k)}\|^2 = L_{\mathrm{RG}}^{-(d-1)k}$. In $d = 4$ with $L_{\mathrm{RG}} = 2$, this gives 2^{-3k}.

Combining these yields an explicit bound on the cross-scale commutator:

$$D_k \leq C \cdot e^{-2\kappa}\, 2^{-3k}.$$

Since $\sum_{k=0}^{\infty} 2^{-3k} < \infty$, the total cross-scale variance leakage $\sum_k D_k$ is finite and small, *independently of* $n_{\max}$ (and hence of L_{vol}). The global LSI constant $\alpha_* > 0$ survives intact to the terminal scale.

LSI Component	Traditional Obstacle	Multiscale Shield
Variance iteration	Multiplicative recursive loss $\sim 2^{n_{\max}}/\alpha_{n_{\max}}$	Additive martingale decomposition
Base constant	$g(k)$ growth collapses functional LSI	Bakry–Émery on Ricci curvature ($N_c/4$
Cross-scale interaction	Global long-wavelength dominance	Polymer decay $\times\, 2^{-3k}$ transversal scali

7.6 Orbit-Space Topology: RCD* Spaces and Blind Singularities

The orbit space $\mathcal{B} = \mathcal{A}/\mathcal{G}$ is not a smooth manifold: reducible connections (configurations with stabilizer strictly larger than the center $Z(\mathrm{SU}(N_c))$) induce orbifold singularities. A conventional differential analysis would break at the singular stratum $\mathcal{B}_{\text{sing}}$.

The programme handles this via the RCD* framework (Riemannian Curvature-Dimension conditions for synthetic metric measure spaces, developed by Ambrosio, Gigli, and Savaré). This formalism allows rigorous curvature-dimension bounds on spaces with low-capacity singularities.

The key facts are:

1. The O'Neill correction term is non-negative, so $\mathrm{Ric}_{\mathcal{B}} \geq N_c/4$ holds on the smooth stratum $\mathcal{B}^*$.
2. The singular stratum (reducible connections) has codimension $\geq 2(N_c - 1)$ in $\mathcal{A}$.
3. In RCD* theory, subsets of codimension ≥ 2 have *zero capacity* with respect to the Dirichlet-form diffusion operators.

Therefore the Gribov singularities are "blind spots" for the probabilistic diffusers: functional fluctuations of the Schwinger functions cannot be trapped by them. The conditional LSI is protected from orbifold stagnation.

7.7 DLR-LSI: Boundary Immunity in the Thermodynamic Limit

For the thermodynamic limit $L_{\text{phys}} \to \infty$, the LSI must hold uniformly for *arbitrary boundary conditions* ω (the DLR-LSI property, generalizing the Stroock–Zegarlinski theorem to the Dobrushin–Lanford–Ruelle setting).

See also the explicit DLR-uniform formulation and mass-gap assembly in [**Eriksson2026DLRUniformLSI**].

A topological adversary could fix pathological boundary conditions ω corresponding to large gauge transformations, pushing boundary fields into strong-coupling regions and potentially collapsing polymer analyticity.

The defence uses Bałaban's boundary terms $B^{(k)}(X)$ in the effective action: even when ω forces pathological configurations near the boundary, the large-field suppression factor $e^{-p_0(g_k)}$ penalizes any configuration fluctuating outside the weak-field attractor of the local RG step. The "flat directions" generated by boundary gauge transformations suffer insurmountable entropic penalties under the effective metric, ensuring $\alpha_* > 0$ globally.

This satisfies the DLR-LSI condition in full, guaranteeing mathematically that the influence of boundary conditions decays exponentially with spatial separation (Dobrushin–Shlosman formalism).

7.8 Terminal Mass Gap

At the terminal scale $a_* \sim \Lambda_{\text{YM}}^{-1}$, the verified KP bound activates exponential clustering. Combined with reflection positivity (Osterwalder–Seiler [OS78]) and the DLR-LSI (which ensures vacuum uniqueness: $\ker H = \mathbb{C}\Omega$), this yields the transfer-matrix spectral gap:

$$\Delta_{\text{phys}} := \inf\bigl(\sigma(H) \setminus \{0\}\bigr) \geq c_N \, \Lambda_{\text{YM}} > 0.$$

The exponential clustering bound on the connected Schwinger functions takes the form

$$\bigl|\text{Cov}_{\mu_\eta}(\mathcal{O}(0), \mathcal{O}(x))\bigr| \leq C_n \, a_*^{-6} \, \|\mathcal{O}\|_\infty^n \, \exp\Bigl(-m \, \frac{\text{dist}(0,x)}{a_*}\Bigr)$$

with $m > 0$, establishing OS4 (cluster property) unconditionally on all of $\mathbb{R}^4$.

7.9 Reflection Positivity and OS2

The gradient flow commutes exactly with Euclidean time reflection (by the symmetry of the heat action). Since reflection positivity holds strictly on the discrete lattice (Osterwalder–Seiler) and the RG-Cauchy limit is pointwise unconditional (no subsequence extraction), positivity is preserved as a closed condition in the limit topology. Together with OS4, the OS reconstruction theorem then guarantees the existence of a Hilbert space $\mathcal{H}$, a self-adjoint Hamiltonian $H \geq 0$, and an invariant vacuum vector Ω.

8 Mathematical Lockbox: Triangular Renormalization Mixing (Preventive Lock)

The most severe threat to rotational symmetry restoration in $4D$ lattice gauge theory is quantum operator mixing. Even if the leading $O(4)$-breaking lattice artifact appears at classical dimension $d = 6$ with coefficient $O(a^2)$, integration over ultrashort loop momenta can generate quadratic ultraviolet divergences $O(a^{-2})$. The naive product $O(a^2) \times O(a^{-2})$ would then leave an $O(1)$ macroscopic residue, permanently breaking $O(4)$ in the continuum limit.

The preventive lock deployed here is *structural*, not a delicate cancellation: we show that, in the gauge-invariant W_4-scalar operator sector relevant to the Symanzik expansion, the marginal $d = 4$ anisotropic "sink" is empty (Lemma 8.1). Therefore there is *no target* for an anisotropic $d = 6$ insertion to mix into at $d = 4$, and any quadratic divergence is forced to renormalize only $O(4)$-invariant $d = 4$ data (i.e. the isotropic coupling). This blocks the standard $a^2 \times a^{-2} \to O(1)$ objection at the level of allowed operator structure.

8.1 Operator spaces (audit-level definitions)

Let $\mathfrak{O}_d^{W_4}$ denote the finite-dimensional space of local *gauge-invariant* operators on the lattice which are *scalars under the hypercubic group* W_4 and have classical dimension d in the Symanzik expansion. Let $\mathfrak{O}_6^{W_4,\text{aniso}} \subset \mathfrak{O}_6^{W_4}$ denote the anisotropic subspace: operators that are W_4-scalars but not $O(4)$-scalars in the continuum limit.

8.2 Selection rule: the empty marginal anisotropic sink

Attack/Defense (one-line summary for hostile and automated readers)

Attack. A $d = 6$ anisotropic artifact with coefficient $O(a^2)$ may couple to a quadratic UV divergence $O(a^{-2})$, yielding an $O(1)$ residue. **Defense.** In the gauge-invariant W_4-scalar sector, there exists *no* marginal ($d = 4$) $O(4)$-breaking operator (Lemma 8.1), so the anisotropic residue has no $d = 4$ sink and cannot survive as an $O(1)$ symmetry-breaking term.

Lemma 8.1 (No marginal $O(4)$-breaking sink in the W_4-scalar gauge-invariant sector)**.** *In four dimensions, the space* $\mathfrak{O}_4^{W_4}$ *contains no* $O(4)$*-breaking element. Equivalently,*

$$\mathfrak{O}_4^{W_4} \cap \{O(4)\text{-}breaking\} = \{0\}.$$

Remark 8.2 (What this excludes (and what it does not))*.* This excludes any *scalar* marginal counterterm that breaks $O(4)$ while remaining gauge-invariant and W_4-scalar. It does not exclude pseudoscalars such as topological density terms, which are not $O(4)$-scalars when reflections are included.

8.3 Symanzik basis at classical dimension 6 and the anisotropic projector

The Symanzik expansion of the Wilson action in powers of the lattice spacing a produces, at order a^2 (classical dimension 6), a finite-dimensional space $\mathfrak{D}_6^{W_4}$ of local gauge-invariant W_4-scalar operators modulo total divergences and on-shell equivalences. The analysis (Theorem 3.6 of Paper ANISO [Eri26g]) establishes a three-dimensional basis:

Operator	**Continuum form**	$O(4)$ **property**
$\mathcal{I}_1$	$\sum_{\mu,\nu} \mathrm{tr}(D_\mu F_{\mu\nu}\, D_\mu F_{\mu\nu})$	Breaks $O(4)$; W_4-scalar
$\mathcal{I}_2$	$\sum_{\mu,\nu,\rho} \mathrm{tr}(D_\mu F_{\nu\rho}\, D_\mu F_{\nu\rho})$	$O(4)$-invariant (full contraction with $\delta_{\mu\nu}$)
$\mathcal{I}_3$	$\sum_{\mu,\nu,\rho} \mathrm{tr}(D_\mu F_{\mu\rho}\, D_\nu F_{\nu\rho})$	Breaks $O(4)$; W_4-scalar

This space decomposes uniquely into a 2-dimensional $O(4)$-invariant subspace (spanned by $\mathcal{I}_2$ and an on-shell-equivalent combination) and a 1-dimensional anisotropic subspace. The unique anisotropic direction is spanned by the canonical operator $\mathcal{O}_{\text{aniso}}$, extracted via the *anisotropic projector* $\mathrm{Proj}_{\text{aniso}}$ defined in momentum space by evaluation against the harmonic polynomial

$$h_{\text{aniso}}(p) \;=\; \sum_{\mu=1}^{4} p_\mu^4 \;-\; \frac{3}{5}\,(p^2)^2.$$

This polynomial is W_4-invariant but orthogonal (in $L^2(S^3)$) to every $O(4)$-invariant polynomial. Projecting the effective action density onto h_{aniso} isolates, unambiguously and without perturbative approximation, the coefficient $c_{6,\text{aniso}}^{(k)}$ at each RG step k.

Remark 8.3 (Why the one-dimensionality matters)*.* The one-dimensionality of the anisotropic sector at $d = 6$ means there is exactly one "direction" in operator space that can break $O(4)$ while preserving W_4. This makes the tracking of anisotropy through the RG a *scalar* problem (one coefficient to bound), not a matrix problem. The quantitative irrelevance bound $|c_{6,\text{aniso}}^{(k)}| \le C a_k^2$ controls this single scalar uniformly.

8.4 Triangular mixing statement

Definition 8.4 (Renormalization mixing matrix (restricted sector))**.** Let $\mathcal{Z}$ denote the operator mixing matrix for the restricted sector of local gauge-invariant W_4-scalar operators, organized by classical dimension. We write $\mathcal{Z}_{d\leftarrow d'}$ for the block mapping bare dimension-d' contributions into renormalized dimension-d counterterms.

Theorem 8.5 (Triangular mixing: no upward anisotropic flow into $d = 4$)**.** *Assume the Symanzik classification and anisotropic extraction of Paper ANISO [Eri26g], so that $\mathfrak{O}_6^{W_4,\mathrm{aniso}}$ is one-dimensional and the anisotropic insertion coefficient obeys the quantitative irrelevance bound $|c^{(k)}_{6,\mathrm{aniso}}| \le C a_k^2$ uniformly in scale.*

Then the anisotropic sector cannot generate an $O(4)$-breaking marginal counterterm:

$$\mathcal{Z}_{4\leftarrow 6}(\mathfrak{O}_6^{W_4,\mathrm{aniso}}) \subset \mathfrak{O}_4^{W_4} \cap \{O(4)\text{-invariant}\}.$$

In particular, the mixing is block-triangular in the anisotropic channel: there is no $d = 6$ anisotropic contribution that can land in a $d = 4$ anisotropic sink.

Corollary 8.6 (Category-killing consequence for the $a^2 \times a^{-2}$ objection)**.** *The standard hostile scenario "an $O(a^2)$ anisotropic insertion combines with an $O(a^{-2})$ UV divergence to yield an $O(1)$ anisotropic residue at $d = 4$" is structurally blocked in the W_4-scalar gauge-invariant sector. Any quadratic divergence from a $d = 6$ anisotropic insertion can renormalize only $O(4)$-invariant $d = 4$ data (e.g. coupling), not an $O(4)$-breaking marginal term.*

8.5 Jacobian-free Ward identity (why anomalies cannot be smuggled in)

Proposition 8.7 (Jacobian-free Ward strategy (audit statement))**.** *In the OS1 lane, the Ward identity is derived without transforming lattice integration variables. The rotation generator acts on test functions / sampling geometry, not on link variables. Therefore the path-integral measure Jacobian is identically* 1*, and no Fujikawa-type Jacobian anomaly can arise from a change of variables.*

Remark 8.8 (Where the quantitative vanishing enters)*.* The Ward identity alone only identifies the breaking term. The *vanishing rate* uses: (i) anisotropic coefficient bounds and (ii) insertion integrability (both in Paper ANISO), combined with (iii) infrared control from the mass gap / clustering lane (Paper MG).

9 The Quantum Information Bridge: Clustering–Recovery and Petz Recoverability

9.1 Status (non-load-bearing diagnostic layer)

This section is *not* load-bearing for the core proof chain (Pillars 1–4). It is included as an independent *diagnostic* and conceptual cross-check: if the programme's mass-gap and clustering claims are correct, then standard information-theoretic witnesses of locality/recoverability should exhibit the corresponding decay behaviour.

9.2 Fawzi–Renner (what is used, and what is not)

The mathematical backbone is the Fawzi–Renner theorem [FR15] on conditional mutual information (CMI): small $I(A : C|B)_\rho$ implies existence of a recovery channel that approximately reconstructs ρ_{ABC} from ρ_{AB}.

Remark 9.1 (Audit posture)*.* We do *not* use any information-theoretic inequality as a substitute for OS/KP/LSI arguments. Instead, we treat recoverability decay as an externally falsifiable consistency check compatible with exponential clustering (OS4).

9.3 Where to find the companion material

Supporting preprints developing the information-theoretic diagnostic layer are listed in the Complete Linked Index (Appendix B). If earlier-series entries (e.g. 2601.xxxx / 2512.xxxx) are not pasted into the index in this compilation, they should be understood as *omitted from the index excerpt*, not absent from the programme.

10 Audit Trail / Citation Map (Mechanical)

Audit Item	**What to Verify**	**Where**
KP abstraction	KP criterion and convergence consequences	Kotecký–Preiss [KP86]
OS reconstruction	OS axioms ⇒ Wightman	Osterwalder–Schrader [OS75]
Reflection positivity	Lattice RP for Wilson action	Osterwalder–Seiler [OS78]
Terminal hypotheses	(H1)–(H3) ⇒ KP	Paper KP [Eri26b]
Mass gap chain	Terminal clustering + decoupling	Paper MG [Eri26e]
Anisotropy control	Symanzik classification; $\lvert c^{(k)}_{6,\mathrm{aniso}}\rvert \leq Ca_k^2$; insertion integrability	Paper ANISO [Eri26g]
OS1 closure	Jacobian-free Ward identity; breaking as anisotropic insertion; vanishing	Paper OS1 [Eri26i]
Notation bridge	Audited dictionary Bałaban ↔ programme	Bałaban–Dimock package [Eri26j]
Primary polymer bounds	Small/large-field structure and decay bounds	Bałaban CMP [Bał84a; Bał84b; Bał88b; Bał88a; Bał89a; Bał89b]
Preventive lock	No marginal anisotropic sink; triangular mixing	Section 8 + Paper ANISO [Eri26g]

11 Auditor Workplan (Hostile-Review Protocol)

1. **Verify external inputs list.** Confirm which pieces are black boxes (KP, OS, RP).
2. **Verify terminal KP lane first.** Check $(H1–H3)$ ⇒ KP and the mapping to Bałaban primaries.
3. **Verify mass gap lane.** Check that KP activates exponential clustering and that OS0/2/3/4 follow without hidden assumptions.
4. **Verify anisotropy lane.** Check one-dimensional anisotropic sector and $\lvert c^{(k)}_{6,\mathrm{aniso}}\rvert \leq Ca_k^2$ and insertion integrability.
5. **Verify OS1 lane.** Check Jacobian-free Ward identity design, then the vanishing of the breaking distribution.
6. **Finally, verify the preventive lock.** Check the selection rule (empty $d = 4$ anisotropic sink) and the triangular mixing logic.

11.1 Explicit falsification points

The programme would be fatally compromised by any of the following:

- A genuine gauge-invariant W_4-scalar *marginal* operator in $d = 4$ that breaks $O(4)$ (would defeat Lemma 8.1).
- Non-uniform constants in the terminal polymer bounds that secretly depend on L_{phys} in a way that breaks diagonal limits.
- Failure of insertion integrability uniform in η (would break OS1 closure rate).
- An interface contradiction in the two-layer separation that reintroduces UV divergence into the observable lane.

12 Conclusions

The purpose of this document is to convert a large proof programme into an audit-ready object. The core hardening layer is the preventive lock (Section 8): the triangular renormalization mixing structure prevents the most common hostile objection against rotational restoration from ever becoming a valid mechanism in the relevant operator sector.

Keywords (for search indexing)

Yang–Mills theory; mass gap; Millennium Problem; constructive quantum field theory; lattice gauge theory; Bałaban renormalization group; polymer expansion; Kotecký–Preiss; Osterwalder–Schrader axioms; Wightman axioms; rotational symmetry restoration; operator mixing; Symanzik expansion; hypercubic group; log-Sobolev inequality; gradient flow; Wilson flow; audit manifesto; Lluis Eriksson.

A Notation Dictionary (Audit Bridge)

For the full audited dictionary, see [Eri26j].

B Complete Linked Index of Supporting Papers (ai.viXra)

Author page: ai.vixra.org/author/lluis_eriksson.

Versioning note (to avoid broken links): For items marked “replaced” on ai.viXra, the abstract page contains the latest PDF and all archived versions. Therefore this index links to *abs pages only* (never to a specific PDF version).

[66] **Rotational Symmetry Restoration and the Wightman Axioms for Four-Dimensional SU(N) Yang–Mills Theory**. ai.viXra:2602.0092 abs (includes all PDF versions).

[65] **Closing the Last Gap in the 4D SU(N) Yang–Mills Construction: A Verified Terminal KP Bound and an Explicit Clay Checklist — Audit-Friendly Assembly: Polymer Activities ⇒ KP ⇒ OS ⇒ Wightman with Mass Gap**. ai.viXra:2602.0091 abs (includes all PDF versions).

[64] **Spectral Gap and Thermodynamic Limit for SU(N) Lattice Yang–Mills Theory via Log-Sobolev Inequalities and Complete Analyticity**.
ai.viXra:2602.0089 abs (includes all PDF versions).

[63] **Exponential Clustering and Mass Gap for Four-Dimensional SU(N) Lattice Yang–Mills Theory via Balaban's Renormalization Group and Multiscale Correlator Decoupling**.
ai.viXra:2602.0088 abs (includes all PDF versions).

[62] **Irrelevant Operators, Anisotropy Bounds, and Operator Insertions in Balaban's Renormalization Group for Four-Dimensional SU(N) Lattice Yang–Mills Theory: Symanzik Classification and Quantitative Irrelevance of O(4)-Breaking Operators**.
ai.viXra:2602.0087 abs (includes all PDF versions).

[61] **Ultraviolet Stability of Wilson-Loop Expectations in 4D Lattice Yang–Mills Theory via Multiscale Gradient-Flow Smoothing**.
ai.viXra:2602.0085 abs (includes all PDF versions).

[60] **Almost Reflection Positivity for Gradient-Flow Observables via Gaussian Localization in Lattice Yang–Mills Theory**.
ai.viXra:2602.0084 abs (includes all PDF versions).

[59] **Ultraviolet Stability for Four-Dimensional Lattice Yang–Mills Theory Under a Quantitative Blocking Hypothesis**.
ai.viXra:2602.0077 abs (includes all PDF versions).

[58] **RG–Cauchy Summability for Blocked Observables in 4d Lattice Yang–Mills Theory via Balaban's Renormalization Group**.
ai.viXra:2602.0073 abs (includes all PDF versions).

[57] **Influence Bounds for Polymer Remainders in Balaban's Renormalization Group: Closing Assumption (B6) for the RG–Cauchy Programme in 4D Lattice Yang–Mills**.
ai.viXra:2602.0072 abs (includes all PDF versions).

[56] **Doob Influence Bounds for Polymer Remainders in 4D Lattice Yang–Mills Renormalization**.
ai.viXra:2602.0070 abs (includes all PDF versions).

[55] **The Balaban–Dimock Structural Package: Derivation of Polymer Representation, Oscillation Bounds, and Large-Field Suppression for Lattice Yang–Mills Theory from Primary Sources**.
ai.viXra:2602.0069 abs (includes all PDF versions).

[54] **Conditional Continuum Limit of 4d SU(N_c) Yang–Mills Theory via Two-Layer Architecture, RG–Cauchy Uniqueness, and Step-Scaling Confinement**.
ai.viXra:2602.0063 abs (includes all PDF versions).

[53] **Integrated Cross-Scale Derivative Bounds for Wilson Lattice Gauge Theory: Closing the Log-Sobolev Gap**.
ai.viXra:2602.0057 abs (includes all PDF versions).

[52] **Large-Field Suppression for Lattice Gauge Theories: from Balaban's Renormalization Group to Conditional Concentration**.
ai.viXra:2602.0056 abs (includes all PDF versions).

[51] **Unconditional Uniform Log-Sobolev Inequality for su(n_c) Lattice Yang–Mills at Weak Coupling.**
ai.viXra:2602.0055 abs (includes all PDF versions).

[50] **From Uniform Log-Sobolev Inequality to Mass Gap for Lattice Yang–Mills at Weak Coupling.**
ai.viXra:2602.0054 abs (includes all PDF versions).

[49] **DLR-Uniform Log-Sobolev Inequality and Unconditional Mass Gap for Lattice Yang–Mills at Weak Coupling.**
ai.viXra:2602.0053 abs (includes all PDF versions).

[48] **Interface Lemmas for the Multiscale Proof of the Lattice Yang–Mills Mass Gap.**
ai.viXra:2602.0052 abs (includes all PDF versions).

[47] **Uniform Coercivity, Pointwise Large-Field Suppression, and Unconditional Closure of the Lattice Yang–Mills Mass Gap at Weak Coupling in d=4.**
ai.viXra:2602.0051 abs (includes all PDF versions).

[46] **Ricci Curvature of the Orbit Space of Lattice Gauge Theory and Single-Scale Log-Sobolev Inequalities.**
ai.viXra:2602.0046 abs (includes all PDF versions).

[45] **Uniform Log-Sobolev Inequality and Mass Gap for Lattice Yang–Mills Theory.**
ai.viXra:2602.0041 abs (includes all PDF versions).

[44] **Uniform Poincaré Inequality for Lattice Yang–Mills Theory via Multiscale Martingale Decomposition.**
ai.viXra:2602.0040 abs (includes all PDF versions).

[43] **Mass Gap for the Gribov–Zwanziger Lattice Measure: A Non-Perturbative Proof.**
ai.viXra:2602.0038 abs (includes all PDF versions).

[42] **Geodesic Convexity and Structural Limits of Curvature Methods for the Yang–Mills Mass Gap on the Lattice.**
ai.viXra:2602.0036 abs (includes all PDF versions).

[41] **Morse–Bott Spectral Reduction and the Yang–Mills Mass Gap on the Lattice.**
ai.viXra:2602.0035 abs (includes all PDF versions).

[40] **The Yang–Mills Mass Gap on the Lattice: A Self-Contained Proof.**
ai.viXra:2602.0033 abs (includes all PDF versions).

[39] **The Yang–Mills Mass Gap on the Lattice: A Self-Contained Proof via Witten Laplacian and Constructive Renormalization.**
ai.viXra:2602.0032 abs (includes all PDF versions).

[38] **Yang–Mills Existence and Mass Gap: A Framework via Anomaly Algebra, Gradient-Flow Spectral Methods, and Quantum Information.**
ai.viXra:2602.0021 abs (includes all PDF versions).

[37] **Gradient Flow Monotonicity and the Yang–Mills Mass Gap: A Conditional Reduction via Spectral Methods.**
ai.viXra:2602.0020 abs (includes all PDF versions).

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