Rotational Symmetry Restoration and the Wightman Axioms for Four-Dimensional $SU(N)$ Yang–Mills Theory

— a conditional OS1 theorem —*

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Abstract

We derive a lattice Ward identity for infinitesimal Euclidean rotations of the Wilson theory, identify the breaking term as a dimension-6 anisotropic operator insertion (in the classification of 2602.0087 v3), and show that the breaking distribution is $O(\eta^2|\log((\Lambda_{\mathrm{YM}}\eta)^{-1})|) \to 0$, establishing axiom OS1 (full $O(4)$ covariance) for subsequential continuum limits — *conditionally on the audited companion inputs*. Version 2 corrects v1's framing: v1 imported the mass gap, OS0/2/3/4, anisotropy and insertion bounds as "unconditional"; per the audited versions these carry the programme ledger's loads (the source-mapped KP block of 2602.0091 v2, (H-LOC), per-scale decoupling and coupling control for 2602.0088 v3; window and structural loads for 2602.0087 v3). The assembled result — a non-trivial Poincaré-covariant Wightman theory with mass gap $\Delta_{\mathrm{phys}} \geq c_N \Lambda_{\mathrm{YM}} > 0$ — therefore holds under the explicit composed hypothesis set, stated in §5; v1's "no unproved hypotheses remain" is withdrawn. The native content survives audit and is machine-verified: the Ward mechanism on an exactly solvable lattice Gaussian model (breaking = $O(\eta^2)$ measured), the $\lambda_{\mu\nu} \neq 0$ mechanism in the exact quartic model (rotations annihilate $O(4)$ invariants, not the hypercubic harmonic), the Lie-algebra-to-group invariance lemma, the symmetric-difference $\eta^2/6$ error constant, and the η^2 log vanishing arithmetic.

1 Introduction and Main Theorem

1.1 The problem

As in v1: the companions establish (conditionally — the audited statuses are attached in §2) exponential clustering with mass gap and OS0/2/3/4 (2602.0088 v3), and the Symanzik classi-

****Version 2.** Audited replacement of v1 within the programme-wide verification pass (2602.0041 v3, 0051–0057 v2, 0063 v3, 0069–0077 v2, 0087 v3, 0088 v3, 0089 v2, 0091 v2). Principal change — **F-UNCOND**: v1 asserted that all imports from the companions were unconditional and concluded "All results are unconditional: no unproved hypotheses remain." Against the audited versions this is withdrawn: the imports (Theorems 2.1–2.5) are retagged to 2602.0088 v3 and 2602.0087 v3 with their ledger loads (KP block per 2602.0091 v2, (H-LOC), per-scale decoupling, coupling control; window and structural loads for the anisotropy bounds), the status table of §5 gains an honest status column, and Theorem 1.1 becomes a *conditional* Wightman/mass-gap theorem with the composed load set stated explicitly. What is native to this paper — the lattice Ward identity mechanism (rotation acting on test functions only), the Symanzik identification of the breaking term, the $\lambda_{\mu\nu} \neq 0$ mechanism, the Lie-algebra-to-group lemma, and the $O(\eta^2)$ discretisation bounds — survives and is machine-verified in exactly solvable settings in the companion suite (`verification/2602-0092/`, 6/6 PASS). With this replacement the citation graph of the Clay chain is fully audited: every node carries named loads and none claims unconditional results.

fication/anisotropy/insertion bounds (2602.0087 v3). The remaining axiom is OS1. This paper supplies it, at the price of the same conditional load set — no more, no less.

1.2 Main result

Theorem 1.1 (Main Theorem — conditional)**.** *Assume the ledger loads of the audited companions: the KP block* $\{(H1),(H2)+\beta_{\mathrm{LF}},(H3),(H2')\}$ *(2602.0091 v2), (H-LOC), the per-scale decoupling import and (H-R*β*)+(H-P0′) (2602.0088 v3), and the structural/window loads of 2602.0087 v3. Then for each* $N \geq 2$ *there exists a non-trivial relativistic quantum field theory* $(\mathcal{H}, U(\Lambda, a), \Omega)$ *satisfying all Wightman axioms, associated with pure* $SU(N)$ *Yang–Mills in four dimensions, with mass gap* $\Delta_{\mathrm{phys}} \geq c_N \Lambda_{\mathrm{YM}} > 0$ *and* $S_4^c \not\equiv 0$*.*

Notation and conventions as in v1 §1.3.

2 Imported Results — audited statuses

v1 stated "All are unconditional." Corrected:

Theorem 2.1 (Mass gap bound; 2602.0088 v3 — conditional)**.** $\mathrm{Cov}_{\mu_\eta}(O(0), O(x)) \leq Ce^{-m|x|/a_*}$ *uniformly in* η, L_{phys}*,* conditional on *the KP block (2602.0091 v2), (H-LOC), the Lemma 6.2 decoupling import, and coupling control.*

Theorem 2.2 (OS0/2/3/4 and distinguished-time reconstruction; 2602.0088 v3)**.** *OS0, OS2, OS3 and* W_4*-covariance of subsequential limits:* unconditional *(lattice level). OS4, vacuum uniqueness, spectral gap, non-triviality:* conditional *(they are consequences of Theorem 2.1).*

Theorem 2.3 (Classification; 2602.0087 v3 — survives)**.** *The on-shell dimension-6 anisotropic sector is one-dimensional, spanned by* $[\mathcal{O}_{\mathrm{aniso}}]$*. (Machine-verified there; representation theory, no load.)*

Theorem 2.4 (Anisotropy coefficient bound; 2602.0087 v3 — windowed/conditional)**.** $|c^{(k)}_{6,\mathrm{aniso}}| \leq Ca_k^2$ *for* $k \leq k_*$*,* $g_k \leq \gamma_0$*, conditional on the structural package (traceable per 2602.0069 v2,* β_{LF} *dichotomy).*

Theorem 2.5 (Insertion integrability; 2602.0087 v3 Thm 6.6 — conditional)**.** *The insertion functional bound holds* given *the clustering input — i.e. it inherits Theorem 2.1's loads.*

3 Lattice Ward Identity for Rotations

Definition 3.1 (local breaking density), Proposition 3.2 (lattice Ward identity, $|E^\eta_n(f)| \leq C_n \eta^2 \|f\|_{W^{1,1}}$) and Remark 3.3 (no reindexing of links: the rotation acts on test functions and plaquette positions only) are retained from v1 — this is the paper's native mechanism. *Audit note:* the proof of Prop. 3.2 in v1 was thin; v2 backs it with an exactly solvable exhibit: for the lattice Gaussian field the entire identity is computable in closed form and the measured breaking $|\langle S^\eta_2, L_{\mu\nu} f\rangle|$ scales as η^2 [suite check 3], and the discretisation error sources of Appendix B carry measured constants ($\eta^2/6$ for the symmetric difference; antisymmetry cancellation in the product rule) [check 4].

Proposition 3.1 (Breaking term identification; v1 Prop. 3.4 — conditional on the slow-field import)**.** $(\delta_{\mu\nu} s_W)(y) = g_0(\eta)^{-2}\eta^2[\lambda_{\mu\nu}\mathcal{O}_{\mathrm{aniso}}(y) + Q^{O(4)}_{\mu\nu}(y) + O(\eta^2)]$*, with* $Q^{O(4)}_{\mu\nu} \in \ker(\mathrm{Proj}_{\mathrm{aniso}})$*. The dimension count and uniqueness use Theorem 2.3 (load-free); the slow-field expansion is the windowed import of 2602.0087 v3.*

4 Ward Identity: Final Form and OS1

Theorem 4.1 (final form) as in v1, with (i) now reading: the insertion bound is Theorem 2.5 — *conditional*. Theorem 4.2 ($L_{\mu\nu}S_n = 0$; the η^2 log vanishing arithmetic is verified with explicit constants [check 5]), Corollary 4.3 (W_4+SO(4)⇒ $O(4)$) and Lemma 4.4 (Lie-algebra annihilation implies group invariance; verified numerically on radial vs non-radial distributions [check 2]) survive as stated. Appendix A's $\lambda_{\mu\nu} \neq 0$: the mechanism — rotations annihilate $O(4)$ invariants but not the hypercubic harmonic, so the rotational variation of a W_4-invariant non-$O(4)$-invariant density has non-vanishing anisotropic projection — is verified exactly in the quartic model [check 1]; the plane-independence via W_4-equivariance is correct. What remains analytic (not machine-checkable): that the specific Wilson-action variation realises the generic case ($\lambda \neq 0$); v2 flags this as a traceable computation, not an axiom.

5 Assembly: Wightman Theory with Mass Gap — conditional

The five steps of v1 are retained with statuses: Step 1 imports: OS0/2/3 unconditional; OS4 conditional (Thm 2.1 loads). Step 2 (OS1, this paper): conditional on Thm 2.5's input + Prop. 3.1's window. Steps 3–5 (OS reconstruction, gap, non-triviality): activate under the composed set. Corrected table:

Axiom	Content	Reference	Status
OS0	Temperedness	0088 v3	unconditional
OS2	Reflection positivity	0088 v3	unconditional
OS3	Symmetry	0088 v3	unconditional
OS4	Cluster property	0088 v3	**conditional**
OS1	$O(4)$ covariance	this paper	**conditional**
Mass gap	$\Delta \geq c_N \Lambda_{\mathrm{YM}}$	0088 v3	**conditional**
Non-triviality	$S_4^c \neq 0$	0088 v3	**conditional**

The composed hypothesis set (the "remaining unproved hypotheses" that v1 said did not exist): $\{$(H1), (H2)+β_{LF}, (H3), (H2′)$\}$ ∪ $\{$(H-LOC), L6.2-import, (H-Rβ), (H-P0′)$\}$ ∪ $\{$0087-structural+window$\}$ ∪ $\{\lambda \neq 0$ traceable computation$\}$. Its consistency and non-emptiness are checked symbolically [check 6].

6 Discussion

Clay relation (corrected): Theorem 1.1 matches the Jaffe–Witten formulation *relative to the composed load set above*. With this v2, the citation graph of the programme is fully audited: every companion is identified, versioned, and carries named loads; no node claims unconditional results. The open problems are now exactly the ledger entries — the honest formulation the programme has been converging to since 2602.0041 v3. *Extensions* as in v1 (other groups, fermions, confinement).

Note added in v2

Finding F-UNCOND: imports retagged to audited versions with loads (§2), status table corrected (§5), "no unproved hypotheses remain" withdrawn and replaced by the explicit composed set, $\lambda \neq 0$ reframed as mechanism (verified) + traceable computation (open), Prop. 3.2's mechanics backed by an exactly solvable exhibit. Native content of v1 otherwise survives with its numbering: Defs/Props 3.1–3.4, Thms 4.1–4.2, Cor 4.3, Lemma 4.4, Appendices A–B. Suite: `verify_2602_0092.py`, 6 deterministic checks, exit 0 iff 6/6.

References

[1] L. Eriksson, ai.viXra:2602.0088 **v3** (clustering/mass gap, conditional).

[2] L. Eriksson, ai.viXra:2602.0087 **v3** (Symanzik classification, conditional/windowed).

[3] L. Eriksson, ai.viXra:2602.0091 **v2** (source-mapped KP block).

[4] L. Eriksson, ai.viXra:2602.0069 **v2** (audited structural bridge).

[5] A. Jaffe, E. Witten, in *The Millennium Prize Problems*, Clay Math. Inst., 2006.

[6] K. Osterwalder, R. Schrader, Comm. Math. Phys. 42 (1975) 281.

[7] K. Osterwalder, E. Seiler, Ann. Phys. 110 (1978) 440.

[8] T. Balaban, Comm. Math. Phys. 102 (1985) 15.

[9] L. Eriksson, ai.viXra:2602.0063 **v3** (continuum capstone; consumer of the $SO(4)$ input via 2602.0087 v3).