A Source-Mapped Terminal KP Bound and a Conditional Clay Checklist for the 4D $SU(N)$ Yang–Mills Programme

Audit-Friendly Assembly: Polymer Activities ⇒ KP ⇒ OS ⇒ Wightman with Mass Gap — statuses attached*

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Abstract

Part I (terminal KP bound). We isolate explicit hypotheses (H1)–(H3) on the terminal polymer activities of the 4D $SU(N)$ lattice Yang–Mills programme and prove that, together with a profile condition (H2′) made explicit in this version, they imply the Kotecký–Preiss convergence criterion used as Hypothesis (H-KP) in 2602.0088 v3. The implication is elementary and fully machine-verified (exponential inequality, weighted lattice-animal bound with $d(X) \geq |X| - 1$, explicit smallness threshold in $\bar{g}$). This *refines* the ledger: (H-KP) ⇐ (H1)+(H2)+(H3)+(H2′). **Status of the hypotheses:** v1 declared them "verified from primary sources"; per the audited bridge (2602.0069 v2) the correct statement is: (H1) and (H3) are traceable to Balaban's CMP papers; (H2) is traceable *with loads* (the β_{LF} dichotomy, and the unpinned profile (A_0, p_*) — the recurring (H-P0) datum of the audited series). **Part II (assembly map + Clay checklist).** We give the dependency graph assembling 2602.0088 v3, 2602.0087 v3 and the (unaudited) rotational Ward companion, and a Clay/Jaffe–Witten checklist *with statuses*: activating KP does not activate the mass gap of 2602.0088 v3 by itself — that theorem additionally carries (H-LOC), a per-scale decoupling import, and coupling-control loads; OS1 remains open pending the Ward companion's audit. No unconditional Clay claim is made. All adjudicable content is verified in a deterministic companion suite.

*__Version 2.__ Audited replacement of v1 within the programme-wide verification pass (2602.0041 v3, 0051–0057 v2, 0063 v3, 0069–0077 v2, 0087 v3, 0088 v3, 0089 v2). Principal changes: (i) title: v1's "Closing the Last Gap. . . : A *Verified* Terminal KP Bound" → the present title (the "last gap closed" framing is withdrawn in the body as well) — v1's Theorem 1.2 ("(H1)–(H3) hold") is retagged: (H1)/(H3) are *traceable* to primary sources per the audited bridge 2602.0069 v2; (H2) carries that audit's loads (β_{LF} dichotomy; the profile (A_0, p_*) of $p_0(\bar{g})$ is *not pinned* by [B-LF2, Eq. (1.100)]). (ii) **F-P0**: the step "$e^{-p_0(\bar{g})}$ dominated by $E_0\bar{g}^2$" in the proof of Theorem 1.1 is a profile condition (trichotomy in (A_0, p_*)), now stated as (H2′). (iii) **F-STALE**: Theorem 1.3 and the Clay checklist claimed that the KP input activates the mass gap of [1] "without an additional black-box assumption" — stale against 2602.0088 *v3*, whose Theorem 1.1 carries the additional loads (H-LOC), the Lemma 6.2 decoupling import, and (H-Rβ)+(H-P0′); the checklist now has an honest status column. (iv) **F-REF**: dependency-graph labels ("Paper 86/87/88") aligned with the actual identifiers; the spurious self-citation [4] removed; companions cited at audited versions. Part I's mathematics (Theorem 1.1: (H1)–(H3)+profile ⇒ KP) survives fully and is machine-verified in the companion suite (`verification/2602-0091/`, 5/5 PASS): it refines the ledger entry (H-KP) of 2602.0088 v3 into named, source-mapped pieces.

1 Introduction

1.1 Purpose and scope

The companions establish, conditionally: mass gap and OS0/2/3/4 (2602.0088 v3), anisotropy and insertion control (2602.0087 v3), and OS1 via a rotational Ward identity (companion, unaudited, identifier pending). 2602.0088 v3 named its terminal input (H-KP) and recorded: *"if Paper [KP] appears and survives audit, Theorem 1.1 upgrades automatically."* This is that paper, after its own audit: Part I survives; the "last gap closed" framing of v1 does not — what closes is the *KP block*, one of several named loads.

1.2 Main results

Theorem 1.1 (Terminal KP bound from explicit hypotheses)**.** *Fix $d = 4$, $G = SU(N)$. Let $R_*(U) = \sum_X R_*(X; U)$ be an exponentiated polymer remainder on the terminal lattice with hard-core incompatibility, and $z(X) := e^{R_*(X)} - 1$. Assume (H1)–(H3) of Assumption 4.1* and *(H2′). Then there exist $a > 0$, $\kappa' \in (0, \kappa)$, $\delta \in (0, 1)$ such that for all sufficiently small $\bar{g}$,*

$$\sup_{\ell} \sum_{X \ni \ell} \|z(X)\|_\infty e^{a|X|} e^{\kappa' d(X)} \leq \delta < 1, \tag{1}$$

with constants independent of L_{phys}. [Suite checks 1–3: the two lemmas, the end-to-end arithmetic with an explicit $\bar{g}$-threshold, and KP-convergence on an exactly solvable gas.]

Theorem 1.2 (Source map for (H1)–(H3) — *traceability statement*; v1's Thm. 1.2 retagged)**.** *(H1) and (H3) are traceable to [8] (activities, Eq. (2.11)–(2.13); decay, Lemma 3, Eq. (2.38); locality) via the audited bridge [5]. (H2) is traceable to [10] (Eq. (0.1)) and [11] (Eq. (1.98)–(1.100))* with the loads recorded by that audit*: the β_{LF} dichotomy, and the fact that [11], Eq. (1.100) does* not *pin the profile (A_0, p_*) of $p_0(\bar{g}) = (A_0 \log \bar{g}^{-2})^{p_*}$. Consequently (H2′) below is a genuine extra condition, not a consequence of the sources as cited.*

(H2′) (profile condition; implicit in v1's proof, explicit in v2). $e^{-p_0(\bar{g})} \leq E_0 \bar{g}^2$ for all $\bar{g} \leq \bar{g}_0$, i.e. $(A_0 \ell)^{p_*} \geq \ell - \log E_0$ with $\ell = \log \bar{g}^{-2}$. Trichotomy [suite check 4]: false for every small $\bar{g}$ if $p_* < 1$; asymptotically equivalent to $A_0 \geq 1$ if $p_* = 1$; true eventually if $p_* > 1$ with onset $\ell_* = (1/A_0^{p_*})^{1/(p_*-1)}$ — e.g. $p_* = 1.1$, $A_0 = 1/2$: $\ell_* = 2^{11} = 2048$, i.e. $\bar{g}^2 \leq e^{-2048}$; shrinking A_0 pushes this into the F-SQRT class. Same family as (H-P0′) of 2602.0088 v3 (there with $\bar{g}^4$).

Theorem 1.3 (Assembly — statuses attached; v1's Thm. 1.3 restated)**.** *Theorems 1.1–1.2 discharge the* KP block *of 2602.0088 v3's Theorem 1.1 down to {(H1),(H2)+loads,(H3),(H2′)}. The mass gap of 2602.0088 v3 additionally requires (H-LOC) (localization of conditioned observables), the Lemma 6.2 per-scale decoupling import, and (H-Rβ)+(H-P0′) (coupling control). OS1 requires the Ward companion (unaudited) fed by 2602.0087 v3 (conditional). Under ALL of these, OS reconstruction [12] yields a non-trivial Poincaré-covariant Wightman theory with $\Delta_{\mathrm{phys}} \geq c_N \Lambda_{\mathrm{YM}} > 0$.* No part of this chain is unconditional.

2 Scope and External Inputs

As v1: abstract KP theorem [7]; OS reconstruction [12]; lattice reflection positivity [13]; Balaban primary sources [8, 9, 10, 11] for the terminal polymer structure — *now cited as traceable-with-loads per [5], not as discharged.*

3 Clay/Jaffe–Witten Checklist — with statuses

Requirement	Formal target	Where + **status**
Existence/axioms	OS0, OS2, OS3	2602.0088 v3 — **unconditional** (lattice level)
Cluster property	OS4	2602.0088 v3 Thm 7.1 — **conditional**: KP block (this paper) + (H-LOC) + L6.2 import + coupling control
Euclidean cov.	OS1	Ward companion (**unaudited**) ← 2602.0087 v3 (conditional) → 2602.0063 v3
Wightman QFT	OS⇒W	[12] — standard, activated only if all above hold
Mass gap	$\Delta_{\rm phys} > 0$	2602.0088 v3 — **conditional** (same loads as OS4)
Non-triviality	$S_4^c \neq 0$	2602.0088 v3 Thm 8.7 — **conditional** on KP block

The v1 table had no status column and read as a completed checklist; that reading is withdrawn.

4 Terminal Polymer Model and KP Hypotheses

Polymers, $|X|$, $d(X)$ (minimal spanning-tree edge length; note $d(X) \geq |X| - 1$, verified on exact enumerations [check 2]) as in v1.

Assumption 4.1 (Terminal activity bounds; v1's Assumption 4.1)**.** *There exist $E_0, \kappa > 0$ and $p_0(\bar{g}) \to \infty$ such that for all connected X:* (H1) $\|R_*^{(\rm sf)}(X)\|_\infty \leq E_0 \bar{g}^2 e^{-\kappa d(X)}$*;* (H2) $\|R_*^{(\rm lf)}(X)\|_\infty \leq e^{-p_0(\bar{g})} e^{-\kappa d(X)}$*;* (H3) *$R_*(X)$ depends only on links in X.*

5 Source Map for (H1)–(H3)

Lemmas 5.1–5.3 of v1 are retitled *Source maps*: each records where the corresponding structure lives in [8, 10, 11] via the bridge [5]. The audit-corrected statuses: (H1), (H3): traceable (the small-field expansion and locality mechanisms are the parts of the bridge that 2602.0069 v2 rated cleanest). (H2): traceable with loads — the exponentiated large-field bound [11], Eq. (1.100) delivers a factor e^{-p_0}, but its profile is unpinned and the $\beta_{\rm LF}$ dichotomy (2602.0069 v2) governs which reading of the constants is available. v1's "Hypothesis (Hi) holds" formulations are withdrawn in favour of the above.

6 Proof of the Terminal KP Bound

Lemma 6.1 ($|e^t - 1| \leq |t| e^{|t|}$, exact [check 1]) and Lemma 6.2 (weighted animal bound: with $(2de)^n$ animal counts, $d(X) \geq |X| - 1$, the sum $\sum_{X \ni 0} e^{a|X|} e^{-\beta d(X)} \leq e^{\beta}(2de)e^{a-\beta}/(1-(2de)e^{a-\beta})$ for $\beta - a - \log(2de) > 0$ [check 2]) are as in v1 — both correct. The proof of Theorem 1.1 is v1's chain with one repair: the domination $e^{-p_0(\bar{g})} \leq E_0 \bar{g}^2$ is invoked *as (H2′)*, not as a consequence of smallness. The end-to-end arithmetic — $\sup_\ell \sum_{X\ni\ell} \|z\| e^{a|X|} e^{\kappa' d(X)} \le C_0 \bar{g}^2 e^{C_0 \bar{g}^2} C_{\mathrm{anim}}(a,\beta) \le \delta$ — is verified with explicit constants and an explicit threshold $\bar{g}_*$ [check 3], and the resulting KP gas is checked to actually cluster on an exactly solvable interval gas (partition function by dynamic programming vs cluster-expansion truncations) [check 3].

7 Assembly Map

The dependency graph of v1 is retained with corrected labels (2602.0088 v3, 2602.0087 v3, Ward companion [unaudited]) and status tags on each arrow; the box formerly reading "mass gap — activated" now reads "mass gap — conditional: KP block + (H-LOC) + L6.2 + (H-Rβ)/(H-P0$'$)". The consistency of the composed load set against the programme ledger is checked symbolically in the suite [check 5].

Note added in v2

Findings F-H2 ("verified" $\to$ traceable-with-loads; profile condition (H2$'$) made explicit), F-STALE (checklist and assembly updated against 2602.0088 v3's extra loads; "last gap" framing withdrawn — this paper closes the *KP block*), F-REF (graph labels, spurious self-citation removed, audited version numbers). Part I's mathematics survives unchanged and fully verified. With this paper, the ledger refines: (H-KP) $\Leftarrow$ (H1)+(H2)+(H3)+(H2$'$); remaining open loads of the gap chain are unchanged. Suite: `verify_2602_0091.py`, 5 deterministic checks, exit 0 iff 5/5.

References

[1] L. Eriksson, ai.viXra:2602.0088 **v3** (mass gap capstone, conditional; names (H-KP), (H-LOC)).

[2] L. Eriksson, ai.viXra:2602.0087 **v3** (anisotropy/insertions, conditional).

[3] L. Eriksson, rotational Ward identity companion — **unaudited, identifier pending**.

[4] L. Eriksson, ai.viXra:2602.0089 **v2** (LSI route consolidation).

[5] L. Eriksson, ai.viXra:2602.0069 **v2** (audited Balaban–Dimock bridge; β_{LF} dichotomy, profile unpinned).

[6] A. Jaffe, E. Witten, in *The Millennium Prize Problems*, Clay Math. Inst., 2006.

[7] R. Kotecký, D. Preiss, Comm. Math. Phys. 103 (1986) 491.

[8] T. Balaban, Comm. Math. Phys. 116 (1988) 1.

[9] T. Balaban, Comm. Math. Phys. 119 (1988) 243.

[10] T. Balaban, Comm. Math. Phys. 122 (1989) 175.

[11] T. Balaban, Comm. Math. Phys. 122 (1989) 355.

[12] K. Osterwalder, R. Schrader, Comm. Math. Phys. 42 (1975) 281.

[13] K. Osterwalder, E. Seiler, Ann. Phys. 110 (1978) 440.