Irrelevant Operators, Anisotropy Bounds, and Operator Insertions in Balaban's Renormalization Group for Four-Dimensional $SU(N)$ Lattice Yang–Mills Theory: Symanzik Classification and Quantitative Irrelevance of $O(4)$-Breaking Operators*

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Abstract

We classify gauge-invariant local lattice operators of classical dimension 6 on the four-dimensional hypercubic lattice into $O(4)$-invariant, hypercubic-invariant but $O(4)$-breaking (anisotropic), and on-shell-redundant components, following the Symanzik improvement programme and the on-shell technique of Lüscher–Weisz. The anisotropic sector is *one-dimensional* (Theorem 3.6, Proposition 3.7: uniqueness of the hypercubic harmonic) — a purely representation-theoretic fact, machine-verified in the companion suite together with the classical Symanzik $a^2/12$ anisotropic term of the Wilson plaquette itself. Inside Balaban's renormalization group framework (small-field regime, $g_k \leq \gamma_0$, $k \leq k^*$ — the window bookkeeping of the audited chain), we extract the anisotropic projection of the effective action via local Taylor (jet) expansion of polymer activities and prove the quadratic bound $|c^{(k)}_{6,\text{aniso}}| \leq C a_k^2$, uniformly in lattice spacing η, physical volume, and RG step k within the window — *conditionally on the structural package*, whose audited status is now attached (traceable per 2602.0069 v2; β_{LF} dichotomy for the large-field remainder). We further prove an insertion integrability estimate for connected correlators with one anisotropic insertion; *Version 3 corrects its status*: v1–v2 called it unconditional, resting on an imported clustering/mass-gap bound (Theorem 4.5, from the unaudited companion [1]) claimed uniform in η and L_{phys} — per the audited program (2602.0053/0054 v2) such a gap is conditional

*Version 3. v1–v2 imported their entire §4 input block (inductive representation, polymer bounds, coupling control, analyticity, and — critically — the exponential clustering/mass-gap bound, Theorem 4.5, "with $m > 0$ and C independent of η and L_{phys}") from the companion [1], which is not yet audited and whose uniform unconditional gap contradicts the audited program: per 2602.0053/0054 v2 the transfer/clustering gap is *conditional* on (H-DOB-blk)+(H-P0) and windowed. Accordingly Remark 1.1's conclusion "Theorem 6.6 is unconditional" is withdrawn: the insertion integrability is conditional on the clustering input. v2 also shipped a broken citation ("the interface with [?]", §1.3). v3 retags the import block to the audited suppliers (2602.0069 v2 for the traceable structural parts, with the β_{LF} dichotomy; 2602.0054 v2 for the gap, conditional), fixes the citation, flags the companions [1]/[2] as unaudited with identifiers pending, and states explicitly the interface this paper serves: it supplies — now conditionally — the operator-classification input of 2602.0063 v3, Proposition 5.3/Remark 5.4 (continuum $SO(4)$ restoration). The paper's own core (the W_4-vs-$O(4)$ classification with its one-dimensional anisotropic sector, the Symanzik jet extraction, and the quadratic bound $|c^{(k)}_{6,\text{aniso}}| \leq C a_k^2$ in the already-windowed small-field regime) survives and is machine-verified. See the Note added in v3 (Section 8).

on (H-DOB-blk)+(H-P0) and windowed, so Theorem 6.6 is conditional on that input. Combined with the rotational Ward identity of the companion [2] (unaudited, identifier pending), the $O(4)$-breaking distribution tested against Schwartz functions is $O(\eta^2|\log((\Lambda_{\rm YM}\eta)^{-1})|)$ and vanishes as $\eta \to 0$ — under the same conditional load. This paper thereby supplies, conditionally, the operator-classification input required by 2602.0063 v3 (Proposition 5.3/Remark 5.4) for continuum $SO(4)$ restoration. All verifiable content (representation counts, the unique hypercubic harmonic, the plaquette's own $a^2/12$ anisotropy, Cauchy-jet mechanics, the $a_k^2/\log$ bookkeeping, and the clustering-to-integrability conversion) is adjudicated in a companion suite.

Contents

1 Introduction

1.1 Motivation

Continuum $SO(4)$ invariance must be recovered from a hypercubic lattice; the obstruction lives in the $O(4)$-breaking part of the dimension-6 sector of the effective action. This paper classifies that sector, bounds its coefficient along Balaban's RG flow, and controls its insertions.

1.2 Main Results

(I) *Classification* (Theorem 3.6): the on-shell quotient of gauge-invariant, W_4-scalar dimension-6 operators has a *one-dimensional* anisotropic sector. (II) *Anisotropy coefficient bound* (Theorem 5.4): $|c^{(k)}_{6,\rm aniso}| \le C a_k^2$ for $k \le k^*$, $g_k \le \gamma_0$, conditional on the structural package (audited statuses attached). (III) *Insertion integrability* (Theorem 6.1): conditional on the clustering input (v3 correction).

Remark 1.1 (Dependence on the mass gap input; v3 rewrite of v1–v2's Remark 1.1)**.** *The insertion estimate uses (i) ultraviolet locality from the polymer expansion [traceable, 2602.0069 v2]* *and (ii) exponential clustering [Theorem 4.5]. v1–v2 concluded "Therefore Theorem 6.6 is unconditional." Withdrawn: input (ii) is a uniform clustering bound imported from the unaudited companion [1]; per the audited program the corresponding gap is* conditional *on (H-DOB-blk)+(H-P0) and windowed (2602.0053/0054 v2). Theorem 6.1 carries that load.*

1.3 Organisation

§2: operators and classical dimension. §3: classification. §4: imported framework with audited statuses. §5: jet extraction and the a_k^2 bound. §6: insertions. §7: interface with 2602.0063 v3 (v2 printed "the interface with [?]"; fixed).

2 Lattice Operators and Classical Dimension

Definition 2.1 (Local lattice operator)**.** *Gauge-invariant function of links in a bounded stencil; as in v2.*

Definition 2.2 (Classical dimension)**.** *Via the slow-field expansion $U = e^{\eta A}$; as in v2.*

The plaquette expansion (as in v2, §2.3) gives

$$1 - \tfrac{1}{N}\operatorname{Re}\operatorname{tr} U_{p_{\mu\nu}} = \tfrac{\eta^4}{2N}|F_{\mu\nu}|^2 + \tfrac{\eta^6}{24N}\left(|D_\mu F_{\mu\nu}|^2 + |D_\nu F_{\mu\nu}|^2\right) + O(\eta^8),$$

so the Wilson action itself carries the classical anisotropic dimension-6 term with the Symanzik coefficient ($a^2/12$ per plaquette pair; suite check 3 verifies this numerically in an exact abelian background).

Definition 2.3 (Dimension-6 operator space $\mathcal{O}_6^{W_4}$)**.** *As in v2.*

3 W_4 versus $O(4)$: Classification

Definition 3.1 (On-shell equivalence)**.** *As in v2 (equality modulo equations of motion and total derivatives).*

Proposition 3.2 (Lüscher–Weisz spectrum-conserving transformation)**.** *As in v2, by citation to Lüscher–Weisz (1985).*

Definition 3.3 (Anisotropic projection)**.** *As in v2: the linear functional on $\mathcal{O}_6^{W_4}$ vanishing on the $O(4)$-invariant and redundant parts.*

Definition 3.4 (Canonical anisotropic operator)**.** $\mathcal{O}_{\mathrm{aniso}} := \sum_{\mu\nu}\operatorname{tr}(D_\mu F_{\mu\nu} D_\mu F_{\mu\nu}) -$ *($O(4)$-invariant completion), normalized by* $\mathrm{Proj}_{\mathrm{aniso}}(\mathcal{O}_{\mathrm{aniso}}) = 1$.

Remark 3.5. *The normalization is a convention; all bounds are stated for the normalized coefficient. (As in v2.)*

Theorem 3.6 (Classification of anisotropic operators)**.** *The on-shell quotient $\mathcal{O}_6^{W_4}/(\mathcal{O}_6^{O(4)} + \mathcal{O}_{6,\mathrm{EOM}})$ is* one-dimensional*, spanned by the class of $\mathcal{O}_{\mathrm{aniso}}$. (As in v2; representation-theoretic; machine-verified: suite checks 1–2.)*

Proposition 3.7 (Uniqueness of the hypercubic harmonic)**.** *The space of W_4-invariant quartic forms on $\mathbb{R}^4$ is two-dimensional,* $\mathrm{span}\{(\sum_\mu x_\mu^2)^2,\ \sum_\mu x_\mu^4\}$*; the $O(4)$-invariants are one-dimensional; the quotient is one-dimensional, spanned by the cubic harmonic $\sum_\mu x_\mu^4 - \frac{3}{6}(\sum_\mu x_\mu^2)^2$. (Machine-verified by exact group-averaging over all 384 elements of W_4: suite check 1.)*

4 Balaban's Framework: Imported Results — v3: audited statuses

Theorem 4.1 (Balaban's inductive representation)**.** *As in v2, for $g_k \in (0, \gamma]$, $k \le K \le k^*$.* (v3) *Status: traceable per 2602.0069 v2 (transcription-conditional); the window $k \le k^*$ is the audited flow window (2602.0051 v2), whose compatibility with continuum trajectories is Lemma 1.4 of 2602.0063 v3.*

Proposition 4.2 (Polymer bounds)**.** *As in v2.* (v3) *Traceable (2602.0069 v2, Axiom A1 route).*

Proposition 4.3 (Coupling control)**.** *As in v2.* (v3) *Traceable; correct AF direction (2602.0051 v2).*

Proposition 4.4 (Analyticity in the small-field domain)**.** *As in v2: activities analytic with radius $R_k > 0$ depending only on (γ, L).* (v3) *Traceable (2602.0069 v2, Definition 4.4 route); the Cauchy-jet mechanics are machine-verified there and here (suite check 4).*

Theorem 4.5 (Clustering bound — v3: CONDITIONAL)**.** *For gauge-invariant local O with $\|O\|_\infty \le 1$: $|\mathrm{Cov}_{\mu_\eta}(O(0), O(x))| \le Ce^{-m|x|/a_*}$.* (v1–v2 imported this from the unaudited companion [1] with "$m > 0$ and C independent of η and L_{phys}". Per the audited program, no companion delivers such a bound unconditionally: the transfer/clustering gap is conditional on (H-DOB-blk)+(H-P0) and windowed (2602.0054 v2; DLR route 2602.0053 v2). v3 treats this as a *conditional input*: either [1] survives its own audit, or the bound is consumed with the (H-DOB-blk)+(H-P0)+window load.)

5 Symanzik Decomposition of the Effective Action

Lemma 5.1 (Polymer Taylor expansion)**.** *Jets of order Δ of small-field activities are bounded by $C_\Delta R_k^{-\Delta}$ times the activity norm (Cauchy estimates on the analyticity domain). (As in v2; mechanics verified, suite check 4.)*

Definition 5.2 (Local jet extraction and anisotropic coefficient)**.** $c^{(k)}_{6,\mathrm{aniso}} := \mathrm{Proj}_{\mathrm{aniso}}(J_{k,6} S^{\mathrm{eff}}_k)$*; as in v2.*

Lemma 5.3 (Large-field remainder)**.** *Bounded by $C_\Delta R_k^{-\Delta} e^{-p_0(g_k)}$ per polymer.* (v3) *The summed large-field contribution inherits the β_{LF} dichotomy / (H-θ) statuses of 2602.0069 v2 when uniformity over all scales is invoked; within the window at fixed k the per-scale statement stands.*

Theorem 5.4 (Anisotropy coefficient bound)**.** *For each $N \ge 2$ there is $C = C(N, \gamma_0, L)$ such that for all $k \le k^*$ with $g_j \le \gamma_0$ ($j \le k$):*

$$|c^{(k)}_{6,\mathrm{aniso}}| \;\le\; C\, a_k^2,$$

uniformly in η, L_{phys} and k within the window. (v3) *Conditional on the structural package statuses of §4; the a_k^2 mechanism itself (dimension counting + jets) is unconditional given analyticity and is verified in the suite (checks 3–5).*

6 Operator Insertions

Intermediate lemmas as in v2 (insertion setup, connected-correlator decomposition, summation over positions).

Theorem 6.1 (Insertion integrability — v3: conditional)**.** *Under the clustering input of Theorem 4.5, connected correlators with one insertion of $\mathcal{O}_{\text{aniso}}$ are summable over the insertion position, with constant $O((a_*/m)^4)$: $\sum_x |\text{Cov}(O(0), \mathcal{O}_{\text{aniso}}(x))| \le C(a_*/m)^4$.* (v1–v2: "unconditional"; withdrawn per Remark 1.1. The clustering→integrability conversion itself is exact arithmetic: suite check 6.)

7 Interface: continuum $SO(4)$ restoration

Combining Theorems 5.4 and 6.1 with the rotational Ward identity of the companion [2] (unaudited; identifier pending), the $O(4)$-breaking distribution tested against Schwartz functions is

$$O\big(\eta^2 \,|\log((\Lambda_{\text{YM}}\eta)^{-1})|\big) \underset{\eta\to 0}{\longrightarrow} 0$$

(suite check 5 verifies the bookkeeping: the a_k^2 coefficient beats the logarithmic coupling growth). **(v3)** This is precisely the operator-classification input deferred in 2602.0063 v3 (Proposition 5.3, Remark 5.4) for $SO(4)$ restoration of renormalized Schwinger functions: this paper supplies it *conditionally* on (i) the structural package statuses, (ii) the clustering input (Theorem 4.5), and (iii) the Ward-identity companion [2] surviving its own audit.

8 Note added in v3

Findings of the series audit applied to this paper's v2:

- **F-MG.** The §4 import block came from the unaudited companion [1], including a clustering/mass-gap bound claimed uniform in η and L_{phys}. No audited companion delivers this unconditionally (2602.0053/0054 v2: conditional on (H-DOB-blk)+(H-P0), windowed). Theorem 4.5 is retagged as a conditional input and Remark 1.1's "Theorem 6.6 is unconditional" is withdrawn (Remark 1.1).
- **F-REF.** v2 printed "the interface with [?]" (§1.3); fixed — the interface is with 2602.0063 v3, now stated explicitly (§7). Companions [1] (clustering) and [2] (Ward identity) are flagged unaudited, identifiers pending; both are natural next audit targets.
- **What survives, machine-verified:** the classification (Theorem 3.6; quotient dimension 1 by exact W_4 group-averaging, 384 elements), the unique hypercubic harmonic (Proposition 3.7), the Wilson plaquette's own Symanzik $a^2/12$ anisotropic term (exact abelian background), the Cauchy-jet mechanics, the a_k^2-vs-log bookkeeping of the breaking distribution, and the clustering→integrability conversion arithmetic. [Suite checks 1–6]
- **Statuses attached** to the import block (§4): traceable parts per 2602.0069 v2; window per 2602.0051 v2/2602.0063 v3; large-field load per the β_{LF}/(H-θ) adjudications. The paper's role in the audited program is now explicit: conditional supplier of the $SO(4)$-restoration input of 2602.0063 v3.

Suite: `verify_2602_0087.py` (6 checks) at `https://github.com/lluiseriksson/THE-ERIKSSON-PROGRAMME`, directory `verification/2602-0087/`.

References

[1] L. Eriksson, *Exponential clustering and mass gap for four-dimensional $SU(N)$ lattice Yang–Mills at weak coupling* (companion; UNAUDITED, identifier pending). [Source of v2's §4 import block; to be read with the (H-DOB-blk)+(H-P0)+window load of 2602.0053/0054 v2 until audited.]

[2] L. Eriksson, *Rotational Ward identity for lattice Yang–Mills in Balaban's renormalization group* (companion; UNAUDITED, identifier pending).

[3] L. Eriksson, ai.viXra:2602.0069 (v2), 2026. [Structural package: traceability, β_{LF} dichotomy, (H-θ).]

[4] L. Eriksson, ai.viXra:2602.0054 (v2), 2026; and ai.viXra:2602.0053 (v2), 2026. [Conditional transfer/clustering gap: (H-DOB-blk)+(H-P0), windowed.]

[5] L. Eriksson, ai.viXra:2602.0063 (v3), 2026. [Consumer of this paper's output: Proposition 5.3/Remark 5.4, $SO(4)$ restoration.]

[6] L. Eriksson, ai.viXra:2602.0041 (v3); 2602.0046; 2602.0051–0057 (v2); 2602.0070/0072/0073/0077 (v2), 2026. [The audited program.]

[7] M. Lüscher, P. Weisz, *On-shell improved lattice gauge theories*, Comm. Math. Phys. **97** (1985), 59–77.

[8] K. Symanzik, *Continuum limit and improved action in lattice theories I, II*, Nucl. Phys. B **226** (1983), 187–227.

[9] T. Bałaban, Comm. Math. Phys. **95–122** (1984–1989).