On the Invariance of Planck Scale Under Special Relativity and the Geometry of Quantum Space

Moninder Singh Modgil[1] and Dnyandeo Dattatray Patil[2]

[1]PhD Physics, IIT Kanpur, India, B.Tech. (Hons.) Aeronautical Engineering, IIT Kharagpur, India, Cosmos Research Labs, Centre for Ontological Science, Meta Quanta Physics and Omega Singularity. email: msmodgil@gmail.com

[2]Electrical and AI Engineering, Cosmos Research Labs. email: cosmoslabsresearch@gmail.com

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Abstract

This work develops a unified and conservative framework for reconciling Planck-scale physics with Special Relativity by shifting the foundational emphasis from symmetry modification to observer admissibility. We demonstrate that invariant Planck scales can coexist with exact local Lorentz invariance when Planck length and Planck time are interpreted as operational lower bounds on spatial and temporal resolution, rather than as ontological spacetime discreteness. Special Relativity is reformulated operationally on curved spacetime through a generalized relativistic factor γ_g, allowing a precise treatment of relativistic kinematics in black-hole, cosmological, and rotating spacetimes without modifying Lorentz transformations or dispersion relations.

We show that event horizons, cosmological expansion, and global rotation generate kinematic phase boundaries that restrict the physical realizability of observers while leaving local inertial physics intact. Planck-scale structure near black-hole horizons is incorporated through geometric regularization rather than symmetry breaking, and black-hole thermodynamics is recovered entirely from local Special Relativity combined with spacetime geometry. The framework further incorporates invariant global bounds on mass, time, and length, leading to a classification theorem for physically admissible observers across all scales.

Information-theoretic limits on entropy, computation, and information recovery are derived as kinematic consequences of admissibility rather than as fundamental postulates. Apparent paradoxes in black-hole complementarity, trans-Planckian physics, and infinite boosts are shown to arise from implicitly assuming inadmissible observers. The resulting picture preserves the empirical successes of Special and General Relativity while providing a unified, observer-centered principle that regulates both ultraviolet and infrared extremes without invoking Lorentz violation, deformed symmetries, or holographic reduction of degrees of freedom.

1 Introduction

The reconciliation of Planck-scale physics with Special Relativity remains one of the central conceptual challenges in fundamental physics. On the one hand, Special Relativity has been confirmed to extraordinary precision and rests on a clear operational foundation rooted in inertial observers and Lorentz invariance. On the other hand, the appearance of invariant Planck scales constructed from $\hbar$, G, and c suggests the existence of fundamental limits on localization, energy density, and temporal resolution. The tension between these two facts has motivated a wide range of proposals, including modified dispersion relations, deformed Lorentz symmetries, spacetime discreteness, and holographic reductions of degrees of freedom. Despite their diversity, many such approaches share a common feature: they modify the kinematical structure of relativity in order to accommodate Planck-scale effects.

In this work a different route is pursued. Rather than modifying Lorentz symmetry or postulating new microscopic dynamics, we ask a more basic question: which observers, frames, and measurements are physically realizable once both local spacetime geometry and global structure are taken into account? The central thesis of this paper is that Planck-scale limits can coexist with exact local Lorentz invariance when they are interpreted as operational bounds on admissible observers, rather than as indicators of symmetry breaking or fundamental spacetime discreteness.

A key ingredient of the analysis is an operational formulation of Special Relativity on curved spacetime. While General Relativity provides the dynamical description of spacetime geometry, many physical processes of interest involve kinematics measured by local observers embedded in curved backgrounds. By generalizing the relativistic factor to include explicit dependence on the spacetime metric, relativistic kinematics can be analyzed consistently in black-hole, cosmological, and rotating spacetimes without altering Lorentz transformations in local inertial frames. This approach allows horizon physics, gravitational redshift, and relativistic motion to be treated within a unified kinematic framework.

Within this setting, Planck length and Planck time emerge as invariant lower bounds on spatial and temporal resolution. These bounds arise from the interplay between quantum localization and gravitational backreaction and are shown to restrict what admissible observers can operationally measure. Importantly, this interpretation is asymmetric: while Planck length and time act as lower bounds on resolution, the Planck mass does not represent a minimum mass scale. Instead, it marks a crossover between regimes dominated by quantum effects and those dominated by gravitational self-interaction. This asymmetry plays a crucial role in formulating consistent admissibility conditions across scales.

The framework is further extended by incorporating invariant global bounds on mass, time, and length. These bounds reflect the finite extent, lifetime, and total mass-energy content of the universe and lead naturally to restrictions on infinite boosts, infinite time dilation, and unbounded information processing. Rather than invalidating Special Relativity, these restrictions limit the physical realizability of certain idealized observers. A

classification theorem for admissible observers is developed, distinguishing locally admissible, globally admissible, and inadmissible classes in a precise geometric manner.

One of the strengths of this observer-centered perspective is its ability to clarify long-standing paradoxes. Event horizons are shown to act as kinematic phase boundaries separating admissible and inadmissible observers, while local inertial physics remains regular. Black-hole thermodynamics and information-theoretic bounds are recovered without invoking Lorentz violation or exotic microscopic structure. Apparent trans-Planckian divergences and information paradoxes are traced to the implicit use of observers that lie outside the admissible domain.

The scope of the analysis spans a wide range of spacetimes, including Schwarzschild, Friedmann–Robertson–Walker, and Gödel geometries, demonstrating that the admissibility principle is robust across curvature, expansion, and rotation. Throughout, Special Relativity is preserved exactly at the local level, and General Relativity is recovered in the appropriate limits. The resulting framework provides a conservative yet conceptually powerful unification of Planck-scale limits, relativistic kinematics on curved spacetime, and global structure.

This paper is organized as follows. The foundational formulation of Special Relativity on curved spacetime and the generalized relativistic factor are introduced first. Planck-scale and global admissibility bounds are then developed and applied to black-hole and cosmological settings. Information-theoretic implications and consistency conditions are analyzed, followed by a comparison with alternative approaches such as deformed symmetries and holography. The paper concludes with a discussion of ontological versus operational interpretations and an outline of open problems and future directions.

Special Relativity (SR), introduced by Einstein in 1905, rests upon two postulates: the invariance of the speed of light and the equivalence of physical laws in all inertial frames. Despite being formulated over a century ago, SR remains a foundational pillar of modern physics. Quantum Gravity (QG) theories, which attempt to unify quantum mechanics and general relativity, introduce new concepts such as the Planck length ℓ_P, defined as

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616 \times 10^{-35} \text{ m}. \tag{1}$$

Here, $\hbar$ is the reduced Planck constant, G is the gravitational constant, and c is the speed of light in vacuum. This length scale often appears as a proposed lower bound on physical length, suggesting a discrete or "pixelated" structure of space at the quantum level [1].

However, this raises a conceptual issue. SR allows arbitrarily high boosts between inertial frames, leading to Lorentz contraction of lengths. If any observer can contract lengths indefinitely, how can there exist a minimum invariant length such as ℓ_P? This question forms the basis of much debate in modern theoretical physics [2].

In this work, we argue in favor of maintaining SR and instead propose that the geometry and topology of space undergo fundamental changes at small scales. We explore multiple frameworks that preserve Lorentz invariance while altering the structure of space to accommodate a minimal length. These include general relativity, quantum field theory in curved spacetime, loop quantum gravity, holographic theories, and noncommutative geometry. Each of these supports the thesis that it is not SR that breaks, but rather that space bends or transforms in a way that is compatible with SR.

2 Special Relativity and Lorentz Invariance

The Lorentz transformations form the core of SR, relating space and time coordinates between inertial frames moving at constant velocity v relative to each other:

$$t' = \gamma \left(t - \frac{vx}{c^2} \right), \quad x' = \gamma(x - vt), \tag{2}$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{3}$$

Under this transformation, the proper length L_0 of an object in its rest frame appears contracted in a moving frame:

$$L = \frac{L_0}{\gamma}. \tag{4}$$

This contraction appears to contradict the idea of an invariant minimum length. However, it is important to note that the Lorentz transformations are valid only in flat spacetime. In curved or quantum spacetime, the assumption of global Lorentz symmetry may no longer hold [3].

Nevertheless, local Lorentz invariance—valid in infinitesimal regions of spacetime—remains a robust principle even in general relativity. This allows SR to persist as a local symmetry, even in the presence of curvature or quantum structure. Therefore, the introduction of a minimal length does not necessarily imply the breakdown of SR [4].

We can also consider the energy-momentum relation in SR:

$$E^2 = p^2 c^2 + m^2 c^4. \tag{5}$$

This equation is invariant under Lorentz transformations. In QG theories, it is often modified at high energies, leading to deformed dispersion relations. However, these modifications can be consistent with a generalized form of Lorentz symmetry, preserving the structure of SR in an appropriate limit [5].

3 Planck-Scale Geometry of Space

At the Planck scale, quantum gravitational effects are expected to dominate, fundamentally altering the smooth, continuous structure of spacetime described by classical theories. The Planck length, given by Equation (1), is believed to set a lower bound on measurable distances. This assertion arises from the interplay of quantum mechanics and general relativity, where attempting to probe spacetime at scales smaller than ℓ_P would require energies so large that they lead to the formation of black holes [22]. Hence, ℓ_P represents a fundamental limit to spatial resolution.

Loop Quantum Gravity (LQG) provides a formalism in which spatial geometry becomes quantized. In this framework, space is represented by a spin network, where geometric operators such as area and volume possess discrete spectra. For example, the area operator in LQG has eigenvalues given by

$$A = 8\pi\gamma\ell_P^2 \sum_i \sqrt{j_i(j_i + 1)}, \tag{6}$$

where γ is the Barbero–Immirzi parameter, ℓ_P is the Planck length, and j_i are spin quantum numbers associated with the edges of the spin network [?]. These discrete eigenvalues suggest that space is composed of indivisible quanta, with minimum measurable area and volume, and imply a granular spatial structure at the Planck scale.

Despite this quantization, LQG maintains local Lorentz invariance in the semiclassical limit. The transition from a discrete quantum geometry to a continuous classical manifold occurs through coarse-graining, where the cumulative behavior of many quantum geometrical entities gives rise to a smooth spacetime background [1]. This supports the idea that Lorentz symmetry, and by extension SR, remains a valid symmetry at observable scales, even if spacetime is discrete at the smallest scales.

Other approaches, such as causal dynamical triangulations (CDT), model spacetime as a sum over piecewise flat geometries. In CDT, spacetime is built from simplices (triangles, tetrahedra, etc.), and a path integral over these geometries is used to compute the quantum gravitational vacuum. Simulations in CDT reveal that a smooth, four-dimensional spacetime emerges at large scales, even though the fundamental constituents are discrete and locally Lorentz-invariant [?].

Noncommutative geometry presents yet another approach to modeling Planck-scale space. In this framework, spacetime coordinates become noncommuting operators:

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \tag{7}$$

where $\theta^{\mu\nu}$ is an antisymmetric tensor that encodes the scale and structure of noncommutativity. This modification induces a minimal length scale in a Lorentz-covariant manner and alters the standard Heisenberg uncertainty principle. It has been shown that the introduction of such a noncommutative structure can preserve deformed versions of Lorentz invariance, such as those seen in κ-Minkowski spacetime [10].

These examples illustrate that the Planck-scale geometry of space can be drastically different from the continuum description, yet they do not necessitate abandoning SR. Instead, the geometric and algebraic structure of space is modified in such a way that Lorentz invariance either remains exactly valid or emerges effectively at large scales.

4 Special Relativity and Event Horizons via the Equivalence Principle

Special Relativity (SR) enters black-hole physics through the equivalence principle, which states that at every spacetime point there exists a locally inertial frame in which the laws of physics reduce to those of SR. This principle implies that even in the vicinity of a black-hole event horizon, the local structure of spacetime is Minkowskian. Mathematically, this is expressed by the condition that at a point p one can choose coordinates such that

$$g_{\mu\nu}(p) = \eta_{\mu\nu}, \quad \partial_\lambda g_{\mu\nu}(p) = 0, \tag{8}$$

where $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ is the Minkowski metric. The Schwarzschild metric,

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{9}$$

possesses an event horizon at $r_s = 2GM/c^2$. Near this radius, one can define locally inertial coordinates in which the metric reduces to SR form. Radial null geodesics satisfy

$$ds^2 = 0, \tag{10}$$

which locally reduces to the SR light-cone condition

$$-c^2 dT^2 + dX^2 = 0. \tag{11}$$

Thus, the event horizon is not a breakdown of SR but a global causal boundary defined by curved spacetime geometry [6, 7].

Time dilation near the horizon can be decomposed into gravitational and kinematic components. The proper time of a stationary observer at radius r is

$$d\tau = dt\sqrt{1 - \frac{2GM}{rc^2}}, \tag{12}$$

while a freely falling observer experiences SR time dilation relative to this stationary frame given by

$$d\tau = dt\sqrt{1 - \frac{v^2}{c^2}}. \tag{13}$$

As $r \to r_s$, the velocity v approaches c, and the dominant contribution to time dilation becomes kinematic, reinforcing the SR interpretation [8].

5 Planck-Scale Structure Near Black-Hole Horizons Without Modifying SR

Quantum gravity suggests that spacetime acquires a discrete structure at the Planck scale $\ell_P = \sqrt{\hbar G/c^3} \approx 1.616 \times 10^{-35}$ m. Near black-hole horizons, the relevant physical scale is set by the surface gravity κ, given by

$$\kappa = \frac{c^4}{4GM}. \tag{14}$$

The near-horizon geometry can be approximated by Rindler spacetime,

$$ds^2 = -\kappa^2 \rho^2 d\tau^2 + d\rho^2 + dy^2 + dz^2, \tag{15}$$

which is locally flat and related to Minkowski spacetime by SR coordinate transformations. The Unruh temperature associated with acceleration a is

$$T = \frac{\hbar a}{2\pi c k_B}, \tag{16}$$

and identifying $a = \kappa$ yields the Hawking temperature

$$T_H = \frac{\hbar c^3}{8\pi GM k_B}. \tag{17}$$

This derivation relies on local SR vacuum structure and does not require Lorentz violation [9].

Planck-scale discreteness can be introduced through geometric quantization rather than symmetry breaking. In loop quantum gravity, area eigenvalues are given by

$$A = 8\pi\gamma\ell_P^2 \sum_i \sqrt{j_i(j_i+1)}, \tag{18}$$

where γ is the Barbero–Immirzi parameter and j_i are spin labels [1]. Near the horizon, the black-hole entropy becomes

$$S = \frac{k_B A}{4\ell_P^2}, \tag{19}$$

indicating that Planck-scale structure modifies geometry while leaving SR intact at the local level.

Noncommutative geometry introduces minimal length through coordinate operators satisfying

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \tag{20}$$

which can preserve Lorentz covariance in a generalized sense [10]. These approaches demonstrate that Planck-scale physics near black-hole horizons can be accommodated by modifying spacetime structure while maintaining SR as the governing local symmetry.

6 Special Relativity on Curved Spacetime and the γ_g Formalism

The special theory of relativity is conventionally formulated on flat Minkowski spacetime with metric $\eta_{\mu\nu}$. However, operationally, special relativistic effects arise from null geodesic structure and light signal exchange. In a curved spacetime with metric $g_{\mu\nu}$, null geodesics remain defined by the invariant condition

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = 0. \tag{21}$$

Following Modgil [11], the relativistic time dilation factor must be generalized by replacing the Minkowski contraction $\eta_{\mu\nu}V^\mu V^\nu$ with its curved spacetime counterpart. For a four-velocity $V^\mu = (1, v, 0, 0)$, the generalized relativistic factor is defined as

$$\gamma_g^{-1} = g_{\mu\nu}V^\mu V^\nu. \tag{22}$$

In flat spacetime this reduces to the standard Lorentz factor

$$\gamma^{-1} = 1 - \frac{v^2}{c^2}. \tag{23}$$

7 Schwarzschild Horizons and Curvature-Modified Kinematics

The Schwarzschild spacetime describing a non-rotating black hole of mass M is given by

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)c^2 dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2 d\Omega^2. \tag{24}$$

The event horizon occurs at

$$r_s = \frac{2GM}{c^2}. \tag{25}$$

For radial motion the generalized relativistic factor becomes

$$\gamma_g = \left(1 - \frac{2GM}{rc^2} - \frac{v^2}{c^2}\right)^{-1/2}. \tag{26}$$

8 Near-Horizon Rindler Limit

Near the horizon define the proper distance

$$\rho = \sqrt{\frac{4GM}{c^2}(r - r_s)}. \tag{27}$$

The metric reduces to

$$ds^2 = -\kappa^2 \rho^2 d\tau^2 + d\rho^2 + r_s^2 d\Omega^2, \tag{28}$$

with surface gravity

$$\kappa = \frac{c^4}{4GM}. \tag{29}$$

9 Planck-Scale Structure at Horizons

The Planck length is

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}}. \tag{30}$$

The Kretschmann scalar is

$$K = \frac{48G^2M^2}{c^4 r^6}. \tag{31}$$

10 Kruskal Extension and γ_g Across the Event Horizon

The Schwarzschild metric exhibits a coordinate singularity at the event horizon $r = r_s = 2GM/c^2$. This coordinate pathology obscures the physical interpretation of relativistic kinematics near the horizon. To analyze horizon crossing using the γ_g formalism, it is necessary to adopt Kruskal–Szekeres coordinates, which render the metric regular across the horizon [12]. Starting from the Schwarzschild line element

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)c^2 dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{32}$$

one defines the tortoise coordinate

$$r_* = r + \frac{2GM}{c^2}\ln\left|\frac{r}{r_s} - 1\right|. \tag{33}$$

Null coordinates $u = t - r_*/c$ and $v = t + r_*/c$ allow construction of Kruskal coordinates

$$U = -e^{-\kappa u}, \quad V = e^{\kappa v}, \tag{34}$$

where the surface gravity $\kappa = c^4/(4GM)$. In these coordinates the metric becomes

$$ds^2 = -\frac{32G^3M^3}{c^6 r}e^{-r/r_s}dU\,dV + r^2 d\Omega^2, \tag{35}$$

which is manifestly regular at $r = r_s$. The generalized relativistic factor

$$\gamma_g^{-1} = g_{\mu\nu}V^\mu V^\nu \tag{36}$$

remains finite for freely falling observers expressed in Kruskal coordinates, demonstrating explicitly that horizon crossing does not entail any breakdown of special relativity [11]. The divergence of γ_g for stationary observers arises from acceleration relative to freely falling frames, not from intrinsic spacetime pathology.

11 Freely Falling Frames and Horizon-Crossing Kinematics

Consider a freely falling observer dropped from rest at infinity. The four-velocity components in Schwarzschild coordinates are

$$V^t = \left(1 - \frac{2GM}{rc^2}\right)^{-1}, \quad V^r = -c\sqrt{\frac{2GM}{r}}, \tag{37}$$

with angular components vanishing. Substituting into the γ_g definition yields

$$\gamma_g^{-1} = g_{tt}(V^t)^2 + g_{rr}(V^r)^2 = 1, \tag{38}$$

showing that proper time equals coordinate time for the infalling observer, consistent with local Minkowski behavior [7]. The event horizon is therefore traversed in finite proper time with no singular kinematic effects.

Relative to a stationary observer, the relative velocity approaches the speed of light as $r \to r_s$, producing

$$\gamma_g = \left(1 - \frac{2GM}{rc^2} - \frac{v^2}{c^2}\right)^{-1/2}, \tag{39}$$

which diverges at the horizon. This divergence encodes gravitational time dilation and redshift entirely within the γ_g framework and preserves the operational meaning of special relativity [8].

12 Stretched Horizon and Planck-Scale Cutoff

The stretched horizon concept replaces the classical event horizon with a timelike surface located a proper distance ℓ_P outside r_s. The proper distance is

$$\Delta\rho = \int_{r_s}^{r_s + \delta r} \left(1 - \frac{2GM}{rc^2}\right)^{-1/2} dr \approx \sqrt{\frac{4GM}{c^2}\delta r}. \tag{40}$$

Setting $\Delta\rho = \ell_P$ gives

$$\delta r \approx \frac{c^2\ell_P^2}{4GM}. \tag{41}$$

At this radius the redshifted local temperature equals the Planck temperature, providing a natural cutoff for semiclassical physics [15]. The γ_g factor evaluated at the stretched horizon remains finite, showing that Planck-scale effects enter through geometric regularization rather than Lorentz violation.

The Hawking temperature

$$T_H = \frac{\hbar c^3}{8\pi GMk_B} \tag{42}$$

arises from SR vacuum fluctuations in the Rindler limit, reinforcing the conclusion that black hole thermodynamics is rooted in special relativity applied locally [14].

13 Consistency with Horizon Entropy and Quantum Geometry

The horizon area is

$$A = 4\pi r_s^2 = \frac{16\pi G^2 M^2}{c^4}. \tag{43}$$

Quantum geometry predicts discrete area spectra

$$A = 8\pi\gamma\ell_P^2 \sum_i \sqrt{j_i(j_i + 1)}, \tag{44}$$

leading to the entropy

$$S = \frac{k_B A}{4\ell_P^2}. \tag{45}$$

The γ_g formalism accommodates this discreteness by embedding SR kinematics in a curved background whose microscopic structure is quantized, preserving Lorentz invariance at the operational level [1]. Thus black hole horizons represent a synthesis of special relativity, spacetime geometry, and quantum structure.

14 Generalized Relativistic Factor γ_g in Friedmann–Robertson–Walker Spacetime

The Friedmann–Robertson–Walker (FRW) spacetime provides the standard geometric description of a homogeneous and isotropic universe. Its line element is given by

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{46}$$

where $a(t)$ is the scale factor and $k = 0, \pm 1$ characterizes the spatial curvature [16]. In the framework of Special Relativity on curved spacetime, the generalized relativistic factor is defined operationally as

$$\gamma_g^{-1} = g_{\mu\nu} V^\mu V^\nu, \tag{47}$$

where V^μ denotes the relative four-velocity between two frames [11].

Consider two observers with relative radial motion described by the coordinate velocity $v = dr/dt$. The corresponding four-velocity components are

$$V^\mu = (1, v, 0, 0). \tag{48}$$

Substituting into the FRW metric yields

$$\gamma_g^{-1} = 1 - \frac{a^2(t)v^2}{1 - kr^2}. \tag{49}$$

Hence,

$$\gamma_g = \frac{1}{\sqrt{1 - \dfrac{a^2(t)v^2}{1 - kr^2}}}. \tag{50}$$

This expression reduces to the standard Lorentz factor in the local Minkowski limit $a(t) \to 1$ and $k \to 0$, as required by the equivalence principle [12].

The condition for imaginary γ_g is given by

$$\frac{a^2(t)v^2}{1 - kr^2} > 1. \tag{51}$$

This inequality can be satisfied for arbitrarily small coordinate velocities when either the scale factor becomes sufficiently large or when the comoving coordinate r approaches the curvature radius for $k = +1$. This demonstrates that imaginary γ_g arises not from superluminal local motion but from the global expansion and geometry of spacetime. The result is consistent with the well-known fact that recession velocities in cosmology can exceed the speed of light without violating Special Relativity, since such velocities are geometric rather than kinematic in origin [17].

To further elucidate the physical meaning, consider the proper radial distance

$$d\ell = a(t)\frac{dr}{\sqrt{1 - kr^2}}. \tag{52}$$

The generalized relativistic factor may then be rewritten as

$$\gamma_g^{-1} = 1 - \left(\frac{1}{c}\frac{d\ell}{dt}\right)^2, \tag{53}$$

showing explicitly that γ_g encodes the comparison between local light cones and the rate at which proper distance changes due to cosmic expansion. The imaginary regime corresponds to the breakdown of global inertial frame comparisons rather than any physical instability.

15 Generalized Relativistic Factor γ_g in Gödel Spacetime

Gödel spacetime represents a homogeneous rotating universe and is described by the metric

$$ds^2 = -\left(dt + e^x dy\right)^2 + dx^2 + \frac{1}{2}e^{2x}dy^2 + dz^2, \tag{54}$$

where the exponential factor encodes global rotation [18]. Unlike FRW spacetime, Gödel spacetime admits closed timelike curves, making it an important testing ground for the limits of relativistic kinematics.

For relative motion along the y-direction with coordinate velocity $v = dy/dt$, the four-velocity is

$$V^\mu = (1, 0, v, 0). \tag{55}$$

Substituting into the metric yields

$$\gamma_g^{-1} = g_{00} + 2g_{0y}v + g_{yy}v^2. \tag{56}$$

Using the explicit metric components,

$$g_{00} = -1, \quad g_{0y} = -e^x, \quad g_{yy} = \frac{1}{2}e^{2x}, \tag{57}$$

one obtains

$$\gamma_g^{-1} = 1 - e^{2x}v^2. \tag{58}$$

Therefore,

$$\gamma_g = \frac{1}{\sqrt{1 - e^{2x}v^2}}. \tag{59}$$

The condition for imaginary γ_g is

$$e^{2x}v^2 > 1. \tag{60}$$

This condition can be satisfied for arbitrarily small velocities at sufficiently large x, reflecting the global rotational structure of Gödel spacetime. The imaginary regime coincides with the region where closed timelike curves appear, demonstrating that imaginary γ_g signals a breakdown of global inertial structure rather than any local violation of Special Relativity [19].

The relativistic mass in curved spacetime is defined as

$$m_g^2 = g_{\mu\nu}P^\mu P^\nu, \tag{61}$$

and becomes imaginary whenever γ_g does. In Gödel spacetime this reflects observer-dependent inertia induced by global rotation, providing a concrete realization of Machian ideas in relativistic cosmology [20].

15.1 Conclusion

The detailed analyses of FRW and Gödel spacetimes demonstrate that imaginary γ_g is a generic feature of curved spacetimes with expansion or rotation. It marks kinematic phase boundaries beyond which classical inertial frames cannot be globally extended. In all cases, local Special Relativity remains exact, and the imaginary regime encodes geometric and observer-dependent effects rather than physical instabilities.

16 Global Admissibility and Invariant Maximum Scales

While local admissibility is governed by geometry, global admissibility is governed by the large-scale structure and topology of spacetime. In [21], it was postulated that spacetime admits invariant maximum values of time T, length L, and mass M, associated with the global properties of the universe. In a de Sitter or closed cosmological model, these quantities may be identified respectively with the lifetime of the universe, its maximum spatial extent, and its total mass.

Standard Special Relativity permits arbitrarily large time dilation, length contraction, and relativistic mass increase,

$$\Delta t' = \gamma \Delta t, \quad \Delta s' = \gamma \Delta s, \quad m' = \gamma m, \tag{62}$$

which contradicts the existence of invariant global maxima. To preserve invariance of T, L, and M, the operational definition of velocity must be modified. The generalized velocity incorporating global constraints is defined as [21]

$$v_{TLM} = v_{\text{Newton}} \left(1 - \frac{\Delta t}{T}\right) \left(1 - \frac{\Delta s}{L}\right) \left(1 - \frac{m}{M}\right). \tag{63}$$

The corresponding generalized relativistic factor becomes

$$\gamma_{TLM} = \frac{1}{\sqrt{1 - \frac{1}{c^2} \left(\frac{\Delta s}{\Delta t}\right)^2 \left(1 - \frac{\Delta t}{T}\right)^2 \left(1 - \frac{\Delta s}{L}\right)^2 \left(1 - \frac{m}{M}\right)^2}}. \tag{64}$$

Global admissibility requires

$$\Delta t \leq T, \quad \Delta s \leq L, \quad m \leq M, \tag{65}$$

ensuring consistency with global spacetime structure.

17 A Classification Theorem for Physically Admissible Observers

The purpose of this section is to formulate a precise and general classification theorem for physically admissible observers, based on the synthesis of Planck-scale limits, Special Relativity formulated on curved spacetime, and the existence of global maximal scales of mass, time, and length. The analysis proceeds without modifying Lorentz symmetry or the local structure of relativistic kinematics, and instead restricts the physically realizable class of observers through geometric and global admissibility conditions.

In relativistic physics, an observer is represented by a timelike worldline equipped with a tetrad that allows operational measurement of spacetime intervals. The admissibility of such an observer is not guaranteed solely by mathematical consistency of the spacetime metric, but must also respect physical limitations arising from both microscopic and macroscopic structure. These limitations enter through invariant bounds on localization and global finiteness, which together constrain the domain of meaningful relativistic transformations.

Local admissibility is governed by the requirement that the observer's worldline remains timelike with respect to the spacetime metric. In curved spacetime, the contraction

of the metric with the relative four-velocity determines whether an observer can consistently define proper time and local inertial measurements [11]. For an observer undergoing relative motion characterized by parameters collectively denoted by $\mathcal{V}$, local admissibility requires

$$\mathcal{A}_{\mathrm{local}}(\mathcal{V}) \equiv g_{\mu\nu} V^{\mu} V^{\nu} > 0. \tag{66}$$

This inequality defines an open region in the space of kinematic configurations. When it is violated, the observer's proper time becomes ill-defined, and the corresponding frame loses physical meaning even though the spacetime geometry itself remains regular [7].

The local admissibility condition may be expressed equivalently in terms of the observer's measurable energy and acceleration scales. For an observer measuring a particle of rest mass m, the measured energy satisfies

$$E = \Gamma(\mathcal{V}) m c^2, \tag{67}$$

where $\Gamma(\mathcal{V})$ denotes the generalized relativistic amplification factor induced by relative motion and curvature. Attempting to probe distances below the Planck length requires energies satisfying

$$E \gtrsim \frac{\hbar c}{\Delta s}, \tag{68}$$

while the associated gravitational radius satisfies

$$r_s \sim \frac{2GE}{c^4}. \tag{69}$$

Equating r_s with Δs yields $\Delta s \sim \ell_P$, demonstrating that the local admissibility region excludes observers whose measurements would require sub-Planckian localization [22]. An analogous argument applies to temporal resolution, excluding observers whose clocks would require intervals shorter than t_P.

Global admissibility introduces independent constraints arising from the finite extent and duration of spacetime. If the universe possesses a finite lifetime T and a finite maximal spatial extent L, then any observer describing a process with characteristic temporal and spatial scales $(\Delta t, \Delta s)$ must satisfy

$$\mathcal{A}_{\mathrm{global}}(\Delta t, \Delta s) \equiv (\Delta t \leq T) \wedge (\Delta s \leq L). \tag{70}$$

Similarly, the existence of a finite total mass-energy content implies an upper bound M on admissible relativistic mass-energy,

$$m_{\mathrm{eff}} \leq M. \tag{71}$$

These conditions are independent of local geometry and reflect global topological and cosmological structure [21].

The combined admissibility condition for an observer is therefore defined as the intersection of local and global admissibility domains,

$$\mathcal{A} = \mathcal{A}_{\mathrm{local}} \cap \mathcal{A}_{\mathrm{global}}. \tag{72}$$

Observers whose kinematic parameters lie outside this intersection are physically inadmissible, even if their worldlines are mathematically well-defined solutions of the spacetime geometry.

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This structure allows a natural classification of observers. Observers satisfying both admissibility conditions everywhere along their worldlines form the class of fully admissible observers. These observers can consistently define local inertial frames, measure proper time, and apply relativistic kinematics without encountering Planck-scale or global inconsistencies. Freely falling observers in regular regions of spacetime belong to this class [12].

Observers that satisfy local admissibility but violate global admissibility form a second class. Such observers can define proper time and local measurements, but their descriptions of processes would exceed global bounds. Examples include observers attempting to define eternal acceleration or infinite time dilation in a universe of finite age. These observers are locally meaningful but globally inadmissible [23].

A third class consists of observers that violate local admissibility. These observers encounter imaginary amplification factors and lose the ability to define proper time or inertial frames. Stationary observers at black-hole horizons or observers attempting rigid rotation beyond critical radii fall into this class. Their inadmissibility arises from local geometric constraints rather than global finiteness [18].

The classification theorem may now be stated formally. An observer $\mathcal{O}$ in a spacetime $(\mathcal{M}, g_{\mu\nu})$ is physically admissible if and only if, for all events along its worldline,

$$g_{\mu\nu} V_{\mathcal{O}}^{\mu} V_{\mathcal{O}}^{\nu} > 0, \tag{73}$$

and for all physical processes described by $\mathcal{O}$,

$$\Delta t_{\mathcal{O}} \leq T, \qquad \Delta s_{\mathcal{O}} \leq L, \qquad m_{\mathcal{O}} \leq M. \tag{74}$$

This theorem establishes that admissibility, rather than symmetry alone, determines the physically meaningful domain of relativistic physics.

The significance of this result is that it preserves the exact local structure of Special Relativity while explaining the emergence of both microscopic and macroscopic bounds. Planck-scale limits arise from the failure of local admissibility, while cosmological bounds arise from the failure of global admissibility. The framework therefore provides a unified and non-perturbative criterion for physical observerhood across all scales [1].

18 Implications for Black-Hole Complementarity and Information

The black-hole information problem arises from the apparent incompatibility between semiclassical gravity and the unitary evolution of quantum states. Hawking radiation appears thermal, suggesting information loss, while quantum mechanics forbids such loss. Black-hole complementarity was proposed to resolve this tension by asserting that no single observer can witness violations of fundamental principles, even though different observers may give mutually incompatible descriptions [24]. Within the framework of observer admissibility developed in this work, complementarity emerges as a direct consequence of kinematic and global constraints on physically realizable observers.

Consider a Schwarzschild black hole of mass M_{BH}. The horizon radius is given by

$$r_h = \frac{2GM_{\mathrm{BH}}}{c^2}. \tag{75}$$

The Hawking temperature associated with this horizon is

$$T_H = \frac{\hbar c^3}{8\pi G k_B M_{\mathrm{BH}}},\tag{76}$$

and the Bekenstein–Hawking entropy is

$$S_{\mathrm{BH}} = \frac{k_B c^3 A}{4\hbar G} = \frac{4\pi k_B G M_{\mathrm{BH}}^2}{\hbar c},\tag{77}$$

where $A = 4\pi r_h^2$ [13, 25]. These relations encode the interplay of gravity, quantum theory, and thermodynamics.

Stationary observers hovering near the horizon experience an unbounded gravitational redshift. The locally measured energy of a quantum of frequency ω_∞ measured at infinity scales as

$$\omega_{\mathrm{loc}} = \frac{\omega_\infty}{\sqrt{1 - \frac{2GM_{\mathrm{BH}}}{rc^2}}}.\tag{78}$$

As $r \to r_h$, ω_{loc} diverges, leading to the trans-Planckian problem. Within the admissibility framework, this divergence signals the breakdown of local admissibility for stationary observers before the horizon is reached. Such observers are therefore physically unrealizable arbitrarily close to the horizon.

Freely falling observers, by contrast, cross the horizon in finite proper time and do not encounter divergent energies. Their worldlines remain admissible, and local inertial physics remains well defined. The distinction between stationary and freely falling observers thus acquires a precise kinematic meaning rooted in admissibility rather than in coordinate pathology.

The information carried by Hawking radiation is detected by asymptotic observers over long timescales. The total evaporation time of the black hole scales as

$$t_{\mathrm{evap}} \sim \frac{5120\pi G^2 M_{\mathrm{BH}}^3}{\hbar c^4},\tag{79}$$

which becomes comparable to cosmological timescales for sufficiently large M_{BH} [26]. Global admissibility requires that this timescale remain bounded by the invariant cosmic time T. Observers attempting to extract information over arbitrarily long durations are therefore globally inadmissible.

Black-hole complementarity asserts that information is both reflected at the horizon and transmitted inward, but never in a way that allows observable cloning. In the admissibility framework, the observers who could potentially witness cloning would need simultaneous access to near-horizon and asymptotic descriptions. Such observers violate either local or global admissibility conditions and are therefore excluded from the physically realizable class. Complementarity thus follows from admissibility rather than from an independent axiom [27].

Recent firewall arguments rely on combining local quantum field theory near the horizon with the assumption that observers can compare early and late Hawking radiation. These arguments implicitly assume observers that are inadmissible under the present framework. The emergence of firewalls can therefore be interpreted as a signal that the assumed observer configuration is physically unrealizable, rather than as evidence for a breakdown of semiclassical gravity at the horizon [29].

In this way, observer admissibility provides a unified resolution of black-hole complementarity and information preservation. Horizons are reinterpreted as kinematic boundaries separating admissible and inadmissible observer classes. Information is preserved globally, while its localization remains observer-dependent within admissible domains. This resolution preserves unitarity, Special Relativity, and semiclassical gravity within their respective domains of validity.

19 Contrast with Doubly Special Relativity, Lorentz Violation, and Holography

The framework developed in this work, based on observer admissibility across microscopic and macroscopic scales, occupies a conceptual position distinct from several influential approaches to Planck-scale physics and gravity. In particular, it differs fundamentally from Doubly Special Relativity, explicit Lorentz-violating theories, and holographic principles, despite superficial similarities in motivation. A careful comparison clarifies both the novelty and the conservative strength of the admissibility-based approach.

Doubly Special Relativity was introduced to reconcile the existence of invariant Planck scales with the relativity principle by modifying the action of Lorentz transformations on energy and momentum space [30,31]. In its prototypical form, DSR postulates a modified dispersion relation of the form

$$E^2 - p^2c^2 = m^2c^4 \left[1 + \eta \left(\frac{E}{E_P} \right) + \mathcal{O} \left(\frac{E^2}{E_P^2} \right) \right], \tag{80}$$

where $E_P = m_P c^2$ is the Planck energy and η is a dimensionless deformation parameter. The modified transformation laws imply nonlinear representations of the Lorentz group, leading to energy-dependent speeds of light,

$$v(E) = \frac{\partial E}{\partial p} \neq c, \tag{81}$$

and to frame-dependent notions of locality.

In contrast, the admissibility framework retains the exact linear Lorentz transformations of Special Relativity in their operational domain. No modification of dispersion relations is required. Instead of deforming the symmetry algebra, the admissibility framework restricts the physically realizable domain of observers and measurements. Planck-scale limits emerge because observers capable of probing trans-Planckian regimes become locally inadmissible before such measurements can be performed. The invariant Planck scales therefore arise as boundaries of physical observability rather than as new invariant arguments of the Lorentz group.

This distinction has direct physical consequences. In DSR, the modified dispersion relation predicts potentially observable time-of-flight delays for high-energy photons,

$$\Delta t \sim \eta \frac{E}{E_P} \frac{L}{c}, \tag{82}$$

which are actively constrained by astrophysical observations [32]. In the admissibility framework, no such effects arise for admissible observers, as local Special Relativity remains exact. Any apparent superluminal or subluminal propagation arises only in descriptions associated with inadmissible frames.

Lorentz-violating theories take a more radical approach by explicitly introducing preferred frames or background tensor fields. A generic Lorentz-violating action contains terms such as

$$\mathcal{L}_{\mathrm{LV}} \supset k^{\mu\nu}\partial_\mu\phi\partial_\nu\phi, \tag{83}$$

where $k^{\mu\nu}$ is a fixed tensor selecting a preferred spacetime direction [33]. Such terms lead to anisotropic propagation, vacuum birefringence, and violations of CPT symmetry. Precision experiments place extremely tight bounds on the coefficients of Lorentz violation [34].

The admissibility framework avoids these issues entirely by preserving exact local Lorentz invariance. No preferred frame is introduced, and no background tensors are required. Apparent violations of Lorentz invariance arise only when one attempts to extend physical descriptions beyond the admissible domain. In this sense, Lorentz symmetry is not broken but delimited by physical realizability.

The holographic principle represents a different conceptual response to Planck-scale physics and gravity. Motivated by black-hole thermodynamics, holography asserts that the number of degrees of freedom in a spatial region scales with its boundary area rather than its volume [27, 35]. This principle is often formalized through entropy bounds,

$$S \leq \frac{k_B c^3 A}{4\hbar G}, \tag{84}$$

and finds explicit realization in the AdS/CFT correspondence [36].

While the admissibility framework is compatible with holographic entropy bounds, it does not require a fundamental reduction of degrees of freedom or a dual boundary description. Instead, entropy bounds arise naturally from global admissibility constraints. The maximal entropy accessible to any observer is limited by both Planck-scale localization bounds and the finite global scales L and T. Observers attempting to encode or retrieve information exceeding these bounds become globally inadmissible.

In this sense, holography can be viewed as an emergent description of admissibility-limited physics rather than as a fundamental principle. The reduction of degrees of freedom reflects the restricted class of admissible observers capable of accessing and comparing information across large spacetime regions. The admissibility framework therefore provides a kinematic underpinning for holographic behavior without requiring a dual non-gravitational theory.

A key conceptual distinction is that both DSR and Lorentz-violating theories modify the structure of spacetime symmetries, while holography modifies the counting of degrees of freedom. The admissibility framework modifies neither. Instead, it modifies the interpretation of physical observability. Symmetries remain exact, degrees of freedom remain locally field-theoretic, but not all formally allowed measurements correspond to physically realizable observations.

This distinction can be summarized by considering the role of infinity. DSR and Lorentz-violating theories regulate ultraviolet infinities by altering kinematics. Holography regulates gravitational entropy by reducing degrees of freedom. The admissibility framework regulates both ultraviolet and infrared infinities by excluding physically unrealizable observers. Infinite boosts, infinite energies, and infinite observation times are mathematical idealizations that lie outside the admissible domain.

The contrast elucidated here highlights the conservative but powerful nature of the admissibility-based approach. It preserves the empirical successes of Special Relativity and quantum field theory while resolving Planck-scale and cosmological paradoxes

through physically motivated restrictions on observerhood. In doing so, it offers a unifying perspective that complements, rather than competes with, existing approaches to quantum gravity.

20 Recovery of Standard Special and General Relativistic Limits

Any framework proposing new organizing principles for relativistic physics must demonstrate that all experimentally verified limits of Special Relativity and General Relativity are recovered in the appropriate regimes. In this section it is shown explicitly that the admissibility-based formulation developed in this work reduces exactly to standard Special Relativity in flat spacetime and to conventional General Relativity in the weak-field and local-inertial limits.

The flat-spacetime limit is obtained when curvature effects vanish and the metric reduces smoothly to the Minkowski form,

$$g_{\mu\nu} \to \eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1). \tag{85}$$

In this limit spacetime intervals reduce to their standard Lorentzian form,

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2, \tag{86}$$

and all admissibility conditions become trivial for timelike worldlines.

Planck-scale constraints decouple when characteristic intervals satisfy

$$\Delta s \gg \ell_P, \qquad \Delta t \gg t_P. \tag{87}$$

Under these conditions localization energies satisfy

$$E \ll E_P, \tag{88}$$

and relativistic kinematics reduces exactly to that of Special Relativity [16].

The recovery of General Relativity follows from the equivalence principle. In sufficiently small neighborhoods of spacetime, the metric may be expanded as

$$g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(R_{\mu\nu\rho\sigma} x^2). \tag{89}$$

For curvature scales satisfying

$$|R_{\mu\nu\rho\sigma}| \ll \ell_P^{-2}, \tag{90}$$

local inertial physics is recovered.

In the weak-field regime the metric components take the form

$$g_{00} \approx 1 + \frac{2\Phi}{c^2}, \tag{91}$$

leading to gravitational time dilation

$$\frac{\Delta\tau}{\Delta t} \approx \sqrt{1 + \frac{2\Phi}{c^2}}, \tag{92}$$

in agreement with experiment [37].

Global admissibility constraints decouple in the limit

$$T, L, M \to \infty, \tag{93}$$

recovering standard relativistic idealizations.

21 Thought Experiments and Operational Tests of Observer Admissibility

The admissibility framework developed in this work is not merely a formal restriction on kinematic variables, but an operational principle governing which measurements, observers, and experimental protocols are physically realizable. This section develops a sequence of idealized thought experiments designed to probe the boundaries of admissibility in concrete relativistic settings. These experiments do not invoke new dynamics or speculative quantum-gravitational effects. Instead, they rely solely on Special Relativity formulated on curved spacetime, Planck-scale localization limits, and global bounds on mass, time, and length.

The first thought experiment concerns extreme Lorentz boosting of a physical clock. Consider an idealized atomic clock of rest-frame period $\Delta\tau_0$ undergoing uniform inertial motion relative to a laboratory frame. In standard Special Relativity, the observed period is

$$\Delta t = \gamma \Delta\tau_0, \tag{94}$$

with $\gamma = (1 - v^2/c^2)^{-1/2}$. Formally, γ may be made arbitrarily large as $v \to c$. However, resolving the ticking of the clock with temporal resolution Δt requires energies satisfying

$$E \gtrsim \frac{\hbar}{\Delta t}. \tag{95}$$

As Δt approaches the Planck time t_P, the corresponding energy approaches the Planck energy E_P. The gravitational radius associated with this energy is

$$r_s \sim \frac{2GE}{c^4}. \tag{96}$$

Requiring that the clock remain localized within its own gravitational radius leads to the inequality

$$\Delta t \gtrsim t_P. \tag{97}$$

Thus the attempt to realize arbitrarily large time dilation fails operationally before Δt can be driven below t_P. The clock remains mathematically definable but physically inadmissible beyond this limit [22].

A second thought experiment considers Lorentz contraction of a rigid rod. In its rest frame the rod has proper length ℓ_0. An observer moving relative to the rod measures a contracted length

$$\ell = \frac{\ell_0}{\gamma}. \tag{98}$$

Formally, ℓ may be made arbitrarily small. Operationally, however, resolving the endpoints of the rod with spatial resolution ℓ requires momenta

$$p \gtrsim \frac{\hbar}{\ell}. \tag{99}$$

The corresponding energy again implies a gravitational radius comparable to ℓ when $\ell \sim \ell_P$. Any observer attempting to measure $\ell < \ell_P$ necessarily introduces gravitational backreaction incompatible with the existence of a localized rod. The rod itself does not disappear, but the observer attempting such a measurement becomes inadmissible.

A third thought experiment involves hovering near a black-hole horizon. Consider a Schwarzschild black hole of mass M_{BH} and horizon radius $r_h = 2GM_{\text{BH}}/c^2$. A stationary observer at radius r measures local energies related to asymptotic energies by

$$E_{\text{loc}} = \frac{E_\infty}{\sqrt{1 - \frac{2GM_{\text{BH}}}{rc^2}}}. \tag{100}$$

As $r \to r_h$, E_{loc} diverges. Maintaining stationarity requires proper acceleration

$$a(r) = \frac{GM_{\text{BH}}}{r^2\sqrt{1 - \frac{2GM_{\text{BH}}}{rc^2}}}, \tag{101}$$

which exceeds any physically realizable value before the horizon is reached. The stationary observer becomes locally inadmissible, while a freely falling observer crossing the horizon remains admissible. The horizon thus marks a boundary in observer admissibility rather than a breakdown of local physics [28].

A fourth thought experiment addresses global time bounds. Consider an observer attempting to define an experiment lasting arbitrarily long proper time. In standard relativity this poses no kinematic difficulty. If, however, spacetime possesses a finite global lifetime T, then any process described by an observer must satisfy

$$\Delta t \leq T. \tag{102}$$

Observers attempting to define measurements extending beyond T are globally inadmissible, even if their local motion remains timelike. This restriction has direct implications for eternal acceleration, infinite computation, and late-time information recovery scenarios [26].

A fifth thought experiment concerns information storage and retrieval. Suppose an observer attempts to store information in a spatial region of size R with total energy E. The Bekenstein bound implies

$$S \leq \frac{2\pi k_B E R}{\hbar c}. \tag{103}$$

Requiring $R \geq \ell_P$ and $E \leq Mc^2$ yields a finite maximum entropy accessible to any admissible observer. Attempts to exceed this bound require either sub-Planckian localization or super-global energies, both of which violate admissibility. Infinite information storage is therefore excluded kinematically rather than dynamically [39].

These thought experiments collectively demonstrate that the admissibility framework provides a unified operational criterion for the breakdown of extreme relativistic idealizations. In each case, the failure arises not from inconsistency of the underlying theory, but from the physical unrealizability of the observer attempting the measurement. Special Relativity and General Relativity remain exact within their domains of admissibility, while Planck-scale and cosmological-scale bounds emerge as limits on observation rather than on law.

22 Information, Entropy, and Global Admissibility Bounds

The interplay between information, entropy, and spacetime structure provides one of the most stringent tests of any proposed extension or reinterpretation of relativistic physics.

In the admissibility framework developed in this work, limits on information storage, processing, and retrieval arise naturally from the same physical principles that constrain observers at Planck and cosmological scales. No modification of quantum mechanics or relativistic dynamics is required. Instead, information-theoretic bounds emerge as kinematic consequences of local and global admissibility.

Information is operationally defined through physical degrees of freedom localized within finite spacetime regions. Consider an admissible observer attempting to encode information within a spatial region of linear size R and total energy E. Quantum mechanics requires that the minimum wavelength associated with this energy satisfies

$$\lambda \gtrsim \frac{\hbar c}{E}. \tag{104}$$

At the same time, gravitational consistency requires that the region avoid collapse into a black hole, imposing

$$R > r_s = \frac{2GE}{c^4}. \tag{105}$$

Combining these inequalities yields

$$R \gtrsim \sqrt{\frac{2G\hbar}{c^3}} \sim \ell_P, \tag{106}$$

demonstrating that Planck-scale localization bounds restrict the spatial density of information for any admissible observer [22].

The entropy S associated with the information stored in such a region is bounded by the Bekenstein bound,

$$S \leq \frac{2\pi k_B E R}{\hbar c}. \tag{107}$$

Substituting the admissibility conditions $R \geq \ell_P$ and $E \leq Mc^2$, where M denotes the global maximal admissible mass-energy, yields

$$S \leq \frac{2\pi k_B M c \ell_P}{\hbar}. \tag{108}$$

This bound is finite and observer-independent, reflecting the combined effect of microscopic and macroscopic admissibility constraints [39].

Temporal aspects of information processing are similarly constrained. Any computation or information retrieval process requires a minimum time per logical operation. The Margolus–Levitin bound implies that a system of energy E cannot perform more than

$$\nu_{\max} = \frac{2E}{\pi\hbar} \tag{109}$$

distinct operations per unit time [40]. For an admissible observer, the total number of operations N_{ops} that can be performed over a duration Δt satisfies

$$N_{\mathrm{ops}} \leq \frac{2E\Delta t}{\pi\hbar}. \tag{110}$$

Imposing the global time bound $\Delta t \leq T$ and energy bound $E \leq Mc^2$ yields

$$N_{\mathrm{ops}} \leq \frac{2Mc^2 T}{\pi\hbar}, \tag{111}$$

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demonstrating that infinite computation is excluded kinematically by admissibility rather than dynamically by dissipation or decoherence.

These bounds acquire particular significance in gravitational settings. For a black hole of mass M_{BH}, the Bekenstein–Hawking entropy is

$$S_{\mathrm{BH}} = \frac{4\pi k_B G M_{\mathrm{BH}}^2}{\hbar c}. \tag{112}$$

Requiring $M_{\mathrm{BH}} \leq M$ ensures that the maximal entropy associated with any black hole accessible to an admissible observer remains finite and consistent with the global entropy budget of the universe [13]. The evaporation time of such a black hole,

$$t_{\mathrm{evap}} \sim \frac{5120\pi G^2 M_{\mathrm{BH}}^3}{\hbar c^4}, \tag{113}$$

must also satisfy $t_{\mathrm{evap}} \leq T$ for the evaporation process to be globally admissible.

The admissibility framework therefore provides a unified interpretation of entropy bounds that is independent of specific microscopic models of quantum gravity. Entropy limits arise not because spacetime possesses a reduced number of fundamental degrees of freedom, but because physically realizable observers cannot access arbitrarily fine localization, arbitrarily large energies, or arbitrarily long times. This perspective is compatible with holographic entropy bounds while not requiring holography as a fundamental principle [27, 35].

Information loss paradoxes are similarly reinterpreted. Apparent violations of unitarity arise when descriptions implicitly assume observers capable of tracking correlations across trans-Planckian scales or cosmological times. Such observers violate admissibility conditions and are therefore excluded. For admissible observers, information is conserved within the accessible domain, and entropy bounds ensure consistency between local measurements and global constraints [26].

In summary, the admissibility framework yields finite, observer-independent bounds on information storage, entropy, and computation by combining Planck-scale localization limits with global bounds on mass, time, and length. These results demonstrate that information-theoretic constraints are not additional postulates but inevitable consequences of relativistic kinematics and spacetime structure when physical realizability is taken seriously.

23   Internal Consistency and No-Go Results

Any framework that restricts the class of physically realizable observers must demonstrate not only empirical adequacy but also strong internal consistency. In particular, the admissibility principle introduced in this work must be shown to be free of hidden contradictions, must preserve causality, and must not secretly reintroduce symmetry breaking or preferred frames under a different guise. This section establishes these properties through a series of no-go results that clarify what the framework does not permit, thereby strengthening the case for its minimal and conservative nature.

The first consistency requirement concerns causal structure. In relativistic physics, causality is encoded in the light-cone structure determined by the spacetime metric. An admissible observer is defined by a timelike worldline satisfying

$$g_{\mu\nu}V^\mu V^\nu > 0, \tag{114}$$

everywhere along its trajectory. Because admissibility explicitly excludes null or spacelike trajectories, all admissible observers are confined to the interior of the local light cone. Consequently, no admissible observer can transmit or receive signals outside the causal structure defined by the metric. This establishes the no-go result that admissibility cannot be used to engineer superluminal signaling or causal loops, even in spacetimes with nontrivial global structure [23].

A second consistency requirement concerns agreement between admissible observers. One might worry that restricting observer classes could lead to incompatible physical descriptions that cannot be reconciled even in principle. To address this, consider two admissible observers $\mathcal{O}_1$ and $\mathcal{O}_2$ whose worldlines intersect at an event p. At p, both observers admit well-defined local inertial frames. The equivalence principle ensures that physical laws reduce locally to those of Special Relativity, and standard Lorentz transformations relate the measurements of $\mathcal{O}_1$ and $\mathcal{O}_2$. Since admissibility does not alter these transformations, all locally measurable quantities at p agree up to standard relativistic effects. No-go results therefore exclude observer-dependent physical laws within the admissible class.

The stability of admissibility under perturbations is another crucial requirement. Physical observers are never idealized; they experience small accelerations, noise, and fluctuations. If admissibility were unstable, infinitesimal perturbations could render an otherwise physical observer inadmissible, undermining predictive power. Let an admissible observer have four-velocity V^μ satisfying the admissibility condition with margin $\epsilon > 0$,

$$g_{\mu\nu}V^\mu V^\nu = \epsilon. \tag{115}$$

Small perturbations δV^μ satisfying

$$|g_{\mu\nu}V^\mu \delta V^\nu| \ll \epsilon \tag{116}$$

leave the observer admissible. This establishes that admissibility defines an open set in the space of observer configurations, and that physically realistic perturbations do not induce pathological behavior. Admissibility therefore possesses the necessary robustness to describe real observers [7].

A further no-go result concerns the possibility that admissibility might be equivalent to a hidden violation of Lorentz invariance. This possibility is excluded by construction. Lorentz symmetry governs the transformation laws between local inertial frames. Admissibility places restrictions only on which frames are physically realizable, not on how frames transform when they exist. All admissible observers are related by standard Lorentz transformations in their domains of overlap. There is therefore no preferred frame, no anisotropy, and no modification of dispersion relations within the admissible domain. This distinguishes admissibility sharply from both explicit Lorentz-violating models and deformed symmetry frameworks [34].

One may also ask whether admissibility introduces new paradoxes of its own. In particular, could there exist situations in which two observers are individually admissible but jointly inconsistent, for example by allowing information duplication or causal contradiction when their descriptions are combined. Such scenarios are excluded by a global no-go theorem. Any attempt to combine descriptions from different observers requires a common operational domain. If such a domain exists, then both observers are admissible there and their descriptions are related by standard relativistic transformations. If no such domain exists, then the observers cannot operationally compare results, and no con-

tradiction arises. Apparent paradoxes therefore signal attempts to combine descriptions across inadmissible domains rather than genuine physical inconsistency [27].

Finally, it is important to demonstrate that admissibility does not trivialize physics by excluding too many observers. In the low-energy, weak-field, and large-scale limits, admissibility constraints decouple smoothly, as shown earlier. The set of admissible observers becomes dense in the space of mathematically allowed observers, and standard relativistic idealizations are recovered. The framework therefore excludes only those observers whose physical realization would require infinite energy, infinite acceleration, sub-Planckian localization, or infinite duration, all of which are already physically implausible even in conventional theories [16].

Taken together, these no-go results establish that the admissibility framework is internally consistent, causally well behaved, stable under perturbations, and conservative with respect to established symmetries. The framework restricts the domain of physical observability without modifying the underlying laws of physics. This internal coherence is essential for any proposal that aims to unify Planck-scale limits, curved-spacetime kinematics, and global cosmological bounds within a single relativistic framework.

24 Ontology versus Operational Meaning of Spacetime and Planck Units

A recurring source of confusion in discussions of Planck-scale physics and relativistic limits arises from an implicit conflation of ontology with operational meaning. The admissibility framework developed in this work makes it essential to disentangle these notions carefully. The framework does not assert that spacetime is fundamentally discrete, nor does it assert that spacetime is merely a mathematical bookkeeping device. Instead, it identifies precise operational limits on what can be physically measured, prepared, or compared by admissible observers, while remaining agnostic about the ultimate microscopic ontology of spacetime.

The distinction may be sharpened by considering the role of Planck units. The Planck length ℓ_P and Planck time t_P arise from the combination of fundamental constants,

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}}, \qquad t_P = \sqrt{\frac{\hbar G}{c^5}}. \tag{117}$$

Within the admissibility framework, these quantities do not label the size of fundamental spacetime "atoms." Instead, they characterize the scale at which any attempt at localization or temporal resolution necessarily triggers gravitational backreaction incompatible with the continued existence of a localized observer. The statement that ℓ_P is a minimum length is therefore operational rather than ontological. It asserts that no admissible observer can resolve distances $\Delta s < \ell_P$, not that shorter distances do not exist in any ontic sense [22].

This operational interpretation becomes especially clear when contrasted with the Planck mass,

$$m_P = \sqrt{\frac{\hbar c}{G}}. \tag{118}$$

Unlike ℓ_P and t_P, the Planck mass does not represent a minimum mass scale. Elementary particles exist with masses many orders of magnitude below m_P. Instead, m_P marks

a crossover between regimes in which quantum effects dominate and regimes in which gravitational self-interaction becomes unavoidable. The asymmetry between (ℓ_P, t_P) and m_P underscores the importance of operational interpretation: admissibility imposes lower bounds on spacetime resolution, but only upper bounds on physically realizable mass-energy for observers.

From this perspective, spacetime geometry retains its classical meaning as a differentiable manifold equipped with a metric, as long as observers remain admissible. The metric encodes causal structure and local inertial properties, and no assumption of discreteness is required. The admissibility framework therefore differs fundamentally from approaches that postulate spacetime discreteness as an ontological necessity. Instead, it asserts that any hypothetical discreteness lies beyond operational access and is therefore irrelevant to physical predictions at the level of admissible observers.

Operational meaning also clarifies the status of curvature singularities. In classical General Relativity, singularities signal the breakdown of the manifold description. In the admissibility framework, however, many singular behaviors arise because observers attempting to probe arbitrarily high curvature become inadmissible before reaching such regimes. The question of whether spacetime is singular at a fundamental level is thus separated from the question of whether singularities are physically observable. Admissibility resolves this tension by restricting observability without altering the underlying equations of motion [23].

The same distinction applies to cosmological questions. The existence of global bounds on time T, length L, and mass-energy M does not imply that spacetime is finitely extended in an ontological sense. Rather, it reflects the fact that no admissible observer can operationally access arbitrarily large durations, distances, or energies. Statements about the global structure of spacetime must therefore be interpreted as statements about the limits of physical description, not as absolute claims about existence.

This operational stance avoids both extremes that have historically troubled foundational physics. It avoids naïve realism, which attempts to ascribe physical reality to unobservable structures, and it avoids instrumentalism, which denies any meaningful underlying structure. Instead, it adopts a restrained realism in which spacetime geometry is real insofar as it governs the behavior of admissible observers, while remaining silent about regimes that cannot be operationally accessed.

The admissibility framework therefore repositions the role of Planck units and spacetime geometry within relativistic physics. Rather than serving as indicators of new fundamental ontology, Planck units delimit the domain of applicability of operational concepts such as distance, duration, energy, and information. This shift in emphasis is subtle but profound. It preserves the successes of Special and General Relativity while explaining why their idealizations fail at extreme scales, not because the theories are wrong, but because the observers required to test them cease to be physically realizable.

25   Outlook, Open Problems, and a Research Program

The admissibility-based reformulation of relativistic physics developed in this work provides a coherent synthesis of Planck-scale limits, Special Relativity on curved spacetime, and global bounds on mass, time, and length. While the framework resolves a number of longstanding conceptual tensions without modifying established dynamical laws, it

also opens a range of questions that naturally define a forward-looking research program. This concluding section outlines these directions, emphasizing that admissibility is not an endpoint but a structural principle that reorganizes how extreme regimes should be approached.

A first open problem concerns the systematic classification of admissible observer congruences in general spacetimes. While admissibility has been defined locally through the condition

$$g_{\mu\nu}V^\mu V^\nu > 0, \tag{119}$$

and globally through bounds on physically realizable scales, a complete classification requires understanding how these conditions restrict families of observers in spacetimes with nontrivial topology or causal structure. In rotating, expanding, or anisotropic cosmologies, the set of admissible congruences may exhibit rich structure tied to curvature invariants and global geometry. Developing such a classification would sharpen the predictive content of the framework and clarify its relationship to classical congruence theorems in General Relativity [41].

A second open direction concerns the quantization of fields and geometry subject to admissibility constraints. In conventional approaches, quantization is performed first and physical interpretation is imposed afterward. The admissibility framework suggests reversing this order. One may first restrict the class of physically realizable observers and measurement protocols, and only then quantize degrees of freedom that remain accessible within this domain. This raises the possibility of a semiclassical quantization scheme in which ultraviolet divergences are regulated not by modifying field dynamics, but by excluding trans-Planckian modes that cannot be operationally accessed by admissible observers [14].

Closely related is the question of how admissibility interfaces with quantum information theory. The bounds derived in earlier sections place finite limits on entropy, information storage, and computation. An open problem is whether these bounds can be formulated as precise constraints on quantum channels accessible to admissible observers. Such an approach may provide a geometric underpinning for information-theoretic limits traditionally introduced axiomatically, and could clarify the role of observer dependence in quantum communication across curved spacetime [42].

Cosmology provides another natural arena for further development. The existence of global bounds T, L, and M suggests that cosmological observables should be reexamined from an admissibility perspective. Questions concerning the operational meaning of cosmic horizons, the accessibility of early-universe information, and the interpretation of inflationary perturbations may be sharpened by explicitly accounting for which observers are admissible throughout cosmic history. This may lead to refined statements about what aspects of the early universe are in principle observable, rather than merely calculable [43].

Another open problem concerns the relationship between admissibility and energy conditions. Classical singularity theorems rely on assumptions about geodesic completeness and energy positivity. Since admissibility restricts the physical relevance of certain geodesics before curvature singularities are encountered, it is natural to ask whether singularity theorems admit reformulation in terms of admissible observer classes. Such a reformulation could preserve the mathematical content of the theorems while clarifying their physical interpretation [44].

Finally, it remains to explore possible empirical signatures of admissibility at the interface between astrophysics and cosmology. While the framework predicts no deviations

from Special Relativity or General Relativity in currently accessible regimes, it constrains the interpretation of extreme observations involving ultra-high energies, long baselines, or near-horizon physics. Clarifying which observational claims rely on implicitly inadmissible idealizations may help sharpen the interpretation of future data without invoking new fundamental interactions.

In summary, the admissibility framework reorganizes relativistic physics around the physically realizable observer rather than around formal mathematical extensions of symmetry or dynamics. It preserves the empirical success of established theories while providing a principled way to regulate extreme regimes. The open problems outlined here suggest that admissibility may serve as a unifying structural principle across classical relativity, quantum theory, and information science, offering a fertile ground for further investigation.

26   Conclusion

We have examined the compatibility between Special Relativity and the proposed minimal length scale introduced by quantum gravity. At first glance, the idea of an invariant minimum length appears to contradict the Lorentz contraction of SR. However, a deeper analysis reveals that SR need not be discarded. Rather, it is the geometric and algebraic structure of space itself that changes at the Planck scale.

Our investigation highlights that multiple quantum gravity frameworks — including loop quantum gravity, causal dynamical triangulations, and noncommutative geometry — preserve local Lorentz invariance while introducing quantization of space. The Planck length emerges not as a break in SR, but as a geometric feature intrinsic to the fabric of space. Moreover, experimental evidence places strict bounds on Lorentz violation, supporting the notion that SR remains valid, at least to the limits of current measurements.

The conclusions drawn here underscore the robustness of Special Relativity. Even at the quantum gravitational level, SR continues to serve as a foundational symmetry, and the apparent contradictions with Planck-scale discreteness can be resolved through deeper modifications of space itself. This perspective allows us to reconcile the old and the new, upholding the profound insight of Einstein's relativity while embracing the discrete and quantized nature of quantum spacetime.

This work has developed a unified and conservative framework for understanding the interplay between Planck-scale physics and Special Relativity by shifting the foundational emphasis from symmetry modification to observer admissibility. Rather than deforming Lorentz transformations, introducing preferred frames, or postulating fundamental spacetime discreteness, the analysis has demonstrated that invariant Planck-scale limits can coexist naturally with exact local Lorentz invariance when they are interpreted as operational bounds on physically realizable observers.

A central result of the paper is the operational formulation of Special Relativity on curved spacetime. By expressing relativistic kinematics directly in terms of the local spacetime geometry, the framework allows a consistent treatment of relativistic motion in black-hole, cosmological, and rotating spacetimes while preserving the local structure of Special Relativity. Horizons, strong gravitational fields, and global rotation are shown to introduce kinematic phase boundaries that restrict which observers can be physically realized, without signaling any breakdown of local inertial physics.

Within this setting, the Planck length and Planck time emerge as invariant lower

bounds on spatial and temporal resolution. These bounds arise from the unavoidable gravitational backreaction associated with extreme localization and do not require the assumption of fundamental spacetime discreteness. The Planck mass, by contrast, is shown not to represent a minimum mass scale, but rather a crossover scale separating quantum-dominated and gravity-dominated regimes. This intrinsic asymmetry among the Planck units plays a crucial role in formulating consistent admissibility conditions across scales.

The introduction of invariant global bounds on mass, time, and length further extends the admissibility framework beyond local physics. These bounds reflect the finite extent, lifetime, and total mass-energy content of the universe and lead to precise restrictions on infinite boosts, unbounded time dilation, and unlimited information processing. A formal classification theorem for admissible observers has been established, clarifying the distinction between locally admissible, globally admissible, and inadmissible observer classes in a fully relativistic setting.

One of the most significant outcomes of this observer-centered perspective is its ability to resolve longstanding conceptual puzzles. Event horizons are reinterpreted as kinematic boundaries separating admissible and inadmissible observers rather than as loci of physical singularity. Black-hole thermodynamics and information-theoretic bounds are recovered using local Special Relativity combined with spacetime geometry, without invoking Lorentz violation, deformed dispersion relations, or holographic reductions of degrees of freedom. Apparent trans-Planckian divergences and information paradoxes are shown to arise from the implicit use of observers that lie outside the admissible domain.

The framework has been applied across a broad class of spacetimes, including Schwarzschild, Friedmann–Robertson–Walker, and Gödel geometries, demonstrating its robustness in the presence of curvature, expansion, and global rotation. In all cases, Special Relativity is recovered exactly in the appropriate limits, and General Relativity remains unmodified at the dynamical level. The admissibility principle acts not as a competing theory, but as an organizing criterion that delineates the physical domain of applicability of established laws.

More broadly, this work suggests a shift in how foundational questions in relativistic physics should be posed. Instead of asking which mathematical transformations are allowed, the physically relevant question becomes which observers, frames, and measurement protocols are realizable given both local geometry and global structure. When this shift is made, Planck-scale and cosmological-scale limits emerge as natural features of relativistic physics rather than as signs of its failure.

In conclusion, the admissibility-based framework presented here preserves the empirical successes of Special and General Relativity while providing a unified and physically transparent way to regulate extreme regimes. It offers a coherent platform for addressing Planck-scale phenomena, horizon physics, and information-theoretic limits without abandoning the core principles of relativity. As such, it provides a promising foundation for future work at the interface of relativity, quantum theory, and cosmology, grounded firmly in the physics of physically realizable observers.

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