A Modified Sieve of Sundaram

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Abstract. A new Twin prime sieve based on a modified sieve of Sundaram is introduced. It sieves through the set of natural numbers n such that 3n is not representable in either of the forms 2ij + i + j or 2ij + i + j +1 for positive integers i, j.

Theorem

For $n \in N$, $6n-1$ and $6n+1$ are prime if and only if $3n$ is not representable in either of the forms $2ij+i+j$ or $2ij+i+j+1$ for positive integers i, j.

Proof

We prove both directions.

(⇒) If one of $6n\pm1$ is composite, then $3n$ has one of the required representations.

Case 1: $6n+1$ composite

Then there exist integers $i, j > 0$ such that

$6n+1= (2i+1)(2j+1)$.

Multiply by 3

$18n +3 = 3(4ij+ 2i + 2j + 1)$.

$18n = 12ij + 6i+ 6j$.

Divide by 6

$3n = 2ij+i+j.$

Case 2: 6n−1 composite

Then there exist integers i, j > 0 such that

$6n-1 = (2i+1)(2j+1).$.

Thus

$6n-1 = 4ij + 2i + 2j +1.$

Add both sides by 1

$6n = 4ij + 2i + 2j +2.$

Divide by 2

$3n = 2ij + i + j +1.$

(⇐) Converse direction

Assume integers i, j > 0,

$3n = 2ij+i+j.$

Multiply by 2 and add 1

$6n +1= 4ij+2i+2j +1.$

Hence

$6n+1 = (2i+1)(2j+1).$

Thus 6n+1 is composite.

Similarly, if

$3n=2ij+i+j+1$,

Multiply by 2 and subtract 1

then

$6n-1= \ 4ij + 2i + 2j + 1= (2i+1)(2j+1)$,

so $6n-1$ is composite.

Conclusion

We have shown

$6n\pm1$ composite $\Leftrightarrow$ $3n=2ij+i+j$ or $3n=2ij+i+j+1$.

Taking negations gives

$6n-1$ and $6n+1$ are prime $\Leftrightarrow$ $3n$ is not representable in those forms.

Reference

[1] Ahmed Diab, Development of sieve of Eratosthenes and sieve of Sundaram's proof, arVix: 2102.06653v2 [math.NT] 3 May 2021.