# Ultraviolet Stability for Four-Dimensional Lattice Yang–Mills Theory: Closing the Bałaban–Doob Circuit under Quantitative Blocking and Decoupling Hypotheses*


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**Abstract**

We prove that the continuum limit of pure $SU(N)$ lattice Yang–Mills theory in four Euclidean dimensions exists on the algebra of blocked observables at fixed finite volume, *conditionally on an explicit hypothesis ledger*: a quantitative regularity hypothesis for the blocking map (squared-oscillation summability, Assumption A — equivalently (H-LIP$^2$), a strengthened form of the (H-LIP) contraction of 2602.0073 v2), the Dobrushin-type decoupling hypothesis (H-DEC/AT) of the sibling audits, and the audited statuses of the Bałaban structural package. The argument assembles: (i) Bałaban's renormalization group program (polymer representation, irrelevance bounds after $\beta$-function extraction, UV stability of effective densities) — traceable per 2602.0069 v2, with the $\beta_{\mathrm{LF}}$ dichotomy for the large-field part; (ii) a Doob-martingale covariance *identity* (exact for every measure) together with a conditional-oscillation influence bound — v1's claim that the oscillation control holds "without product-measure hypotheses" is withdrawn: v1's Remark 2.3 correctly rejected Efron–Stein for the non-product interpolating measures, but the Doob-oscillation Lemma 1.5 of v1 fails for the same reason (two-spin counterexample, 2602.0070 v2); the repair is (H-DEC); (iii) the RG–Cauchy summability framework of 2602.0073 v2, consumed with its full ledger (including (H-$\theta$)/F-SQRT for the truncation errors). Under the ledger, the telescopic state sequence converges and the resulting state $\omega_L \in (\mathcal{A}_\ell^{\mathrm{block}})^*$ is gauge-invariant, Euclidean-covariant (hypercubic), and positive. Osterwalder–Schrader reconstruction, the thermodynamic limit, and the mass gap remain open, as in v1. All mechanical steps — the exact covariance identity, both counterexample adjudications, the Assumption A mechanics on explicit blocking maps (including a sharpness example), and the corrected Proposition 6.1 arithmetic — are machine-verified in a companion suite.


*Version 2. v1's Lemma 1.5 asserted the oscillation control of the Doob seminorm "for any probability measure" — the statement refuted, with a two-spin counterexample, in the audit of the companion 2602.0070 (its v1 Lemma 3.4): for non-product measures the Doob increment collects influence transmitted through correlations and is *not* bounded by the raw single-link oscillation. The irony is recorded honestly: v1's Remark 2.3 correctly rejected the Efron–Stein seminorm because the interpolating measures are not product — and then used a Doob-oscillation bound that fails for exactly the same reason. The invariant content of the audit is that non-productness bites at the oscillation→increment step in *either* formulation; the repair, in either formulation, is the Dobrushin-type decoupling hypothesis (H-DEC/AT) (2602.0070/0072 v2). v2 corrects Lemma 1.5 (conditional-oscillation form + (H-DEC)), restates Proposition 6.1 and Theorem 1.6 conditionally, removes an editorial TODO left in v1's Remark 1.3, downgrades the abstract's "all hypotheses . . . are discharged" to the audited conditional statuses, and updates the companion references (2602.0069/0070/0072/0073 v2, 2602.0063 v3, and the audited chain). See the Note added in v2 (Section 9).

# Contents



# 1 Introduction and Main Result

## 1.1 Setting

Four-torus of physical side $L$, lattice spacings $a_k = a_0 2^{-k}$, Wilson action at Balaban-matched couplings; blocked observables at fixed physical scale $\ell > 0$ via iterated blocking $Q_{\ell,k}$ (as in 2602.0073 v2, whose notation we follow).

## 1.2 Blocked observables

**Definition 1.1** (Blocked observable algebra)**.** $\mathcal{A}_\ell^{\mathrm{block}}$*: families* $O^{(k)} = F \circ Q_{\ell,k}$ *with gauge-invariant* $F$*,* $\|F\|_\infty \le 1$*; the bounded Lipschitz subclass* $\mathrm{BL}(\mathcal{A}_\ell^{\mathrm{block}})$ *additionally has* $\mathrm{Lip}(F) < \infty$ *(Definition 1.4).*

**Assumption A** (Squared-oscillation summability of the blocking — (H-LIP$^2$))**.** *There exists* $C_Q < \infty$ *(depending on* $\ell$ *and the blocking scheme, not on* $k$*) such that for every bounded Lipschitz* $F$ *on the coarse configuration space,*

$$\sum_{e \in \Lambda_k^1} \mathrm{osc}_e\big(F \circ Q_{\ell,k}\big)^2 \;\le\; C_Q \,\mathrm{Lip}(F)^2. \tag{1}$$

**Remark 1.2.** *Assumption A is stronger than the global bound* $\mathrm{Lip}(Q_{\ell,k}) \le 1$*: it is a per-link* $\ell^2$ *budget. It is implied by the one-step contraction (H-LIP) of 2602.0073 v2 (Assumption 3.6 there) together with locality of the blocking stencil; the companion suite (check 3) verifies* (1) *on explicit local averaging maps and exhibits a* non-local *map violating it (sharpness).*

**Remark 1.3** (Status; v2 rewrite of v1's Remark 1.3)**.** *Assumption A is the precise blocking input for the Doob approach at fixed* $\ell$*. For Bałaban's averaging maps it is expected to follow from quantitative locality and smoothing of the averaging operation; this remains to be extracted from the primary sources* — an open interface task recorded as such *(v1's remark here ended with an editorial TODO, now removed and logged in the NOTES).*

**Definition 1.4** (Bounded Lipschitz subclass)**.** *As in v1.*

**Lemma 1.5** (Doob seminorm and oscillations — v2 CORRECTED)**.** *(a)* (Conditional oscillation; exact for every $\nu$.) *With* $\widetilde{\mathrm{osc}}_i(f)$ *the oscillation of* $u \mapsto E_\nu[f \mid \mathcal{F}_{i-1}, U_{e_i} = u]$*,*

$$\sigma_\nu(f)^2 \;\le\; \tfrac{1}{4}\sum_i \widetilde{\mathrm{osc}}_i(f)^2. \tag{2}$$

*(b)* (Raw oscillation; product measures, or (H-DEC).) *For product* $\nu$*,* $\widetilde{\mathrm{osc}}_i \le \mathrm{osc}_{e_i}$ *and hence* $\sigma_\nu(f)^2 \le \frac{1}{4}\sum_e \mathrm{osc}_e(f)^2$*; for general* $\nu$ *this* fails *(two-spin counterexample: increment* 1*, oscillation* 0*; 2602.0070 v2), and under* (H-DEC) *(influence-leakage matrix* $\|b\| \le b_*$*) one has*

$$\sigma_\nu(f)^2 \;\le\; \tfrac{1}{4}(1+b_*)^2 \sum_e \mathrm{osc}_e(f)^2. \tag{3}$$

(v1's Lemma 1.5 asserted (b) without (H-DEC) "for any probability measure", with the one-line proof "each martingale increment is bounded by $\frac{1}{2}\,\mathrm{osc}_{e_i}(f)$" — precisely the step refuted in the sibling audit. Suite check 2.)

### 1.3 Statement of the main theorem

**Theorem 1.6** (UV closure under quantitative blocking and decoupling; v2)**.** *Assume Assumption A ((H-LIP*$^2$*)),* (H-DEC/AT) *uniformly along the interpolations, and the Bałaban package (A)/(B)/(B5)/(B6) with its audited statuses (traceable/conditional per 2602.0069/0072 v2;* $\beta_{\mathrm{LF}}$ *dichotomy for the large-field part; (H-θ)/F-SQRT for truncation errors, 2602.0073 v2). Then for every* $O \in \mathrm{BL}(\mathcal{A}_\ell^{\mathrm{block}})$ *the telescopic sequence* $\langle O^{(k)}\rangle_k$ *converges as* $k \to \infty$*, and the limit functional* $\omega_L \in (\mathcal{A}_\ell^{\mathrm{block}})^*$ *is gauge-invariant, hypercubic-covariant, and positive.* (v1 stated this with Assumption A as the only explicit hypothesis and "all [other] hypotheses . . . discharged"; the discharge is conditional, as itemized.)

### 1.4 Logical architecture

Layer 1: the RG–Cauchy framework of 2602.0073 v2 (its Theorem 9.2, consumed with its ledger). Layer 2: discharges of (A), (B), (B5), (B6) by citation to the traceability companion 2602.0069 v2 and the influence companions 2602.0070/0072 v2 (Sections 3–6). Layer 3: assembly (Section 7).

### 1.5 Scope and limitations

Fixed finite volume; blocked observables only; OS reconstruction, thermodynamic limit, mass gap open (Section 8); cf. 2602.0063 v3 for the continuum capstone that consumes this circuit's output as (H-CAUCHY).

## 2 The RG–Cauchy Summability Framework

### 2.1 Telescopic decomposition

As in v1 (= 2602.0073 v2, Lemma A1 route: exact RG identity makes the one-step comparison a genuine scale comparison).

### 2.2 Influence-based covariance control

**Definition 2.1** (Doob influence seminorm). $\sigma_\nu(f)^2 := \sum_i E_\nu[(E[f|\mathcal{F}_i] - E[f|\mathcal{F}_{i-1}])^2]$ *for a fixed link ordering.*

**Lemma 2.2** (Doob covariance bound). *For any* $\nu$ *and* $f, g \in L^2(\nu)$: $\mathrm{Cov}_\nu(f, g) = \sum_i E[\Delta_i f\, \Delta_i g]$, *hence* $|\mathrm{Cov}_\nu(f, g)| \leq \sigma_\nu(f)\sigma_\nu(g)$. An identity — exact for every measure; survives v1 unchanged (suite check 1).

**Remark 2.3** (Why not Efron–Stein — v1's correct observation, now with its citation). *The Efron–Stein seminorm satisfies the covariance bound only for product measures; since* $\nu_{k,t}$ *is not product, the Doob formulation is used.* (v2) *This observation of v1 was correct and is now backed by the explicit counterexample and audit of 2602.0072 v2 — but the* same *non-productness defeats the raw Doob-oscillation bound (Lemma 1.5(b)); neither formulation is decoupling-free. The invariant lesson: the covariance step is free (Doob identity), the oscillation step costs* (H-DEC/AT) *in either currency.*

**Lemma 2.4** (Locality across scales for blocked observables). *As in v1: blocked observables at physical scale* $\ell$ *depend on the fine links through local stencils, so per-link oscillations carry the* $(a_k/\ell)^2$*-type damping budgeted by Assumption A.*

**Theorem 2.5** (RG–Cauchy summability, Doob version; conditional). *= 2602.0073 v2, Theorem 9.2, with the Doob-form influence control of Lemma 1.5(a)+(b) under* (H-DEC)*; consumed here with its full ledger.*

## 3 Discharge of (A): Polymer Representation

**Proposition 3.1** (Polymer representation — composite). *As in v1, by citation ([9] Eq. (2.41), [11] Eqs. (1.98)–(1.100)).* (v2) *Status: traceable, conditional on transcription (2602.0069 v2).*

## 4 Discharge of (B): Per-Link Oscillation Decay

**Lemma 4.1** (Dimension gap and dimension-6 remainder). *As in v1 (Ward–Takahashi + antisymmetry + irrelevance bound).*

**Proposition 4.2** (Discharge of (B)). $\mathrm{osc}_e(K_k(X)) \leq C_{\mathrm{osc}} L^{-2k} |X|^p e^{-\kappa|X|}$, *by citation to [7,9] via the oscillation–analyticity bridge.* (v2) *Status: traceable (2602.0069 v2, §10; the bridge's Cauchy mechanics are machine-verified there); the fine-volume direction bookkeeping of its Remark 10.4 applies.*

## 5 Discharge of (B5): Large-Field Suppression

**Proposition 5.1** (Discharge of (B5))**.** *As in v1, by citation to [10,11] (R-operation, small factors).* (v2) *Status: conditional — the small-factor constant carries the $\beta_{\mathrm{LF}}$ dichotomy (2602.0069 v2, Remark 8.7): $O(1)$ reading, or (H-P0); and summed truncation errors inherit (H-$\theta$)/F-SQRT (2602.0073 v2).*

## 6 Discharge of (B6): Doob Influence Bound

**Proposition 6.1** (Discharge of (B6); v2: conditional)**.** *Assume* (H-DEC) *along the interpolation. Then there is $C_\sigma > 0$ with $\sup_t \sigma_{\nu_{k,t}}(V_k^{\mathrm{irr}}) \le C_\sigma$ for all $k \ge \ell$: by the polymer decomposition (Proposition 3.1), Lemma 1.5(b), Proposition 4.2 and the lattice-animal bound,*

$$\sigma_{\nu_{k,t}}(V_k^{\mathrm{irr}})^2 \;\le\; \tfrac{1}{4}(1+b_*)^2 \sum_e \Big( \sum_{X \ni e} \mathrm{osc}_e(K_k(X)) \Big)^2 \;\le\; (1+b_*)^2 (L/a_0)^4 S_*^2, \tag{4}$$

*by the exact $M \cdot 2^{-4k}$ scale cancellation.* (v1 asserted this without (H-DEC) via its Lemma 1.5; corrected. Suite check 4.)

## 7 Assembly: Proof of the Main Theorem

*Proof of Theorem 1.6.* Verify the hypotheses of Theorem 2.5: (A) by Proposition 3.1 [traceable]; (B) by Proposition 4.2 [traceable]; (B5) by Proposition 5.1 [conditional: $\beta_{\mathrm{LF}}$/(H-$\theta$)]; (B6) by Proposition 6.1 [conditional: (H-DEC)]; the blocking budget by Assumption A [(H-LIP$^2$), input]. Conclude convergence of the telescopic sequence; gauge invariance, hypercubic covariance and positivity of $\omega_L$ pass to the limit as in v1. □

## 8 Discussion and Remaining Gaps

As in v1, with the audited cross-references: OS/RP and reconstruction — 2602.0063 v3 (conditional); thermodynamic limit — open; mass gap — requires the transfer-gap input (2602.0054 v2, conditional on (H-DOB-blk)+(H-P0)); nontriviality — conditional on confinement (2602.0063 v3, Layer 2).

## 9 Note added in v2

Findings of the series audit applied to this assembly paper:

- **F-L15.** v1's Lemma 1.5 ($\sigma_\nu^2 \le \frac{1}{4}\sum \mathrm{osc}_e^2$ "for any probability measure") is the statement refuted in 2602.0070 v2 (two-spin counterexample: increment 1, oscillation 0). Corrected to Lemma 1.5: conditional oscillation (exact) + (H-DEC) for the raw form. Proposition 6.1 and Theorem 1.6 carry the hypothesis. [Suite check 2]
- **The instructive irony** (recorded, not hidden): v1's Remark 2.3 *correctly* rejected Efron–Stein because $\nu_{k,t}$ is not product — the very insight of the 2602.0072 v2 audit — and then used a Doob-oscillation bound that fails for the same reason. The invariant content: non-productness bites at the oscillation→increment step in either formulation; the covariance identity is the free part. [Checks 1–2]

- **F-TODO.** v1's Remark 1.3 shipped with an editorial note ("provide an explicit citation/lemma here once extracted from [3] ..."). Removed; the open interface task (deriving Assumption A from Bałaban's averaging locality) is stated as such.

- **Discharge statuses.** "All hypotheses ... are discharged with explicit citations" $\to$ (A)/(B) traceable [2602.0069 v2], (B5) conditional [$\beta_{\mathrm{LF}}$, (H-$\theta$)/F-SQRT], (B6) conditional [(H-DEC)], blocking budget = Assumption A [(H-LIP$^2$), open interface task]. References updated: 2602.0069/0070/0072/0073 (v2), 2602.0063 (v3), audited chain 2602.0041 v3 + 0051–0057 v2; 2602.0072 is now cited (v1's Remark 2.3 alluded to the ES issue without citation).

- **What survives, machine-verified:** the Doob covariance identity (any $\nu$); Assumption A mechanics on explicit local averaging maps with a non-local sharpness counterexample; the corrected (B6) arithmetic with the $M2^{-4k}$ cancellation; the consistency of Theorem 1.6 with the 2602.0073 v2 ledger (same delivered class, $\eta = 2$). [Checks 1, 3–5]

Suite: `verify_2602_0077.py` (5 checks) at `https://github.com/lluiseriksson/THE-ERIKSSON-PROGRAMME`, directory `verification/2602-0077/`.

## References


[1] T. Bałaban, Comm. Math. Phys. **95–122** (1984–1989) (equation-level citations in ai.viXra:2602.0069 (v2)).

[2] J. Dimock, arXiv:1108.1335; 1212.5562; 1304.0705.

[3] L. Eriksson, ai.viXra:2602.0069 (v2), 2026. [Structural package; $\beta_{\mathrm{LF}}$ dichotomy; (H-$\theta$).]

[4] L. Eriksson, ai.viXra:2602.0070 (v2), 2026. [Doob companion; the counterexample refuting v1's Lemma 1.5; (H-DEC).]

[5] L. Eriksson, ai.viXra:2602.0072 (v2), 2026. [Efron–Stein companion; (H-AT); the product-only tensorisation adjudication behind Remark 2.3.]

[6] L. Eriksson, ai.viXra:2602.0073 (v2), 2026. [RG–Cauchy Master; the ledger consumed by Theorem 2.5; F-SQRT.]

[7] L. Eriksson, ai.viXra:2602.0063 (v3), 2026. [Continuum capstone; (H-CAUCHY) = its Assumption 3.5.]

[8] L. Eriksson, ai.viXra:2602.0041 (v3); 2602.0046; 2602.0051–0057 (v2), 2026. [Audited IR/LSI chain.]

[9] A. Jaffe, E. Witten, *Quantum Yang–Mills theory*, Clay Millennium Prize Problems, 2000.