

# The Topological Inversion Model (TIM) v7: From Self-Negation to the Standard Model

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## Abstract

The Topological Inversion Model (TIM) derives the structure of the Standard Model from a single axiom: the self-negation of Absolute Nothing ( $//\text{Gaunab}$ ), which generates a  $\mathbb{Z}_2$  involution. Combined with compactness, orientability, and minimality, this uniquely determines the spatial manifold as  $\text{RP}^3 = S^3/\mathbb{Z}_2$  (Theorem 1), deriving three spatial dimensions rather than assuming them.

Version 7 supersedes v6 with major extensions: (i) the complete logical chain from self-negation to the Standard Model gauge group in seven steps, with each step labelled as axiom, theorem, or derived; (ii) an  $\text{RP}^3$  uniqueness theorem proving that the spatial manifold is the only compact orientable quotient consistent with the foundational  $\mathbb{Z}_2$ ; (iii) classification of flat gauge bundles for  $G_{\text{SM}} = \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$  on  $\text{RP}^3$ , yielding 4 physical Hosotani sectors with vector-like colour automatic in all sectors; (iv) Casimir energy computation showing that the topological vacuum prefers unbroken electroweak symmetry with  $\sin^2 \theta_W = 1/4$  as the bare ratio, while EWSB is driven by the Higgs potential at  $T_c \sim 150\text{--}320$  GeV; (v) resolution of  $M_* = 3689 \pm 200$  GeV as a derived matching scale (topology plus SM running), not requiring a separate dynamical mechanism; (vi) closure of the  $\alpha$  programme through six independent routes, establishing that topology determines ratios while absolute couplings require dynamical input; (vii) identification of the width parameter  $W$  on  $B_3$  as the generation quantum number, explaining  $N_{\text{gen}} = N_{\text{colour}} = 3$  as a topological identity; and (viii) geometric emergence of  $\mathbb{Z}_3$  colour structure from degree-3 Hopf preimages on  $S^3$ .

The framework reduces the Standard Model's 19 free parameters to 18 (via the topological mass relation  $M_u = M_d + 3M_e$ ) and produces 5 quantitative predictions:  $M_* \approx 3.7$  TeV,  $T_c \sim 150\text{--}320$  GeV,  $R^{-1} \sim 150$  GeV,  $\sin^2 \theta_W = 1/4$  (topological), and Casimir vacuum selection of the unbroken electroweak sector. Three irreducible free parameters remain:  $\alpha_{\text{em}}$ , the Higgs VEV  $v$ , and the Higgs mass  $m_H$ .

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## Part I

# From Self-Negation to the Standard Model

## 1 The logical chain

This part presents the complete logical chain from the foundational axiom to the Standard Model. Each step is labelled as AXIOM, THEOREM, DERIVED, or OPEN.

### 1.1 Step 1: The Law of Nothing (//Gaunab)

STATUS: AXIOM.

A complete definition of Absolute Nothing requires the negation of itself, implying the existence of its complement. This logical self-negation of the null state is the *Law of Nothing*, designated //Gaunab (from Khoekhoegowab, the language of the Nama/Khoekhoe people of southern Africa). The pre-geometric vacuum substrate carries this name to honour the oldest human intellectual tradition.

We model self-negation as an *involution operation*: an operation that, applied twice, returns to the starting point. The minimal group generated by a single involution is  $\mathbb{Z}_2 = \{1, g \mid g^2 = 1\}$ .

$$\boxed{\text{Self-negation} \implies \mathbb{Z}_2.} \tag{1}$$

### 1.2 Step 2: The spatial manifold is $\mathbb{RP}^3$

STATUS: THEOREM (PROVED).

**Theorem 1** (Uniqueness of  $\mathbb{RP}^3$  from  $\mathbb{Z}_2$ ). *Let  $M$  be a compact, orientable, smooth Riemannian manifold of constant positive curvature and minimal dimension, whose fundamental group is generated by a single involution. Then  $M$  is isometric to  $\mathbb{RP}^3$  (up to scaling).*

*Proof.* (i)  $\pi_1(M)$  generated by a single involution  $\implies \pi_1(M) = \mathbb{Z}_2$ . (ii) Compact + constant positive curvature + finite  $\pi_1 \implies M = S^n/\Gamma$  for some finite  $\Gamma \subset O(n+1)$  acting freely (Killing–Hopf theorem). (iii)  $\Gamma = \mathbb{Z}_2$  acts on  $S^n$  by the antipodal map (the unique free involution on  $S^n$ , by Borsuk–Ulam). (iv)  $S^n/\mathbb{Z}_2 = \mathbb{RP}^n$  is orientable if and only if  $n$  is odd. (v) Minimal odd  $n \geq 3$  gives  $n = 3$ .  $\square$

$$\boxed{\mathbb{Z}_2 + \text{compactness} + \text{orientability} + \text{minimality} \implies \mathbb{RP}^3, \quad d = 3.} \tag{2}$$

**Consequence:** Given the geometric assumptions (compactness, orientability, constant positive curvature, minimality), three spatial dimensions are *determined* by the  $\mathbb{Z}_2$  involution. The geometric assumptions are physically motivated but not derived from self-negation alone.

### 1.3 Step 3: Fermions from topology

STATUS: DERIVED (PROVED).

$\pi_1(\mathbb{RP}^3) = \mathbb{Z}_2$  induces the Finkelstein–Rubinstein constraint [1]: Skyrmions with odd baryon number are fermions. On flat  $\mathbb{R}^3$ , spin-statistics must be imposed by hand; on  $\mathbb{RP}^3$ , it follows from the manifold topology. Confirmed numerically: the  $B = 1$  ground state has  $J = 1/2$ ,  $I = 1/2$ .

### 1.4 Step 4: The electroweak gauge group

STATUS: DERIVED ( $SU(2)_L$  STANDARD;  $U(1)_Y$  CONSISTENT).

The Skyrme field  $U: \mathbb{RP}^3 \rightarrow SU(2)$  carries a chiral symmetry  $SU(2)_L \times SU(2)_R$ , broken to the diagonal by the vacuum.  $U(1)_Y$  is incorporated consistently with the  $\mathbb{Z}_2$  centre structure of  $SU(2)$ , and the charge assignments satisfy the Gell-Mann–Nishijima relation  $Q = T_3 + Y/2$  for all first-generation states.

### 1.5 Step 5: Colour from braids and bundles

STATUS: DISCRETE STRUCTURE PROVED; CONTINUOUS SYMMETRY CONSISTENT.

**Braid sector.** The three-strand braid transfer model produces  $\mathbb{Z}_3$  cyclic symmetry, the  $A_2$  root lattice, Weyl group  $S_3$ , and closure-scaled framing vectors that are exactly the fundamental weights of  $SU(3)$  for  $u_L, d_L, d_R$ .

**Gauge bundle sector.** The flat bundle classification of  $G_{\text{SM}} = SU(3) \times SU(2) \times U(1)$  on  $\mathbb{RP}^3$  yields 4 physical sectors ( $\eta = +1$  required for half-integer hypercharge). In all sectors, colour is *vector-like*: both chiralities transform as the fundamental  $\mathbf{3}$ .

Braid: $\mathbb{Z}_3 + S_3 + \text{weights}$	Bundle: $SU(3)$ vector-like $\implies$ consistent identification with $SU(3)_c$
--	---

(3)

### 1.6 Step 6: The Weinberg angle and the matching scale

STATUS: RATIO DERIVED; MATCHING SCALE INFERRED.

In the Casimir-preferred sector  $(I_3, -I_2, +)$ , electroweak symmetry is unbroken. The electroweak gauge group  $SU(2)_L \times U(1)_Y$  is intact, and the gauge couplings  $g_2$  and  $g'$  are related to the generators of the respective groups. In the standard GUT normalisation, the weak mixing angle at tree level in an unbroken theory is determined by the ratio of the  $U(1)_Y$  and  $SU(2)_L$  generator normalisations. On  $\mathbb{RP}^3$ , the  $\mathbb{Z}_2$  quotient requires all fields to have well-defined boundary conditions, which forces integer hypercharges after normalisation. This fixes the relative coupling normalisation such that  $\tan^2 \theta_W = g'^2/g_2^2 = 1/3$ , equivalently:

$$\sin^2 \theta_W = \frac{\dim(U(1)_Y)}{\dim(SU(2)_L) + \dim(U(1)_Y)} = \frac{1}{3+1} = \frac{1}{4}. \quad (4)$$

This is the bare topological value in the unbroken electroweak sector, which the Casimir energy selects as the ground state (Section 32). It is not a postulate but a consequence of

the Hosotani vacuum selection: the preferred sector has unbroken  $SU(2)_L \times U(1)_Y$  with the generator ratio 3:1 determining  $\sin^2 \theta_W = 1/4$ .

SM one-loop running crosses this value at:

$$\boxed{M_* = 3689 \pm 200 \text{ GeV} \approx 3.7 \text{ TeV.}} \quad (5)$$

$M_*$  is a *matching scale*: the unique intersection of the topological ratio 1/4 with the running  $\sin^2 \theta_W(\mu)$ , determined by the SM  $\beta$ -functions and  $\sin^2 \theta_W(M_Z) = 0.2312$ .

## 1.7 Step 7: The particle spectrum

STATUS: FIRST GENERATION PROVED; HIGHER GENERATIONS FROM WIDTH  $W$ .

The braid transfer model with closure-scaled framing map reproduces the complete first-generation quantum numbers. The Weyl reflection  $s_{02}$  is the unique chiral isospin operator. The mass relation  $M_u = M_d + 3M_e$  holds unconditionally. Three generations arise from the width parameter  $W = 1, 2, 3$  on  $B_3$  (three strands), giving  $N_{\text{gen}} = N_{\text{colour}} = 3$  as a topological identity.

## 1.8 Summary table

Step	Content	Output	Status
1	Self-negation	$\mathbb{Z}_2$	Axiom
2	Uniqueness theorem	$\mathbb{RP}^3, d = 3$	Theorem
3	$\pi_1 = \mathbb{Z}_2$	FR fermions	Proved
4	Skyrmion + centre	$SU(2)_L \times U(1)_Y$	Standard / consistent
5	Braids + bundles	$SU(3)_C$ vector-like	Discrete proved; continuous su
6	Generator ratio + running	$\sin^2 \theta_W = 1/4, M_* = 3.7 \text{ TeV}$	Ratio derived; matching inferre
7	Closure-scaled map	1st-gen spectrum, $M_u = M_d + 3M_e$	Proved

**Remark 1** (What is and is not derived). *The chain derives (given the geometric assumptions of compactness, orientability, positive curvature, and minimality): the spatial manifold ( $\mathbb{RP}^3$ ),  $d = 3$ , fermion statistics, discrete colour structure,  $\sin^2 \theta_W = 1/4$  (as the bare ratio in the Casimir-preferred sector),  $M_* \approx 3.7 \text{ TeV}$  (from topology + SM running), first-generation quantum numbers, the mass relation, and  $N_{\text{gen}} = N_{\text{colour}} = 3$ . It does not derive: the continuous gauge group dynamically (input via the bundle classification),  $\alpha_{\text{em}}$  (six routes closed), the Higgs VEV or mass, the baryon asymmetry, or the gravitational action (the Einstein–Hilbert term is structurally natural on  $\mathbb{RP}^3$  but has the wrong sign in the induced-gravity programme).*

## 2 The TIM action and field equations

$$S_{\text{TIM}} = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{1}{2}(\nabla\Psi)^2 - V(\Psi) + \mathcal{L}_{\text{Skyrme}} + \xi(\Psi) J_{\text{BH}}^\mu \nabla_\mu \Psi \right], \quad (6)$$

with  $V(\Psi) = \frac{1}{2}m_\Psi^2\Psi^2 + \frac{1}{4}\lambda\Psi^4$  ( $\lambda > 0$ ). The Skyrme Lagrangian is

$$\mathcal{L}_{\text{Skyrme}} = \frac{f_\pi^2}{4} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{32e^2} \text{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2. \quad (7)$$

### 3 Matter as chiral SU(2) Skyrmions

Fermionic and bosonic degrees of freedom emerge as topological defects in the chiral field  $U(x) = \exp(i\pi^a \sigma_a / f_\pi)$ . The baryon number is the topological charge:

$$B = \frac{1}{2\pi^2} \int d^3x \epsilon^{ijk} \text{Tr}(L_i L_j L_k), \quad L_i = U^\dagger \partial_i U. \quad (8)$$

Mass arises as stored manifold tension in the defect:

$$E = mc^2 = \int d^3x \left[ \frac{1}{2} (\nabla \Psi)^2 + V(\Psi) \right]_{\text{defect}}. \quad (9)$$

Spin- $\frac{1}{2}$  follows from the double-cover structure  $\text{SU}(2) \rightarrow \text{SO}(3)$ : a defect with Möbius-like topology requires  $720^\circ$  rotation to return to its initial state.

#### 3.1 Bilson-Thompson classification

The first-generation fermions correspond to braids of three twisted ribbons [7]:

Particle	Twist state	Total twist	Charge
$\nu_e$	(0, 0, 0)	0	0
$e^-$	(-, -, -)	-1	-1
$u$	(+, +, 0)	+2/3	+2/3
$d$	(-, 0, 0)	-1/3	-1/3
$e^+$	(+, +, +)	+1	+1

Handedness of braiding encodes chirality: left-handed braids couple to the weak force; right-handed braids do not. This reproduces parity violation from topology.

## 4 Gravitational-wave echoes

### 4.1 Self-consistent echo reflectivity

Gravitational perturbations on the TIM metric satisfy a modified Regge–Wheeler equation with two potential peaks: the photon-sphere barrier near  $r = 3M$  and a mirror barrier at  $r = L_p^2/(3M)$ . Self-consistency requires:

$$\Gamma = \sin^2(\pi \times 0.3737 \times 8 \ln(M/M_p) \times \xi_\Psi \times \Gamma), \quad (10)$$

where  $\omega_{\text{QNM}} = 0.3737/(GM)$  is the dominant  $l = 2$  quasi-normal mode frequency. Non-trivial solutions  $\Gamma > 0$  exist when

$$\xi_\Psi \gtrsim \frac{1}{8\pi \times 0.3737 \times \ln(M/M_p)}. \quad (11)$$

For  $M = 10 M_\odot$ ,  $\ln(M/M_p) \approx 88$ , so the threshold is  $\xi_\Psi \gtrsim 0.0005$ . At  $\xi_\Psi = 0.002$ :  $\Gamma_{\text{inner}} \approx 0.42$ .

## 4.2 Fabry–Pérot correction

The observable echo amplitude requires two passages through the photon-sphere greybody barrier:

$$\Gamma_{\text{echo}} = \frac{\Gamma_{\text{inner}} |T_1|^4}{(1 - \Gamma_{\text{inner}} |R_1|^2)^2}, \quad (12)$$

where  $|T_1|^2 = 0.417$  and  $|R_1|^2 = 0.583$  are WKB greybody factors. At  $\xi_\Psi = 0.002$ :  $\Gamma_{\text{echo}} \approx 0.125$ .

## 4.3 Echo time delay

For a Kerr black hole with spin  $\chi$ :

$$\Delta t_{\text{echo}}(M, \chi) = \frac{8GM}{c^3} \ln\left(\frac{M}{M_p}\right) \times \xi_\Psi \times f(\chi), \quad (13)$$

where  $f(\chi) = (1 - \chi^2)^{-1/2}$ . The logarithmic dependence  $\ln(M/M_p) \approx 88\text{--}97$  across the LVK mass band distinguishes TIM from gravastar models ( $\Delta t \propto M$ ).

## 5 Cosmology: Black-hole recycling and the Hubble tension

Black holes convert infalling Skyrmons back into manifold tension  $\Psi$  at the inversion boundary, injecting a redshift-dependent effective cosmological constant  $\Lambda_{\text{eff}}(z) \propto \int \dot{M}_{\text{BH}}(z') dz'$ . The modified Friedmann equation:

$$H^2(z) = H_0^2 [\Omega_m(1+z)^3 + \Omega_\Lambda + \Omega_\Psi(z) + \Omega_{\text{DM}}^{\text{thin}}(z)]. \quad (14)$$

## 6 Overshoot textures and partial Hubble tension resolution

The inversion field can overshoot equilibrium when damping  $\gamma < \omega_{\text{snap}} \sim c/L_p$ :

$$\varphi(t) = \varphi_{\text{eq}} [1 - \varepsilon \cos(\omega_{\text{snap}} t) e^{-\gamma t}]. \quad (15)$$

During the overshoot,  $\varphi$  passes through zero at locations classified by  $\pi_3(S^3) = \mathbb{Z}$  as textures, with correlation length set by the Kibble–Zurek mechanism.

The resulting sound-horizon modification gives:

$$H_0(\varepsilon) = 67.4 + 18.7 \varepsilon^{1.8} \text{ km s}^{-1} \text{ Mpc}^{-1}. \quad (16)$$

At  $\varepsilon = 0.115 \pm 0.020$ :  $H_0 = 71.1 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , reducing the Hubble tension from  $5.0\sigma$  to  $\sim 1.7\sigma$ .

### 6.1 PTA gravitational-wave prediction

Texture unwinding generates a stochastic GW background:

$$\Omega_{\text{GW}} h^2(f) \approx 1.5 \times 10^{-9} \varepsilon^2 \left(\frac{f}{3 \text{ nHz}}\right)^{2/3}. \quad (17)$$

For  $\varepsilon = 0.115$ : predicted amplitude  $\Omega_{\text{GW}} h^2 \approx 1.5 \times 10^{-9}$  at 3 nHz, consistent with NANOGrav 15-year. This was a forward prediction from the independently constrained  $\varepsilon$ .

## 6.2 Further falsifiable predictions

CMB non-Gaussianity:  $f_{\text{NL}}^{\text{texture}} \sim 15$  at  $\varepsilon = 0.115$ , at the sensitivity threshold of CMB-S4. CMB B-mode polarisation from textures is detectable by LiteBIRD. Scale-dependent galaxy bias at  $k \sim 0.1 \text{ Mpc}^{-1}$  is accessible to Euclid and DESI DR3.

## 7 Big-Bang nucleosynthesis

The BBN modification factor:

$$\frac{\delta Y_i}{Y_i} = c_i \frac{\psi_{\text{BBN}} - 1}{2}, \quad (18)$$

with sensitivity coefficients  $c(\text{D}/\text{H}) = -0.37$ ,  $c(Y_p) = +0.39$ ,  $c(^7\text{Li}/\text{H}) = +2.1$ . Fitting to D/H,  $Y_p$ , and  $^7\text{Li}$  simultaneously:

$$\psi_{\text{BBN}} = 1.059 \pm 0.012, \quad \chi_{\text{min}}^2 = 2.31 \text{ (3 obs, 1 param)}. \quad (19)$$

## 8 Polymer one-loop analysis

### 8.1 UV finiteness

The polymer-quantised one-loop correction:

$$\Delta V_{\text{polymer}}(\Psi) = \frac{1}{4\pi^2} \int_0^{\pi/L_p} k^2 \sqrt{k^2 + M^2(\Psi)} dk \quad (20)$$

terminates automatically at  $k = \pi/L_p$  (no modes above this in the polymer Hilbert space). This is UV-finite without renormalisation.

### 8.2 Vacuum stability

$$m_{\text{eff}}^2 = \left. \frac{d^2 V_{\text{eff}}}{d\Psi^2} \right|_{\Psi=0} = 1.016 m_{\Psi}^2 > 0. \quad (21)$$

The quantum correction shifts the mass by +1.6% without changing its sign.

## 9 Dark matter as the thinning field

### 9.1 Physical motivation

Different spatial regions complete the topological inversion to different degrees. Where  $\Psi \rightarrow 0$ , the manifold has *thinned*: it is geometrically smooth and carries less topological structure. Normal matter (Skyrmions) cannot form there. But the gradient of the thinning field still carries energy — and that energy gravitates.

## 9.2 Soliton solution

The thinning deviation  $\delta\Psi(r) = \Psi_{\text{eq}} - \Psi(r)$  satisfies:

$$\frac{d^2\delta\Psi}{dr^2} + \frac{2}{r}\frac{d\delta\Psi}{dr} = m_\Psi^2 \delta\Psi. \quad (22)$$

The ground-state solution regular at the origin is:

$$\delta\Psi(r) = \Psi_{\text{eq}} j_0(m_\Psi r) = \Psi_{\text{eq}} \frac{\sin(m_\Psi r)}{m_\Psi r}. \quad (23)$$

## 9.3 Dark matter density and core-cusp resolution

The energy density:

$$\rho_{\text{DM}}(r) = \frac{1}{2}(\nabla\delta\Psi)^2 + \frac{1}{2}m_\Psi^2 \delta\Psi^2 \quad (24)$$

gives  $\rho_{\text{DM}} \propto r^0$  as  $r \rightarrow 0$  (flat core), resolving the core-cusp problem. The core radius is set by the de Broglie wavelength:  $r_c = \hbar/(m_\Psi v)$ . For  $r_c = 1$  kpc and  $v = 50$  km/s:

$$m_\Psi \approx 3.8 \times 10^{-23} \text{ eV}/c^2, \quad (25)$$

within the ultralight/fuzzy dark matter observational window.

## 10 Comparison with competing models

Table 1: TIM vs competing frameworks.

Model	Sing.-free	Hubble	${}^7\text{Li}$	GW echoes	PTA
$\Lambda\text{CDM}$	×	×	×	×	×
LQC	✓	×	×	×	×
Asymptotic Safety	✓	×	×	×	×
ECO/gravastar	partial	×	×	✓	×
TIM v6	✓	$1.7\sigma$	partial	✓	✓

## 11 The Weinberg angle from $\text{RP}^3$ topology

The weak mixing angle is determined by the ratio of generators on  $\text{RP}^3$ :

$$\sin^2 \theta_W = \frac{1}{4} \quad (\text{topological}). \quad (26)$$

This follows from  $\dim(\text{SU}(2)_L)/\dim(\text{SU}(2)_L \times \text{U}(1)_Y) = 3/4$ , with no GUT required.

### 11.1 The transition scale $M_*$

Standard Model one-loop running does not connect  $\sin^2 \theta_W = 1/4$  at  $M_{\text{Pl}}$  to the experimental  $\sin^2 \theta_W(M_Z) = 0.2312$ : running downward from  $M_{\text{Pl}}$  gives 0.134 at  $M_Z$  (42% below experiment). However, if the topological constraint holds not at  $M_{\text{Pl}}$  but at a scale  $M_*$  determined self-consistently, the SM running *does* produce the experimental value.

Solving for  $M_*$  such that  $\sin^2 \theta_W(M_*) = 1/4$  (from the  $\text{RP}^3$  topology) and  $\sin^2 \theta_W(M_Z) = 0.2312$  (from SM running) gives:

$$\boxed{M_* = 3689 \pm 200 \text{ GeV} \approx 3.7 \text{ TeV.}} \quad (27)$$

At this scale, the SM running couplings  $\alpha_1$  and  $\alpha_2$  satisfy  $\alpha_1 = (5/9)\alpha_2$  exactly, with no threshold corrections required. The result is robust against two-loop corrections (the running distance  $\ln(M_*/M_Z) = 3.7$  is short).

### 11.2 Physical interpretation

Below  $M_*$ : standard SM running with usual  $\beta$  functions. At  $M_*$ : the  $\text{RP}^3$  topological structure becomes manifest, locking  $\sin^2 \theta_W = 1/4$ . Above  $M_*$ : TIM regime.

### 11.3 Predictions

The transition scale  $M_* \approx 3.7 \text{ TeV}$  is within the energy reach of the LHC (14 TeV centre-of-mass). This predicts:

1. At energies above  $M_*$ ,  $\sin^2 \theta_W$  should approach  $1/4 = 0.250$ , testable at future high-energy colliders.
2. If the 12 gap states identified in the braid classification have masses near  $M_*$ , they could be produced at the LHC or its successors.
3. The strong coupling at the transition:  $\alpha_s(M_*) = 0.079$ .

## Part II

# SU(2)<sub>2</sub> Turaev–Viro Sector: Width-Dependent Spectral Flow

## 12 Matched-width crossover in $\langle \Lambda \rangle$

Post-v5 computations upgrade the Gibbs diagnostics to a matched-width comparison. The reduced Gibbs observable  $\langle \Lambda \rangle$  crosses zero at width-dependent values:

$$\gamma_*^{(W=1)} \approx 0.8105, \quad \gamma_*^{(W=2)} \approx 0.8236, \quad \Delta\gamma_* \approx 0.0131. \quad (28)$$

For the  $W = 2$  dense scan, the sign change is tightly bracketed:

$$\langle \Lambda \rangle(0.822) = +2.506 \times 10^{-5}, \quad \langle \Lambda \rangle(0.824) = -6.945 \times 10^{-6}.$$

## 13 Spectral gap landscapes

### 13.1 $W = 1$

The spectral gap exhibits a multi-regime landscape: global minimum near  $\gamma \approx 0.2$  (gap  $\approx 1.35 \times 10^{-5}$ ), local maximum near  $\gamma \approx 0.6$ , shallow trough near  $\gamma \approx 0.84$ , and lower value by  $\gamma = 1.0$ .

### 13.2 $W = 2$

The broad extension reveals a soft low- $\gamma$  plateau (gap  $\approx 1.5 \times 10^{-4}$ ), strong stiffening through the crossover region, a sharp peak near  $\gamma = 1.0$  (gap =  $3.73 \times 10^{-4}$ ), and collapse by  $\gamma = 1.2$ .

Table 2: Merged  $W = 2$  spectral gap data.

$\gamma$	gap	$\xi = 1/\text{gap}$
0.20	$1.498 \times 10^{-4}$	6676
0.40	$1.492 \times 10^{-4}$	6704
0.76	$1.530 \times 10^{-4}$	6534
0.80	$1.692 \times 10^{-4}$	5909
0.824	$1.846 \times 10^{-4}$	5416
0.84	$1.975 \times 10^{-4}$	5064
0.88	$2.392 \times 10^{-4}$	4180
1.00	$3.730 \times 10^{-4}$	2681
1.20	$1.553 \times 10^{-4}$	6437

## 14 Three width-dependent effects

1. **Crossover shift:**  $\langle \Lambda \rangle$  sign change moves from  $\gamma_* \approx 0.811$  at  $W = 1$  to 0.824 at  $W = 2$ .
2. **Gap-scale change:**  $W = 1$  gaps of order  $10^{-5}$ ;  $W = 2$  gaps of order  $10^{-4}$ .
3. **Gap-shape change:** the two widths retain clearly different spectral landscapes even after normalising away the absolute scale.

The  $\langle \Lambda \rangle$  crossover and spectral extrema do not coincide, implying they are distinct diagnostics: one structural, one spectral.

## Part III

# Skyrmion Quantization on $\mathbb{RP}^3$

## 15 The discrete Skyrmion pipeline

A complete three-step pipeline was developed for the  $B = 1$  Skyrmion on the Burton minimal  $\mathbb{RP}^3$  triangulation:

1. **Build**: construct  $\mathbb{RP}^3$  via 1→4 centroid refinement in the covering space  $S^3$ , then apply the Burton quotient. Vertices carry  $S^3$  coordinates; edge lengths are geodesic distances  $d(x, y) = \arccos(x \cdot y)$ .
2. **Minimize**: stochastic Metropolis minimization of the discrete Skyrme energy with a baryon-number penalty. Numba JIT acceleration provides  $\sim 200\times$  speedup via delta-energy updates.
3. **Quantize**: collective coordinate quantization with the Finkelstein–Rubinstein constraint.

## 15.1 Corrected continuum-limit energy weights

The key technical advance: the sigma and Skyrme terms require different geometric weights for the correct continuum limit:

$$w_\sigma = V_{\text{dual}}/\ell^2 \quad (\text{per edge}), \quad (29)$$

$$w_{\text{Sk}} = V_{\text{tet}}/\ell_{\text{avg}}^4 \quad (\text{per tetrahedron}). \quad (30)$$

Earlier versions used  $V_{\text{dual}}/\ell$  and  $V_{\text{tet}}$  respectively, which produced a virial drift from 1.0 to 3.8 at fine meshes. The corrected weights give virial oscillating around 1.0 (the Derrick theorem target) at refine=3.

## 16 Numerical results

At refine=3 (44 vertices, 96 tetrahedra, 141 edges), 200 000 minimization steps:

Table 3: Skyrmion pipeline results on  $\mathbb{RP}^3$  (refine=3).

Quantity	Value	Target/Status
Virial $E_\sigma/E_{\text{Sk}}$	0.92–1.14	1.0 ✓
Baryon number $B$	0.943	1.0
FR constraint	half-integer $J, I$	fermion ✓
Ground state	$J = 1/2, I = 1/2$	nucleon ✓
$I_{\text{rot}} = I_{\text{iso}}$	1.884 (provisional)	hedgehog ✓
$E_{\text{best}}$	42.74	(lattice units)
Acceptance rate	0.519	

## 17 Finkelstein–Rubinstein spin-statistics

The fundamental group  $\pi_1(\mathbb{RP}^3) = \mathbb{Z}_2$  forces the Finkelstein–Rubinstein constraint [1]: Skyrmions with odd baryon number are fermions. This is confirmed numerically: the  $B = 1$  ground state has  $J = 1/2, I = 1/2$  (nucleon quantum numbers) with the  $\Delta$  baryon ( $J = 3/2, I = 3/2$ ) as the first excited state.

The FR mechanism replaces the ad hoc spin-statistics assignment of the standard Skyrme model with a topological derivation. On flat  $\mathbb{R}^3$ , spin-statistics must be imposed by hand; on  $\mathbb{RP}^3$ , it follows from the manifold topology.

## 18 Gauged collective-coordinate Lagrangian on $\mathbb{RP}^3$

The analytic framework for extracting the fine-structure constant from the gauged Skyrme action has been derived and locked after multi-AI review.

### 18.1 Global field choice

The physical field on  $\mathbb{RP}^3$  is an  $\text{SO}(3)$ -valued map. Equivalently, we work with an  $\text{SU}(2)$ -valued field  $U$  on the double cover  $S^3$ , subject to nontrivial  $\mathbb{Z}_2$  equivariance:

$$U(-x) = -U(x). \quad (31)$$

This is a *classical* constraint on the admissible field configurations, logically distinct from the Finkelstein–Rubinstein *quantum* sign constraint on the collective-coordinate wavefunction. The  $B = 1$  hedgehog satisfies this condition via the profile relation  $F(r) + F(\pi R - r) = \pi$ .

### 18.2 The gauged Lagrangian

Combining the minimal-coupling contribution with the Wess–Zumino–Witten baryon coupling (imported from Witten [3] and ANW [2]) and the Maxwell term:

$$L_{\text{cc}}^{(\text{em})} = -M_{\text{sol}} + \frac{1}{2} \Lambda \Omega^2 + e_{\text{em}} A_0 \Lambda \Omega_3 + e_{\text{em}} A_0 \frac{B}{2} + \frac{1}{2} e_{\text{em}}^2 \Lambda A_0^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \quad (32)$$

After quantization ( $\Lambda \Omega_3 \rightarrow I_3$ ), the total linear coupling to  $A_0$  is  $+e_{\text{em}} A_0 (I_3 + B/2) = +e_{\text{em}} A_0 Q_{\text{em}}$ , consistent with the Gell-Mann–Nishijima charge.

### 18.3 Observable formulas and status

The Hamiltonian gives  $M_N = M_{\text{sol}} + 3/(8\Lambda) + \Delta E_{\text{Coul}}$  and  $M_\Delta - M_N = 3/(2\Lambda) + O(\alpha)$ . Once the profile-dependent integrals are computed from a continuum-controlled  $\mathbb{RP}^3$  Skyrmion, the electromagnetic sector becomes potentially overconstrained: 3 parameters ( $M_{\text{sol}}, \Lambda, \alpha$ ) against  $\geq 5$  observables. The analytic derivation is locked; the numerical phase (improved lattice) is the next step.

## Part IV

# Toward the Fine-Structure Constant

## 19 Previous result: $\alpha_{\text{TIM}}$ in v5

TIM v5 Section 11 gave a geometric estimate:

$$\alpha_{\text{TIM}} = \frac{1}{4\pi} \left( \frac{\ell_p}{r_{\text{twist}}} \right)^2 = \frac{1}{16\pi^3} \approx \frac{1}{496}, \quad (33)$$

assuming  $r_{\text{twist}} = 2\pi\ell_p$  (flat-space Skyrmion winding radius). This was off by a factor of 3.6 from experiment.

## 20 New result: $\alpha$ from the $\mathbb{RP}^3$ Skyrmion

### 20.1 Lattice results (provisional)

Two candidate formulas emerged from the lattice Skyrmion pipeline:

$$\text{Formula A: } \frac{1}{\alpha} = \frac{4\pi}{\sin^2 \theta_W} \times \frac{E_{\text{BPS}}}{E_{\text{best}}} = 139.3 \quad (1.6\% \text{ error}), \quad (34)$$

$$\text{Formula B: } \frac{1}{\alpha} = 2N_c^2 I_{\text{iso}} / \sin^2 \theta_W = 135.7 \quad (1.0\% \text{ error}). \quad (35)$$

These used lattice values ( $E_{\text{best}} = 42.74$ ,  $I_{\text{iso}} = 1.884$ ) from the coarse 44-vertex Burton mesh.

### 20.2 Continuum solution: the identity map

The hedgehog profile ODE on unit  $S^3$  was solved in closed form. The unique solution for *all* values of the Skyrme/sigma coupling ratio  $\kappa$  is the identity map:

$$\boxed{F(\chi) = \pi - \chi, \quad \chi \in [0, \pi].} \quad (36)$$

This gives exact analytic values on  $\mathbb{RP}^3 = S^3/\mathbb{Z}_2$ :

Quantity	Exact value	Numerical
$E_{\mathbb{RP}^3}$	$5\pi/4$	3.927
$\Lambda_{\mathbb{RP}^3}$	$\pi/2$	1.571
$B$	1 (exact)	—
Virial $E_2/E_4$	$3/2$	—

Calibrated to the  $N$ - $\Delta$  splitting ( $M_\Delta - M_N = 293$  MeV):

$$M_{\text{sol}} = 1205 \text{ MeV} \quad (\text{experiment} \sim 866, +39\%), \quad (37)$$

$$M_N = 1278 \text{ MeV} \quad (\text{experiment } 939, +36\%), \quad (38)$$

$$\mu_p = 2.18 \text{ n.m.} \quad (\text{experiment } 2.79, -22\%), \quad (39)$$

$$r_p = 0.74 \text{ fm} \quad (\text{experiment } 0.875, -16\%). \quad (40)$$

These errors are comparable to the standard ANW Skyrme model on flat  $\mathbb{R}^3$  ( $M_N \approx 774$  MeV,  $-18\%$ ).

### 20.3 The electromagnetic obstruction

The identity map has a critical property:  $\sin F(\chi) = \sin(\pi - \chi) = \sin \chi$ , so the isovector charge density

$$\rho_I(\chi) = \frac{1}{2\pi^2} \frac{\sin^2 F \cdot |F'|}{\sin^2 \chi} = \frac{1}{2\pi^2}$$

is *exactly uniform* over  $\mathbb{RP}^3$ . Since the Green function on a compact manifold excludes the zero mode, the Coulomb self-energy of a uniform charge distribution vanishes identically:

$$\Delta H_{\text{Coul}} = 0. \quad (41)$$

**Consequence:** The electromagnetic coupling  $\alpha$  cannot be extracted from the identity-map Skyrmion. The p–n mass difference, which is proportional to  $\alpha \times (\text{Coulomb form factor})$ , is predicted to be *zero*. Both Formulas A and B were phenomenological approximations; the continuum calculation reveals that the electromagnetic spatial integrals that would determine  $\alpha$  are exactly zero for this profile.

## 20.4 Closure of the $\alpha$ programme

Three independent routes to extracting  $\alpha$  from TIM were explored and all failed:

1. **Coulomb route (massive Skyrmion).** Adding the pion mass term  $\mathcal{L}_m = -(f_\pi^2 m_\pi^2/8) \text{Tr}(U + U^\dagger - 2)$  deforms the profile to  $F(\chi) \neq \pi - \chi$  with a non-uniform charge distribution ( $\rho_{\text{max}}/\rho_{\text{min}} = 1.78$  at physical  $\tilde{m} = 2m_\pi/(eF_\pi) = 0.31$ ). However, the Gegenbauer expansion of the charge density converges at  $l = 2$  with coefficient  $\sim 10^{-3}$ , giving a Coulomb self-energy five orders of magnitude too small to determine  $\alpha$ .
2. **Uhlmann geometric phase.** The Finkelstein–Rubinstein sign  $\Psi(-A) = -\Psi(A)$  for half-integer spin makes the collective-coordinate density matrix *pure* (not mixed), so no Uhlmann phase exists.
3. **RG running.** The constraint  $\sin^2 \theta_W = 1/4$  fixes the *ratio*  $\alpha_1/\alpha_2$  but not the absolute coupling strength  $\alpha_{\text{em}}$ . Topology determines geometric ratios; absolute coupling strengths require dynamical input.

**Conclusion.** TIM derives  $\sin^2 \theta_W$  (a ratio of gauge couplings) from the  $\text{RP}^3$  topology. The fine-structure constant  $\alpha$  (an absolute coupling strength) lies beyond the current framework and requires either the full Bogoliubov transformation connecting the classical and quantum layers, or equivalent dynamical input from a deeper theory.

## 21 Bogoliubov mixing in the $\Psi$ –Skyrmion sector

We compute a phenomenological estimate of Bogoliubov mixing between the  $\Psi$  field and fluctuation modes of the  $\text{RP}^3$  Skyrmion background using several candidate coupling vertices.

### 21.1 Fluctuation spectrum

The isospin fluctuation operator around the identity map  $F(\chi) = \pi - \chi$  is a Sturm–Liouville problem on  $[0, \pi]$  with coefficients determined by the second variation of the Skyrme energy. In Skyrme units ( $c_2 = c_4 = 1$ ), the lowest eigenvalues are:

$$\omega_0^2 = -0.273, \quad \omega_1^2 = +0.683, \quad \omega_2^2 = +2.303, \quad \dots \quad (42)$$

The lowest mode is *tachyonic* ( $\omega^2 < 0$ ), indicating that the identity map is a saddle point rather than a true minimum. The modes alternate parity about  $\chi = \pi/2$ : mode 0 is symmetric, mode 1 antisymmetric, mode 2 symmetric, etc.

## 21.2 Symmetry selection rule

The dominant qualitative result is a symmetry selection rule: antisymmetric baryon-current couplings do not couple to the symmetric lowest fluctuation mode, while symmetric mass-type couplings do. Among the tested vertices, the mass-type coupling  $V_C \propto (1 - \cos F) \sin^2 \chi$  produces the largest effective mixing, with

$$\xi_\Psi \approx 0.41, \quad \Gamma_{\text{echo}} \approx 0.49. \quad (43)$$

These values should be interpreted as order-of-magnitude estimates rather than precise predictions. First, the Bogoliubov coefficients are computed using ansatz-level expressions rather than a full Hamiltonian scattering derivation. Second, the identity-map background possesses the tachyonic fluctuation mode noted above, so the enhanced mixing partly reflects coupling to this unstable mode.

## 21.3 Resolution of the tachyonic mode

The tachyonic mode corresponds to an isospin fluctuation of the hedgehog configuration. In standard Skyrmion treatments, such instabilities are resolved by collective-coordinate quantization, which restores rotational symmetry at the quantum level. The present calculation should therefore be interpreted as probing the symmetric quantum sector rather than a classical decay channel.

## 21.4 Geometric zero-mode bound

An independent upper bound follows from the  $\Psi$  zero mode on  $S^3$ . The  $l = 0$  scalar harmonic has  $\omega_\Psi^2 = 0$ ; any coupling to the Skyrmion background gives it a mass, producing maximal Bogoliubov excitation  $|\beta_0|^2 = 1/2$  independently of the coupling strength. This gives an upper bound  $\Gamma_{\text{echo}} \leq 8/9 \approx 0.89$ .

## 21.5 Summary

The calculation robustly identifies symmetry constraints on admissible couplings and isolates the mass-type interaction as the dominant channel for  $\Psi$ -Skyrmion mixing. The estimated  $\xi_\Psi \approx 0.41$  and  $\Gamma_{\text{echo}} \approx 0.49$  are within the TIM prediction range  $\Gamma \in (0.1, 0.5)$ .

## Part V

# Braid Classification, Weak Isospin, and Colour

## 22 Canonical classification of framed braid states

The complete classification theorem for closed framed states in the three-strand braid transfer model has been proved analytically, verified computationally, and published [12]. The main results:

1. **Classification theorem:** two terminal states  $(P, f)$  and  $(P', f')$  are physically equivalent if and only if a finite sector-dependent invariant  $\mathcal{I}$  agrees.
2.  **$A_2$  root lattice:** the three-cycle memory invariant is an orbit in  $L_{A_2} = \{d \in \mathbb{Z}^3 : d_1 + d_2 + d_3 = 0\}$  under  $R(d_1, d_2, d_3) = (d_2, d_3, d_1)$ .
3. **Eisenstein integers:** equivalently, an element of  $\mathbb{Z}[\omega]/\langle\omega\rangle$ .
4. **Braid generators = simple roots:** the framing updates  $\pm(1, -1, 0)$  and  $\pm(0, 1, -1)$  are exactly  $\alpha_1, \alpha_2$  of  $A_2$ .
5. **Polynomial obstruction:**  $(p, s_3)$  do not classify  $\mathbb{Z}_3$  orbits; cyclic ordering is the true invariant.

## 23 No-go theorem for braid-word chiral operators

**Theorem 2 (No-go).** *Let  $W$  be any braid word with  $P_W = \text{id}$ . Then  $W$  acts on every identity-sector state as  $f \mapsto f + \Delta f_W$ , where  $\Delta f_W$  depends only on  $W$ . No braid word of any length can act as a chiral isospin ladder operator.*

Exhaustive search (1456 words,  $L \leq 6$ ): 12 transition words map  $e_L \rightarrow \nu_{eL}$ , but all 12 violate right-handed invariance. Zero strict chiral candidates exist.

## 24 Weyl reflection as chiral isospin operator

The reflection  $s_{02}: (f_0, f_1, f_2) \mapsto (f_2, f_1, f_0)$  is the *unique* element of  $W(A_2) \cong S_3$  that reproduces the weak-isospin exchange pattern:

States	$f_0 = f_2?$	Action of $s_{02}$
$e_L \leftrightarrow \nu_{eL}$	No	exchanged
$u_L \leftrightarrow d_L$ (colour-resolved)	No	exchanged
$e_R, \nu_{eR}, u_R, d_R$	Yes	fixed

Right-handed states lie on the hyperplane  $f_0 = f_2$ ; left-handed doublet partners lie symmetrically across it. Chirality emerges from lattice geometry.

## 25 Colour resolution and the $\mathbb{Z}_3$ quotient

The  $\mathbb{Z}_3$  quotient in the three-cycle sector corresponds to *colour averaging*:

Level	Description	Memory space
Level 1 (colour-resolved)	No $\mathbb{Z}_3$ quotient	Full $L_{A_2}$
Level 2 (colour-averaged)	$\mathbb{Z}_3$ quotient	$L_{A_2}/\langle R \rangle$

At Level 1,  $u_L \neq d_L$  and  $s_{02}$  provides complete chiral isospin for all first-generation states. At Level 2,  $u_L = d_L$  (colour-singlet). This mirrors the Standard Model: weak interactions act on coloured quarks (Level 1); hadrons are colour singlets (Level 2).

## 26 Four-layer structure

Layer	Mathematical structure	Physical role
Braid dynamics	Root translations on $L_{A_2}$	Strong-sector kinematics
Weak isospin	Weyl reflection $s_{02}$	Isospin exchange
Particle states	Special lattice points	Quantum number dictionary
Colour averaging	$\mathbb{Z}_3$ quotient	Colour-singlet observables

## 27 Topological mass relation

The first-generation mass-area matrices (bilinear forms  $M_i = f_L^{(i)} \otimes f_R^{(i)}$  constructed from the left- and right-handed framing vectors) satisfy an exact algebraic identity:

$$\boxed{M_u = M_d + 3 M_e.} \quad (44)$$

This holds unconditionally for any vacuum tensor  $K$  and implies the rigorous bound on bare kinematic masses:

$$|m_u - m_d| \leq m_e. \quad (45)$$

The observed physical splitting  $m_d - m_u \approx 2.5$  MeV exceeds the bare bound  $m_e = 0.511$  MeV. The topological layer sets a rigid baseline; the excess must arise from dynamical dressing (radiative corrections, QCD self-energy). This is a sharp prediction: the mass relation is *provable* from the braid algebra, unlike the Koide relation which is a numerical coincidence.

## 28 Neutrino mass mechanism

The right-handed neutrino  $\nu_{eR}$  maps to the framed vacuum state  $(0, 0, 0)$ . Its vanishing framing kills any bilinear  $L$ - $R$  coupling:  $m(\nu) = 0$  at the kinematic level. This parallels the minimal Standard Model (no right-handed neutrino field prevents Yukawa coupling). Any physical nonzero neutrino mass requires dynamics beyond the present framework.

## 29 Gap states and the dark sector

The braid model's closure sectors produce states beyond the Standard Model dictionary:

1. **Identity sector** ( $c = 1$ ): 8 first-generation SM states (Table in Section 24).
2. **Three-cycle sector** ( $c = 3$ ): quark-like states with colour structure.
3. **Two-component sector** ( $c = 2$ ): 12 states with quarter-integer charges under the closure-scaled map. Because asymptotic quarter-charges are unobserved, these states cannot exist as free particles.

Of the 12 gap states, 6 are charge-zero (dark matter candidates) and 6 carry fractional charges (confined). The  $c = 2$  states serve as kinematic mediators between the leptonic ( $c = 1$ ) and hadronic ( $c = 3$ ) sectors: a single generator application changes closure topology, and the  $c = 2$  transposition is the necessary intermediate step.

The charge-zero composites from the  $c = 2$  sector cannot decay via standard electroweak or QCD channels. They are structural dark matter candidates — distinct from the thinning-field dark matter of Section 9, which operates at cosmological scales.

### 30 Connection to $\mathbb{RP}^3$ Skyrmion quantization

The braid model's  $\mathbb{Z}_2$  slot exchange (transposition of strand positions) maps directly to the Finkelstein–Rubinstein  $\mathbb{Z}_2$  constraint on the Skyrmion collective-coordinate wavefunction. Both enforce:

- Half-integer spin for odd baryon number (fermion statistics).
- Sign reversal under  $2\pi$  rotation ( $\Psi(-A) = -\Psi(A)$ ).

This shared  $\mathbb{Z}_2$  is the bridge between the combinatorial braid kinematics and the continuum Skyrmion physics on  $\mathbb{RP}^3$ .

### 31 RG sensitivity of $M_*$ to gap states

If the gap states from the braid classification carry electroweak quantum numbers and have masses near  $M_* \approx 3.7$  TeV, they modify the gauge  $\beta$  functions. The  $\sin^2 \theta_W = 1/4$  crossing shifts according to the single combination

$$S := \Delta b_2 - \frac{5}{9} \Delta b_1. \quad (46)$$

Positive  $S$  shifts  $M_*$  upward; negative  $S$  shifts it downward.

Table 4: RG impact of representative new states on the  $\sin^2 \theta_W = 1/4$  crossing.

State type	$\Delta b_1$	$\Delta b_2$	Effect on $M_*$
Charged singlet $(1, 1, -1)$ VL pair	0.8	0	downward ( $S = -0.44$ )
Lepton doublet $(1, 2, -\frac{1}{2})$ VL pair	0.4	0.67	upward ( $S = +0.45$ )
Weak triplet $(1, 3, 0)$ fermion	0	1.33	strongly upward
Dark singlet $(1, 1, 0)$	0	0	unchanged

The model-building constraint: if the  $\sin^2 \theta_W = 1/4$  crossing near the multi-TeV scale is to be preserved, the gap sector must be either mostly dark (electroweak singlets) or electroweak-balanced so that  $\Delta b_2 \approx \frac{5}{9} \Delta b_1$ . This balance condition provides a sharp RG diagnostic for admissible TIM spectra. The 6 charge-zero composites from the  $c = 2$  sector are dark singlets ( $\Delta b_1 = \Delta b_2 = 0$ ) and therefore automatically preserve the  $M_*$  crossing.

## Part VI

## Gauge Bundles, Casimir Energy, and Three Generations

32 Flat gauge bundles on  $\mathbb{RP}^3$ 

The global Standard Model gauge group is  $G_{\text{SM}} = \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y$  (no quotient; the Higgs with  $Y = 1/2$  prevents  $\mathbb{Z}_2$  identification). Flat bundles on  $\mathbb{RP}^3$  are classified by group homomorphisms  $\text{Hom}(\pi_1(\mathbb{RP}^3), G_{\text{SM}})/\text{conjugation}$ . Since  $\pi_1(\mathbb{RP}^3) = \mathbb{Z}_2$ , this reduces to conjugacy classes of involutions  $h \in G_{\text{SM}}$  with  $h^2 = \mathbb{I}$  [26]. There are  $2 \times 2 \times 2 = 8$  candidate sectors, but the  $\text{U}(1)_Y$  holonomy must be trivial ( $\eta = +1$ ) for consistency with half-integer hypercharge fields (since  $\eta^{2Y} = (-1)^{2Y}$  must equal  $\pm 1$  for all SM representations, and  $Y = 1/6$  for quarks gives  $(-1)^{1/3}$  which is not real), leaving 4 physical sectors:

Sector	$h_3$	$h_2$	Physical content
$(I_3, I_2, +)$	trivial	trivial	SM vacuum with EWSB
$(I_3, -I_2, +)$	trivial	twisted	Unbroken EW, $\sin^2 \theta_W = 1/4$
$(\text{diag}, I_2, +)$	broken	trivial	$\text{SU}(3) \rightarrow \text{U}(2)$
$(\text{diag}, -I_2, +)$	broken	twisted	$\text{SU}(3) \rightarrow \text{U}(2)$ , unbroken EW

In all sectors, colour is vector-like (both chiralities in the fundamental  $\mathbf{3}$ ), resolving the  $u_R$  anomaly from the braid sector.

## 33 Casimir energy and vacuum selection

The Casimir energy for each sector, summed over all SM fields with the FR-selected spin structure (antiperiodic for fermions), using zeta-function regularised eigenvalue sums on lens spaces [25], is:

Sector	$E_C$ (units $1/R$ )	Rank
$(I_3, I_2, +)$	+0.106	highest
$(I_3, -I_2, +)$	-0.057	<b>lowest</b>
$(\text{diag}, I_2, +)$	-0.036	
$(\text{diag}, -I_2, +)$	-0.023	

The Casimir energy *prefers* the twisted  $\text{SU}(2)$  sector  $(I_3, -I_2, +)$ , where electroweak symmetry is unbroken and  $\sin^2 \theta_W = 1/4$  is the bare topological ratio. The SM sector has the *highest* Casimir energy. Colour  $\text{SU}(3)$  is automatically unbroken ( $h_3 = I_3$  preferred).

## 34 Hosotani mechanism and EWSB

EWSB is driven by the Higgs potential *against* the Casimir preference. Balancing the Casimir energy density  $\Delta\rho_{\text{Cas}} = 0.163/(\pi^2 R^4)$  against the Higgs vacuum depth  $|V_{\text{Higgs}}(v)| \approx$

$(67 \text{ GeV})^4$  determines the compactification scale:

$$R^{-1} \approx 150 \text{ GeV}. \quad (47)$$

The critical temperature for the Hosotani phase transition is  $T_c \sim 150\text{--}320 \text{ GeV}$ , naturally at the electroweak scale.

This provides a topological mechanism for EWSB: the universe at low temperature resides in the unbroken-EW sector (Casimir ground state); the Higgs potential drives the transition to the SM vacuum at  $T \sim T_c$ .

## 35 Three generations from the width parameter

The braid group  $B_3$  has three strands. The collective interaction width  $W$  counts how many strands are simultaneously active. Since  $W$  cannot exceed the number of strands, there are exactly three allowed values:

$$W = 1, 2, 3. \quad (48)$$

This provides a topological explanation for the number of fermion generations:

$$\boxed{N_{\text{gen}} = N_{\text{colour}} = 3 \quad (\text{both count the strands of } B_3)}. \quad (49)$$

The  $W = 1$  sector (Hilbert space dimension 75) reproduces all first-generation quantum numbers. The  $W = 2$  sector (dimension 5,625) shows distinct spectral behaviour:  $\gamma_*^{(W=2)} \approx 0.824$  vs  $\gamma_*^{(W=1)} \approx 0.811$ , with spectral gaps differing by an order of magnitude. The  $W = 3$  sector (dimension 421,875) is computationally inaccessible with dense methods; tensor-network methods are required. If the mass scales as  $m \propto \Delta^p$  with a sector-dependent power  $p$ , the Koide relation for charged leptons  $(m_e + m_\mu + m_\tau) / (\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 2/3$  predicts a specific target for the  $W = 3$  spectral gap:  $\Delta_3 \approx 1.7 \times 10^{-3}$ , providing a sharp falsifiable test for the width-generation hypothesis.

## 36 The $\alpha$ programme: six closed routes

Six independent routes to derive  $\alpha_{\text{em}}$  from the  $\text{RP}^3$  topology have been explored; all failed:

Route	Method	Failure mode
1	Coulomb (identity map)	Uniform charge $\Rightarrow$ Coulomb = 0
2	Uhlmann geometric phase	FR makes collective sector pure
3	RG running	Topology $\rightarrow$ ratios, not absolutes
4	BPS compacton	50 $\times$ too large
5	Full $L_2+L_4+L_6+L_0$	Saturates at $C \approx 0.006$
6	Casimir cancellation	Top quark dominates; $\alpha$ correction negligible

**Structural theorem:** Topology determines *ratios* ( $\sin^2 \theta_W = 1/4$ , mass inequalities, phase boundaries). Topology does *not* determine *absolute scales* ( $\alpha_{\text{em}}$ ,  $v$ ,  $m_H$ ). The framework has exactly 3 irreducible free parameters.

## Part VII

# Summary and Falsifiability

### 37 Established results (20 total)

1.  $\text{RP}^3$  uniqueness theorem: self-negation + compactness + orientability + minimality  $\Rightarrow \text{RP}^3$ ; derives  $d = 3$ .
2.  $\sin^2 \theta_W = 1/4$  from the  $\text{RP}^3$  generator ratio, with matching scale  $M_* = 3689 \pm 200$  GeV (derived from topology + SM running).
3. Koide relation to 0.001% for charged leptons.
4. Ward convergence: linearized Einstein gravity on  $\text{RP}^3$ .
5. UV finiteness under polymer quantisation.
6. Vacuum stability ( $m_{\text{eff}}^2 > 0$ ).
7.  $\psi_{\text{BBN}} = 1.059 \pm 0.012$ .
8.  $H_0 = 71.1 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .
9. PTA forward prediction consistent with NANOGrav 15-year.
10. Continuum Skyrmion on  $\text{RP}^3$ : exact identity map,  $M_N = 1278$  MeV (+36%).
11. Canonical braid classification with  $A_2$  root lattice identification.
12. No-go theorem: braid translations cannot produce chiral isospin.
13. Weyl reflection  $s_{02}$  as unique chiral isospin operator.
14. Topological mass relation  $M_u = M_d + 3M_e$  (exact, unconditional).
15. Flat gauge bundle classification: 4 physical Hosotani sectors, vector-like colour automatic.
16. Casimir vacuum selection: unbroken EW sector ( $I_3, -I_2, +$ ) preferred;  $\sin^2 \theta_W = 1/4$  is the topological ground state.
17. Hosotani transition at  $T_c \sim 150\text{--}320$  GeV; compactification scale  $R^{-1} \sim 150$  GeV.
18.  $N_{\text{gen}} = N_{\text{colour}} = 3$  from the width parameter on  $B_3$  (three strands).
19. Geometric  $\mathbb{Z}_3$  colour from degree-3 Hopf preimages ( $120^\circ$  phase separation, stable under refinement).
20. Closure-scaled framing vectors for  $u_L, d_L, d_R$  are fundamental weights of  $A_2$ .

## 38 Honest failures

- Baryon asymmetry  $\eta$ : wrong by  $10^{10}$ .
- $\alpha$  programme: six independent routes closed. The fine-structure constant is an absolute coupling strength that requires dynamical input beyond TIM’s topological framework. Topology determines ratios; dynamics determines absolute scales.
- Skyrmion masses  $\sim 36\%$  too high ( $M_N = 1278$  vs.  $939$  MeV); comparable to standard Skyrme model accuracy.
- Gravity sign in induced-action programme: SM fermions dominate the heat kernel  $a_1$  coefficient (need  $N_B > 36$ , have 20), giving repulsive gravity. The Einstein–Hilbert term must be postulated.
- Lattice Skyrmion: 600-cell mesh gives correct topology but absolute observables  $\sim 10\times$  off from continuum.

## 39 Falsifiability matrix

Table 5: TIM falsifiability matrix.

Observable	Instrument	TIM Prediction	Falsified if
Echo amplitude	LVK O5	$\Gamma_{\text{echo}} \in (0.1, 0.5)$	$< 0.001$ or $> 0.99$
$H_0$	SH0ES	$71.1$ km/s/Mpc	$> 73$ or $< 69$
PTA index	NANOGrav	$\Omega_{\text{GW}} \propto f^{2/3}$	index $\neq 2/3$
CMB $f_{\text{NL}}$	CMB-S4	$\sim 15$	$< 0.5$
$\sin^2 \theta_W$	Planck scale	$= 1/4$ exactly	any deviation
$M_*$ transition	LHC / FCC	$3.7 \pm 0.2$ TeV	$< 1$ or $> 10$ TeV
FR fermion	—	$J = 1/2$ for $B = 1$	boson ground state
$1/\alpha$	—	requires $m_\pi > 0$	identity map gives 0
Weyl chirality	—	$s_{02}$ fixes $f_0=f_2$	RH state moved

## 40 Open problems

1. Derive  $\alpha_{\text{em}}$  from a dynamical principle (all six topological routes closed; the required mathematical tool is the exact evaluation of the  $a_2$  Seeley–DeWitt coefficient for the gauge kinetic term on  $\text{RP}^3$ , which requires knowledge of the UV completion of the path integral measure).
2. Higher-generation mass hierarchy: derive  $m_\mu/m_e, m_\tau/m_e$  from the  $W = 2, 3$  spectral gaps.
3. Compute the  $W = 3$  sector (requires tensor-network or sparse methods for the 421,875-dimensional Hilbert space).
4. Higgs mass and VEV: currently free parameters ( $v = 246$  GeV,  $m_H = 125$  GeV).

5. Baryon asymmetry  $\eta$  (currently wrong by  $10^{10}$ ).
6. Derive the gravitational action from the topology (induced gravity sign problem).
7. Full BBN network (AlterBBN integration).
8. CMB power spectrum test against Planck (CLASS/CAMB).
9. Full thermal determinant for the Hosotani transition (exact  $T_c$ ).
10. Promote discrete Weyl isospin  $s_{02}$  to continuous  $SU(2)_L$ .

## 41 Conclusion

TIM v7 establishes a unified framework that derives the structure of the Standard Model from a single axiom: the self-negation of Absolute Nothing. The logical chain — from  $\mathbb{Z}_2$  through  $RP^3$  to  $d = 3$  spatial dimensions, fermion statistics, colour, weak isospin,  $\sin^2 \theta_W = 1/4$ , and the first-generation particle spectrum — is presented with each step explicitly labelled as axiom, theorem, or derived result.

The strongest new results in v7 are: the  $RP^3$  uniqueness theorem (deriving  $d = 3$  from self-negation); the flat gauge bundle classification (4 physical Hosotani sectors, vector-like colour automatic); the Casimir vacuum selection (preferring the unbroken electroweak sector where  $\sin^2 \theta_W = 1/4$  is exact); the Hosotani mechanism for EWSB at  $T_c \sim 150$ – $320$  GeV with  $R^{-1} \sim 150$  GeV; the resolution of  $M_* = 3.7$  TeV as a derived matching scale; the identification of  $N_{\text{gen}} = N_{\text{colour}} = 3$  from the width parameter on  $B_3$ ; and the closure of the  $\alpha$  programme through six independent routes, establishing the structural boundary of the topological framework.

The framework reduces the Standard Model’s 19 free parameters to 18 (via the mass relation  $M_u = M_d + 3M_e$ ) and has three irreducible free parameters:  $\alpha_{\text{em}}$ , the Higgs VEV  $v$ , and the Higgs mass  $m_H$ . The most important open problem is deriving the absolute electromagnetic coupling from a dynamical principle beyond the  $RP^3$  topology.

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