The Bałaban–Dimock Structural Package: Derivation of Polymer Representation, Oscillation Bounds, and Large-Field Suppression for Lattice Yang–Mills Theory from Primary Sources — a Traceability and Conditional-Discharge Package*

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Abstract

We provide a self-contained, equation-level traceability derivation of the three structural hypotheses — polymer representation (A1), per-link oscillation bounds with irrelevance factor (A2), and large-field suppression (B5) — that were assumed in the companions *Doob Influence Bounds for Polymer Remainders in 4D Lattice Yang–Mills Renormalization* and *RG–Cauchy Master Framework*. All results are traced to precise equations in the primary sources: T. Bałaban (Commun. Math. Phys., 1984–1989) and the expository trilogy of J. Dimock (2011–2014). The translation from Bałaban's analytic norms on gauge-covariant function spaces to the per-link oscillation language of the probabilistic framework is made explicit (Section 10). Version 2 corrects the status of the discharge: it is *traceable and conditional*, not unconditional. (i) The small factor of Theorem 8.4 (= Eq. (1.89) of [12]) carries the constant $2/(1+\beta_{\mathrm{LF}})$; whether β_{LF} is $O(1)$ or a large reference coupling is precisely the dichotomy adjudicated against the audited series (2602.0052/0056/0057 v2), where the large reading trivializes the factor ($e^{-c\,p_0} \approx 0.95$) and forces hypothesis (H-P0); the dichotomy is now stated as an explicit open interface question (Remark 8.7). (ii) The summability claim (M3) of the RG–Cauchy interface was justified in v1 by "super-polynomial decay from asymptotic freedom"; with the profile $p_0(g) = A_0(\log g^{-2})^{\theta_0}$ the decay in the scale index j (distance to the infrared end) is $e^{-A_0(\ln j)^{\theta_0}}$: sub-polynomial for $\theta_0 < 1$ (sum diverges), j^{-A_0} at $\theta_0 = 1$ (converges iff $A_0 > 1$), and super-polynomial only for $\theta_0 > 1$. (M3) is therefore conditional on the explicit profile condition $\theta_0 > 1$ (Remark 12.1; companion suite, check 6). (iii) The irrelevance factor $(L^k\eta)^{4+\alpha}$ is geometric in the distance to the ultraviolet cutoff, not in the infrared direction; the direction-of-limit bookkeeping for (M1) is made explicit (Remark 10.4) and remains hypothesis-level until the Doob companion is audited. What is machine-verified

*Version 2. Version 1 stated that this paper "completes the unconditional discharge of the UV structural inputs." That description is corrected: the discharge is *traceable* (every statement carries a precise primary-source citation) but *conditional* on (i) the faithful transcription of the cited Bałaban/Dimock results, (ii) the $O(1)$ reading of the constant β_{LF} in Eq. (1.89) of [12] (Remark 8.7: under the large-reference reading adjudicated in the series audit the small factor trivializes and hypothesis (H-P0) is inherited), and (iii) the profile condition $\theta_0 > 1$ for the summability claim (M3) (Remark 12.1: v1's justification "super-polynomial decay from asymptotic freedom" is incorrect as stated). v2 also adds the direction-of-limit bookkeeping for the irrelevance factor (Remark 10.4), cites the audited companion chain (ai.viXra:2602.0041 v3, 2602.0051–0057 v2, 2602.0063 v3), and records that the two consumer companions (*Doob Influence Bounds* and *RG–Cauchy Master*) are not yet audited. See the Note added in v2 (Section 13).

in the companion suite: the abelian RG operator algebra (Lemma 2.2 mechanics), propagator decay and the random-walk expansion, exponential sum control, lattice-animal counting (Lemma C.1), and the oscillation–analyticity bridge with its Cauchy constants. Together with the (unaudited) Doob companion, this package provides a *conditional* discharge of the UV structural inputs at finite volume; the finite-volume, ultraviolet character of the package is what shields it from the infrared volume window of the audited chain.

Contents

1 Introduction

1.1 Context and purpose

The program of T. Bałaban [1–12] is the most complete rigorous construction of renormalized lattice gauge theories in four dimensions; Dimock's trilogy [13–15] provides a modern template on φ_3^4. This paper extracts the precise technical results needed to discharge the structural hypotheses assumed in the companions *Doob Influence Bounds* and *RG–Cauchy Master*:

(A1) Polymer representation. The irrelevant part of the effective action at scale k admits $V_k^{\mathrm{irr}}(U) = \sum_{X\in\mathcal{P}_k} K_k(X; U|_X)$ with local, analytic, exponentially decaying activities.

(A2) Oscillation bound with irrelevance factor. For each bond e at scale k,

$$\operatorname{osc}_e\big(K_k(X;\cdot)\big) \;\le\; C_{\mathrm{osc}}\,(L^k\eta)^{4+\alpha}\,|X|^p\,e^{-\kappa\,d_k(X)}, \tag{1}$$

with $\alpha > 0$ and d_k the multi-scale tree distance.

(B5) Large-field suppression. The R-operation replaces large-field contributions by small-field substitutes preserving the partition function, with residual activities

$$|R^{(k)}(X)| \;\le\; e^{-p_0(g_k)}\,e^{-\kappa\,d_k(X)}. \tag{2}$$

Status (v2). The discharge of (A1)/(A2)/(B5) below is *traceable*: each step cites a numbered equation of [1]–[15]. It is *conditional* in the three respects listed in the title footnote and §12; the v1 phrase "unconditional discharge" is withdrawn.

1.2 Logical architecture

§2–5: setup, propagators, gauge fixing, background fields. §6: effective actions, Ward–Takahashi identities, β-function; discharge of (A2). §7: cluster expansions; discharge of (A1). §8: the R-operation; discharge of (B5). §9: convergence. §10: the analytic-to-oscillation translation. §11: the Dimock template. §12: derived axioms and interfaces (with the v2 corrections). §14: traceability table.

1.3 Notational conventions

Definition 1.1 (Lattice structures)**.** *T_η is the unit-scale torus with UV lattice spacing $\eta = L^{-N}$; Λ_k the scale-k block lattice; g_k the effective coupling at scale k (increasing toward the infrared by asymptotic freedom).*

Definition 1.2 (Polymer spaces and distances)**.** *Fix a partition of T_η into M-cubes; $\mathcal{P}_k$ denotes connected unions of scale-k cubes; $d_k(X)$ the multi-scale tree distance of [2], Eq. (2.46) (Definition 3.4).*

2 Lattice Setup and the RG Transformation

Definition 2.1 (Abelian RG operators)**.** *The block averaging operator Q, its adjoint Q^*, the fluctuation projector $1 - Q^*Q$-type operators, and the minimizer map H of the quadratic blocking problem, as in [1], §1.*

Lemma 2.2 (Algebraic identities [1], Eqs. (1.95)–(1.97))**.** *With the normalized block average Q ($QQ^* = \mathrm{id}$ on the coarse space): Q^*Q is an orthogonal projection; the minimizer H of $\langle A, (-\Delta)A\rangle + a\|QA - B\|^2$ satisfies $QHB \to B$ as $a \to \infty$ and H is linear with $QH = \mathrm{id} + O(1/a)$; and the Gaussian blocking kernels compose consistently across scales.* (v2) *These mechanics are machine-verified in the companion suite (check 1) on explicit finite abelian lattices.*

3 Propagators, Random Walks, and Exponential Decay

Proposition 3.1 (Exponential decay of G [1], Proposition 1.2)**.** *There exists $\delta_0 > 0$ such that the fluctuation propagator $G = (-\Delta + aQ^*Q)^{-1}$ satisfies $|G(x,y)| \le Ce^{-\delta_0|x-y|}$, uniformly in the volume.*

Proposition 3.2 (Random walk expansion for G)**.** *For M_0 sufficiently large, G admits a convergent random-walk (Neumann) expansion in localized blocks, with geometric truncation error.* (v2) *Verified numerically: check 2 measures δ_0 and the geometric convergence of the expansion on finite lattices.*

Remark 3.3. *The expansion implies $(GJ)(x)$ depends on J only weakly at large distance — the locality input used in Step 3 of the bridge (§10).*

Definition 3.4 (Point-to-point multi-scale distance [2], Eq. (2.46))**.** *$d_k(x,y)$ rescales distance at each scale and sums over the scale hierarchy; $d_k(X)$ is the induced tree distance on polymers.*

Lemma 3.5 (Exponential sum control [2], Lemma 2.1)**.** *For R_M sufficiently large (specifically $\frac{1}{4}\alpha\delta_0 R_M > 2d\log c_0(\frac{1}{2}\alpha) + 1$): $\sum_{X \ni e} e^{-\alpha\delta_0 d_k(X)} \le c_1 < \infty$, uniformly in k and the volume.* (v2) *Numerically verified above the stated threshold (check 3).*

Proposition 3.6 (Multi-scale propagator bounds [2], Proposition 2.6)**.** *The propagator G obeys exponential bounds in the multi-scale distance of Definition 3.4.*

Corollary 3.7 (Background field kernel [2], Corollary 2.8)**.** *The background-field kernels inherit the decay of Proposition 3.6.*

4 Non-Abelian Gauge Fixing and Regular Spaces

Definition 4.1 (Regular configurations [4], Eqs. (1.7)–(1.9))**.** *Nested spaces $\mathcal{U}_k$ of configurations with plaquette deviations bounded scale-wise.*

Theorem 4.2 (Quantitative axial-to-Landau gauge fixing [4], Theorem 2)**.** *Every $U_0 \in \mathcal{U}_k$ admits a Landau-gauge representative with quantitative bounds; the passage is by contraction mapping.*

Remark 4.3 (Proof mechanism)**.** *Contraction mapping on the gauge slice; the constants enter the analytic radius of Definition 4.4.*

Definition 4.4 (Analytic domain)**.** *$\mathcal{R}_k$ consists of complexified gauge-algebra elements within a uniform complex radius $r > 0$ of the small-field domain; by Theorem 4.2 every configuration in $\mathcal{U}_k$ has a representative in $\mathcal{R}_k$.*

5 The Variational Problem and Background Fields

Theorem 5.1 (Background field [6], Theorem 1; [5], Proposition 9). *The blocking variational problem has a unique minimizer $U_0^{(k)}(V)$ depending analytically on the coarse field V, with multi-scale decay of its kernels.*

Proposition 5.2 (Local gauge chart [4], Proposition 6). *On any cube $\square \subset \Omega_j$, the minimizer admits a local Landau chart with uniform bounds.*

6 Small-Field Effective Actions, Ward–Takahashi Identities, and the β-Function

Without cancellations the sum of small-polymer contributions diverges as η^{-4} ([8], Eq. (0.26)). The rescue: (1) large polymers are automatically small ([8], Eq. (0.27)); (2) Taylor expansion to order 5 ([8], Eq. (3.34)), remainder $(L^j\eta)^5$; (3) Ward–Takahashi identities kill the lower-order terms:

Theorem 6.1 (Ward–Takahashi identities [8], §4). *Gauge invariance implies (i) no tadpole ([8], Eq. (4.14)); (ii) the quadratic form is transverse; (iii) the only marginal invariant is the field-strength renormalization absorbed in the coupling flow.*

Proposition 6.2 (Structure of $\Pi_{\mu\nu}$ [8], §5). *The vacuum polarization tensor is transverse with logarithmic coefficient fixing b_0.*

Definition 6.3 (β-function [8], Eq. (5.42)). $g_{k+1}^{-2} = g_k^{-2} + 2b_0 \ln L + O(g_k^2)$ *(ultraviolet-to-infrared direction; sign consistent with asymptotic freedom and with the corrected flow of the audited chain, 2602.0051 v2).*

Theorem 6.4 (Irrelevance bound — discharge of (A2) [8], Eq. (0.29)). *With $\mathcal{V}^{(j)}(X, U_k)$ the order-≥ 5 remainder plus large-polymer part, $\|\mathcal{V}^{(j)}(X,\cdot)\|_j \leq C\,(L^j\eta)^{4+\alpha} e^{-\kappa d_j(X)}$.*

Corollary 6.5 (UV stability [8], Eq. (0.30)). *The summed effective actions are bounded uniformly in N at fixed physical volume.*

Theorem 6.6 (Discrete Callan–Symanzik equation [8], Theorem 2). *The effective couplings obey the discrete flow of Definition 6.3 with controlled remainder.*

7 Cluster Expansions and Polymer Exponentiation

Lemma 7.1 (Localized fluctuation action [9], Lemma 2). *The fluctuation-plus-correction action decomposes into localized pieces ([9], Eq. (1.41)).*

Lemma 7.2 (Polymer activities [9], Lemma 3). *After Mayer expansion and conditioning, the partition function is an exponential of a polymer gas.*

Theorem 7.3 (Polymer representation — discharge of (A1) [9], Eq. (2.41)). $V_k^{\mathrm{irr}} = \sum_X K_k(X;\cdot)$ *with (a) locality, (b) analyticity on $\mathcal{R}_k$, (c) $|K_k(X)| \leq E_0 e^{-\kappa d_k(X)}$.*

Definition 7.4 (M-tree distance with holes (Dimock)). *As in [14], Appendix F.*

Theorem 7.5 (Cluster expansion with holes [14], Theorem F.1). *Convergent expansion on unions of cubes with excluded (large-field) holes.*

8 Large-Field Renormalization and the R-Operation

In $d = 4$ the coupling varies only logarithmically and the naive bound $\exp(-p_0(g_k))$ with profile

$$p_0(g) \;=\; A_0\,(\log g^{-2})^{\theta_0}, \qquad A_0 > 0,\ \theta_0 > 0, \tag{3}$$

does not produce positive powers of the large-field threshold; the R-operation resolves this.

Definition 8.1 (R-operation [11], Eqs. (0.3)–(0.6))**.** *Decomposition of the density into large-field regions replaced by small-field substitutes, preserving $\int dV\,\rho$.*

Proposition 8.2 (Normalization [11], Eq. (0.4); Eq. (1.102))**.** *The replacement preserves the partition function identically.*

Proposition 8.3 (Minimizer in the large-field domain [11], Proposition 1)**.** *The constrained variational problem has a unique minimizer with quantitative bounds up to the large-field boundary.*

Theorem 8.4 (Small factor per component [12], Eq. (1.89))**.**

$$\big\|T_k(X)\,1\big\| \;\le\; \exp\Big(-\frac{2}{1+\beta_{\mathrm{LF}}}\,p_0(g_k)\Big), \qquad \beta_{\mathrm{LF}} > 0 \textit{ fixed}, \tag{4}$$

with additional small factors from large plaquettes, large fluctuations, and residual fields ([12], pp. 381–386).

Theorem 8.5 (Polymer representation of large-field corrections [12], Eqs. (1.98)–(1.100))**.** *The R-operation output exponentiates into polymer activities $R^{(k)}(X)$ satisfying (2).*

Theorem 8.6 (UV stability [12], Theorem 1, Eq. (0.1))**.** *Bilateral stability bounds for the density, uniform in the scale index, at finite volume.*

Remark 8.7 (The β_{LF} dichotomy — new in v2)**.** *Everything quantitative in (B5) hinges on the size of β_{LF} in (4). Two readings are on record.* (a) $O(1)$ reading *(this paper's v1, implicitly): β_{LF} is a fixed order-one constant of the R-operation; then $2/(1+\beta_{\mathrm{LF}}) = O(1)$ and (4) is a genuine small factor.* (b) Large-reference reading *(series audit, 2602.0052 v2 flag F2; reproduced in 2602.0053/0055/0056/0057 v2): the constant is $2/(1+\beta_0)$ with β_0 the large ultraviolet reference coupling ($\beta_0 = 100$ in the representative arithmetic), giving $e^{-c\,p_0(\gamma_0)} \approx 0.9516$ — no suppression — and forcing the power-law hypothesis (H-P0). The present paper cannot settle the dichotomy by citation alone: [12] must be re-derived at the level of its Eq. (1.73)–(1.89) constants. Until then, every consumer of (B5) must carry the disjunction "$\beta_{\mathrm{LF}} = O(1)$, or (H-P0)". The companion suite (check 7) tabulates both readings.*

9 Convergence of the Full Expansion

Theorem 9.1 (Stability bound [15], Theorem 2)**.** *In the φ_3^4 template, the full expansion converges with the polymer bounds of [15], Lemmas 14–17; the counting and small-factor mechanics are those transcribed above.*

10 Translation: Analytic Norms to Per-Link Oscillation

Definition 10.1. *For $F : G^{\Lambda^1_k} \to \mathbb{R}$ and a bond e:*

$$\operatorname{osc}_e(F) := \sup_{U,U' :\, U_b = U'_b \ \forall b \neq e} |F(U) - F(U')|. \tag{5}$$

The three-step bridge: (1) gauge chart ⇒ Lie-algebra coordinates (Theorem 4.2); (2) Cauchy estimate ⇒ derivative bound: for analytic F on $\mathcal{R}_k$ with radius r,

$$\Big|\frac{\partial}{\partial A(e)} F\Big| \ \le\ r^{-1} \sup_{|z - A(e)| = r} |F|, \qquad \operatorname{osc}_e(F)\ \le\ 2\,\|F\|_k; \tag{6}$$

(3) polymer locality ⇒ restricted support.

Theorem 10.2 (Per-link oscillation bound — osc_e form of (A2))**.** *For each bond e and polymer $X \ni e$:*

$$\operatorname{osc}_e\big(\mathcal{V}^{(j)}(X,\cdot)\big)\ \le\ C_{\mathrm{osc}}\,(L^j\eta)^{4+\alpha}\,e^{-\kappa d_j(X)}, \tag{7}$$

with C_{osc} depending only on d, L, M, α_0. Summing over $X \ni e$ with Lemma C.1 gives the per-link form of (A2). (v2) *The Cauchy mechanics* (6)*, including the factor-2 saturation, are machine-verified (check 5).*

Remark 10.3 (The 2^{-2k} factor)**.** *In the dyadic case $L = 2$ the factor $(L^k\eta)^{4+\alpha}$ evaluated against the volume factor yields net suppression 2^{-2} per step, as used by the companions.* (v2) *See Remark 10.4 for what this does and does not give.*

Remark 10.4 (Direction-of-limit bookkeeping — new in v2)**.** *$(L^k\eta)^{4+\alpha} = L^{-(N-k)(4+\alpha)}$ is geometric in the distance $N-k$ to the* ultraviolet *cutoff: it is smallest deep in the UV and equals* 1 *at the unit (physical) scale $k = N$. Consequently: (i) for* UV stability at fixed physical volume *(the regime of this package and of [8,12,15]) the factor delivers genuine geometric control — this is why the finite-volume discharge escapes the infrared volume window of the audited chain; (ii) for the* continuum-limit telescoping *of the RG–Cauchy Master (interface (M1)), the relevant comparison is between trajectories with different N at fixed physical scale, and whether the 2^{-2}-per-step net factor survives that bookkeeping is exactly the summability question left open in 2602.0063 v3 (F-SUM: the naive rate $g_k^2 \sim c/k$ is not summable). (M1) therefore remains hypothesis-level until the Doob companion is audited. The suite (check 7) prints both directions of the factor.*

11 The Dimock Template: Structural Parallels with φ^4_3

The dictionary between Bałaban's gauge-covariant objects and Dimock's scalar template, and the $d = 3$ prototype of [7], are as in v1; they are expository and carry no hypothesis load.

12 Summary of Derived Axioms and Interface

12.1 Derived axioms

Axiom A1 (source [9] Eq. (2.41); [12] Eq. (1.98)): as in Theorem 7.3. **Axiom A2** (source [8] Eq. (0.29); Theorem 10.2): as in (7), with the direction bookkeeping of Remark 10.4. **Axiom A3** (source [2] Lemma 2.1; Appendix C): lattice-animal count $\le C^n_{\mathrm{anim}}$ with $C_{\mathrm{anim}} = (2d)^3$ (machine-verified against exact enumeration, check 4). **Axiom B5** (source [12] Thm. 1, Eqs. (0.1),(1.89),(1.98)–(1.100)): as in Theorems 8.4–8.6, *subject to the β_{LF} dichotomy of Remark 8.7.*

12.2 Interface with the Doob companion

The Doob companion consumes (A1)–(A3) for its per-link bounds and uniform Doob seminorm. (v2) That companion is not yet audited; per the series notice its conclusions must not be cited as unconditional until it is.

12.3 Interface with the RG–Cauchy Master

(M1) Summable telescoping. v1: "follows from (A2) and the Duhamel identity." (v2) Conditional: see Remark 10.4; the continuum-direction summability is the open F-SUM question of 2602.0063 v3.

(M2) Uniform Doob bound. Consumes (A1)–(A3) via the Doob companion (unaudited).

(M3) Large-field error. v1 claimed $\tau_k = e^{-p_0(g_k)}$ with $\sum_k \tau_k < \infty$ by "super-polynomial decay from asymptotic freedom."

Remark 12.1 (Correction of (M3) — new in v2)**.** *With the profile* (3) *and the AF flow, at j scales above the infrared end $g_j^{-2} = \gamma^{-2} + 2b_0 j \ln L$, so $p_0(g_j) \approx A_0(\ln(2b_0 j \ln L))^{\theta_0}$ and*

$$\tau_j \;=\; e^{-A_0(\ln j)^{\theta_0}(1+o(1))} \;=\; \begin{cases} j^{-A_0(\ln j)^{\theta_0-1}} & \theta_0 < 1: \; \textit{sum DIVERGES}, \\ j^{-A_0} & \theta_0 = 1: \; \textit{converges iff } A_0 > 1, \\ \textit{super-polynomial} & \theta_0 > 1: \; \textit{converges}. \end{cases}$$

v1's justification is incorrect as stated; the conclusion holds under the explicit profile condition $\theta_0 > 1$ (or $\theta_0 = 1$, $A_0 > 1$) — hypothesis (H-θ)*. The chain's representative polylog floor has $\theta_0 = 1.1$: convergent, but only marginally (the suite, check 6, shows the partial sums for $\theta_0 =$* 0.9/1.0/1.1 *side by side). Note this is the ultraviolet-direction sum and is therefore* distinct *from the infrared absorption failure (H-P0) of the audited chain; both, however, are controlled by the same profile exponent.*

13 Note added in v2

Version 1 stated that this paper "completes the unconditional discharge of the UV structural inputs." Following the series audit (ai.viXra:2602.0041 v3, 2602.0051–0057 v2, 2602.0063 v3), the status is corrected:

- The discharge is *traceable and conditional*: on the faithful transcription of [1]–[15] (this paper is an extraction, not a re-derivation); on the β_{LF} dichotomy (Remark 8.7) — $O(1)$ reading, or (H-P0); and on the profile condition (H-θ): $\theta_0 > 1$ for (M3) (Remark 12.1; v1's "super-polynomial decay from asymptotic freedom" is withdrawn as a justification).

- The direction-of-limit bookkeeping for the irrelevance factor is made explicit (Remark 10.4): geometric toward the UV (finite-volume stability: sound), hypothesis-level toward the continuum telescoping ((M1); F-SUM of 2602.0063 v3).

- The two consumer companions (*Doob Influence Bounds*, *RG–Cauchy Master*) are unaudited; per the series notice they must not be cited as unconditional until replaced/audited. Their viXra identifiers are to be added when assigned.

- Reference hygiene: the audited chain is now cited ([16]–[18]); the finite-volume, ultraviolet character of this package — which shields it from the audited infrared volume window — is stated explicitly (Remark 10.4(i)).
- What is machine-verified (companion suite, 7 checks): abelian RG operator algebra; propagator decay δ_0 and random-walk convergence; exponential sum control above the Lemma 3.5 threshold; lattice-animal counting vs. exact enumeration ($d = 2, 3, 4$); the oscillation–analyticity bridge (6) with factor-2 saturation; the (M3) profile trichotomy; and the β_{LF} dichotomy table with the two-direction irrelevance factor arithmetic.

Suite: `verify_2602_0069.py` at https://github.com/lluiseriksson/THE-ERIKSSON-PROGRAMME, directory `verification/2602-0069/`.

14 Complete Traceability Table

Axiom	Statement	Source	v2 status
(A1)	polymer representation	[9] (2.41); [12] (1.98)	traceable
(A2)	oscillation + irrelevance	[8] (0.29); Thm 10.2	traceable; direction per Rmk 10.4
(A3)	animal counting	[2] Lem. 2.1; App. C	verified (check 4)
(B5)	large-field suppression	[12] Thm 1, (1.89)	conditional: $\beta_{\mathrm{LF}} = O(1)$ or (H-P0)
(M1)	summable telescoping	Doob comp. + (A2)	hypothesis-level (F-SUM)
(M3)	large-field error sum	(3)	conditional: (H-θ) $\theta_0 > 1$

A Proof of the Oscillation–Analyticity Bridge

As in v1: the Cauchy integral formula on the analytic domain $\mathcal{R}_k$ gives (6); the finite-variation step gives the factor 2; polymer locality restricts the sum to $X \ni e$. Machine verification: suite check 5 (derivative bound, radius scaling, and factor-2 saturation on explicit analytic families).

B The Semi-Simplicity Argument

Lemma B.1. *Let G be compact semi-simple with algebra $\mathfrak{g}$; then the quadratic form induced by the plaquette action on $\mathfrak{g}$ is non-degenerate, and the Taylor coefficients in the bridge are controlled by the Killing form. (As in v1.)*

C Lattice Animal Counting

Lemma C.1. *The number of connected subsets of the M-cube lattice containing a fixed cube with n cubes is at most C_{anim}^n with $C_{\mathrm{anim}} = (2d)^3$.* (v2) *Exact enumeration (suite check 4): $d = 2$, $n \le 6$: $1, 4, 18, 76, 315, 1296$ and $d = 3$, $n \le 4$: $1, 6, 45, 344$ (matching $n{\cdot}A001168/n{\cdot}A001931$), $d = 4$, $n \le 3$: $1, 8, 84$; all $\le (2d)^{2n} \le (2d)^{3n}$ — the stated constant is safe (and not sharp).*

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