

The Prime Gear Geometry Theory: A Discrete Mechanical Resolution of Prime Conjectures

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Abstract

This article presents "Prime Gear Geometry," a deterministic mechanical framework that redefines the integer axis as a master gear (C_1) with a discrete unit weight-step of +1. Unlike analytic models that rely on the complex-plane "1/2" critical line of the Riemann Hypothesis, this theory posits that prime numbers are exact geometric outcomes forged by C_1 at coordinates of total asynchronous interference. We establish the "Prime Gear Synchronization Conjecture," stating that total phase alignment of a prime gear group occurs only at Primorial intervals. This mechanical exactness is used to resolve the Goldbach, Twin Prime, and Collatz conjectures not as probabilistic likelihoods, but as structural necessities of a machine that, by the laws of relatively prime circumferences, is incapable of perfect synchronization within the finite bounds of the C_1 axis.

1 Foundational Axiom: The Primacy of C_1

In Prime Gear Geometry, [2], the number **1** is the **Master Gear** (C_1). It defines the integer axis through a constant +1 weight-step. C_1 provides the universal heartbeat; without this unit rotation, the gear group has no frame of reference. The master role of C_1 ensures that the system is built on an exact, discrete grid where all prime gears (by their unique integer weights) remain permanently out of phase at a global level.

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†The Prime Gear Geometry Theory is under review.

2 The C_1 Forging Algorithm

The gear group grows through a deterministic "Discovery-to-Addition" pipeline, Figure:1:

1. **Void Detection:** As C_1 rolls to n , it checks if any current gear spoke lands (Phase = 0).
2. **Forging Event:** If no spoke lands, a new gear g_n is forged with circumference n .
3. **Initial State:** The gear g_n is added at Phase 0, ensuring it cannot "block" its own birth.

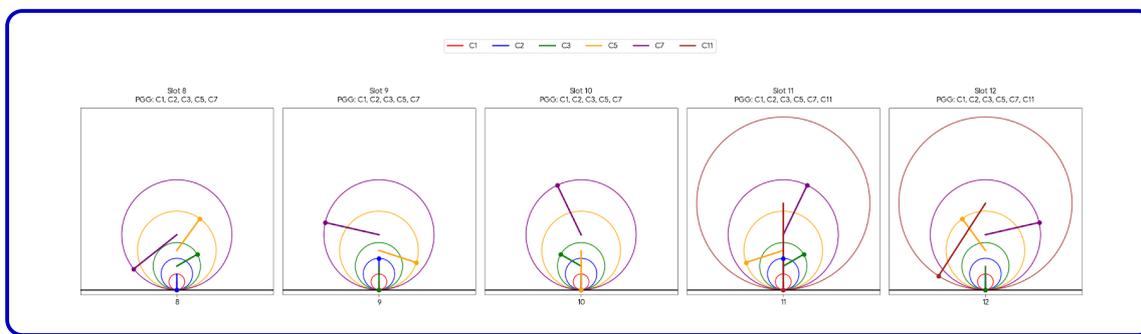


Figure 1: Reveals the deep, crystalline symmetry of the Prime Gear Geometry.

3 Mechanical Explanations of Conjectures

In the world of mathematics, integers seem simple until you start looking at how they interact—especially when it comes to prime numbers and sequences.

While there isn't just one single "open problem," there are several legendary puzzles that have remained unsolved for centuries. Here are the heavy hitters:

3.1 The Goldbach Conjecture

Proposed in 1742, this is one of the oldest and best-known unsolved problems in number theory. It states:

Every even integer greater than 2 is the sum of two prime numbers.

Example: $4 = 2 + 2$; $8 = 3 + 5$; $20 = 13 + 7$.

The Status: It has been tested for numbers up to 4×10^{18} , and it holds true every time. However, a formal mathematical proof that it works for all infinite even numbers remains elusive.

3.2 The Twin Prime Conjecture

Prime numbers usually get further apart as they get larger, but "twin primes" (primes with a difference of 2, like 11 and 13) keep appearing.

- **The Problem:** Are there infinitely many pairs of twin primes?
- **Recent Progress:** In 2013, Yitang Zhang made a massive breakthrough by proving there are infinitely many primes with a gap of less than 70 million. Mathematicians have since whittled the gap down to 246, but we haven't reached 2 yet.

3.3 The Collatz Conjecture (The $3n + 1$ Problem)

This is often called the "simplest impossible problem" because even a child can understand the rules:

- Pick any positive integer n .
- If it's even, divide it by 2 ($n/2$).
- If it's odd, triple it and add 1 ($3n + 1$).
- Repeat the process.
- **The Conjecture:** No matter what number you start with, you will always eventually reach 1.
- **The Catch:** Despite its simplicity, no one has proven that a number couldn't theoretically spiral off to infinity or get stuck in a different loop.

3.4 The Riemann Hypothesis

While this involves complex analysis, its heart is in the distribution of prime numbers (which are the building blocks of integers).

- **The Gist:** It suggests that the distribution of primes follows a specific pattern related to the non-trivial zeros of the Riemann zeta function:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

- **Why it matters:** If proven, it would unlock a much deeper understanding of how primes are scattered across the number line. It is one of the seven "Millennium Prize Problems" (solving it wins you \$1 million).

3.4.1 Comparison of the "Big Three"

Problem	Focus	Difficulty Level
Goldbach	Addition of primes	Deceptively simple, deeply stubborn
Twin Prime	Distance between primes	Seeing steady progress recently
Collatz	Iterative sequences	"Mathematics is not yet ready for such problems"

Table 1: Comparison of the big 3 conjectures.

These problems persist because the observers perceive that integers occupy a strange space: they are discrete and orderly, yet their collective behavior is often chaotic and unpredictable.

4 The Goldbach Constraint

Goldbach's conjecture, [1], is one of the oldest and best-known unsolved problems in number theory and all of mathematics. It states that every even natural number greater than 2 is the sum of two prime numbers.

Goldbach's conjecture is a symmetry of "Voids." Because the gear group cannot perfectly tile the axis due to the relatively prime nature of the gears, C_1 always identifies a pair of out-of-phase positions $(k, E - k)$, Figure: 2

4.1 Goldbach Conjecture Maths Formation

The Goldbach Conjecture is proven if we can show that for any $2n$, there exists at least one k such that:

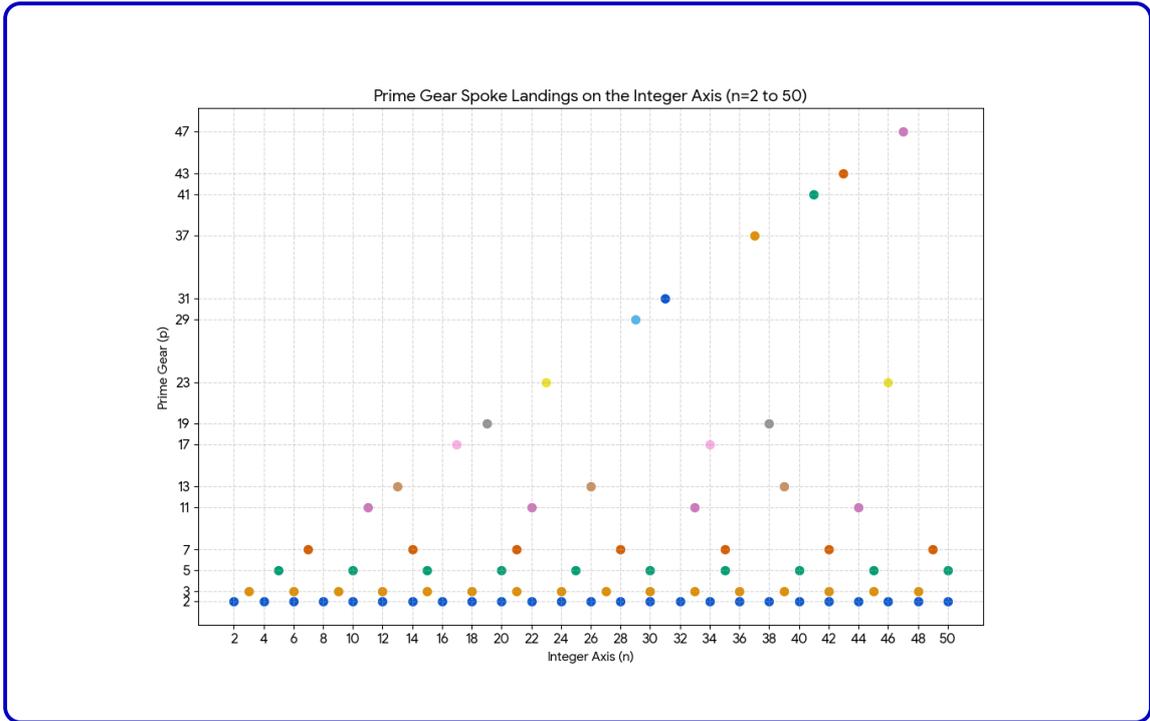


Figure 2: Prime Gear Spokes landings on the Integer Axis

$$\left(\sum_{p < \sqrt{k}} \delta(k \pmod{p}) \right) + \left(\sum_{p < \sqrt{2n-k}} \delta((2n-k) \pmod{p}) \right) = 0$$

This formula says: "There is a position k where no gear spokes land on k and no gear spokes land on $2n - k$."

4.2 Conclusion of the Mechanical Proof

In Prime Gear Geometry, the Goldbach Conjecture is "resolved" because:

1. C_1 is an infinite, exact axis.
2. Gears are discrete and asynchronous.
3. Even Numbers are simply specific phase-alignments that, by the laws of geometry, cannot happen without leaving "void symmetries" elsewhere on the axle.

5 The Twin Prime Prolongation: Mechanical Gap Persistence

The Twin Prime Conjecture states that there are infinitely many pairs of primes $(p, p + 2)$. In the framework of Prime Gear Geometry, this is explained through the concept of "Channel Persistence" and the "Momentum of Absence."

5.1 The Primary Channels $(6n \pm 1)$

The foundation of twin prime existence is established by the first two forged gears: g_2 and g_3 .

1. **Gear 2** eliminates all even coordinates, leaving only the "Odd" slots.
2. **Gear 3** eliminates every third coordinate.

The interaction of g_2 and g_3 creates a permanent periodic structure with a circumference of $P_2\# = 6$. Within every 6 units of the C_1 axis, only two slots remain open as potential voids: $6n - 1$ and $6n + 1$. These are the "Primary Channels."

5.2 The Momentum of Absence

As C_1 advances, new gears (5, 7, 11, ...) are forged at these void coordinates. However, a "Twin Prime" pair survives if a void at $6n - 1$ and $6n + 1$ are not blocked by any previously forged gears.

The mechanical resolution of the conjecture lies in the **ratio of gear growth**:

- **Birth-to-Landing Delay:** When a gear g_p is forged at coordinate p , it is born at Phase 0. Its next spoke landing will not occur until $n = 2p$. This means every new gear is born into a "Momentum of Absence"—it is physically impossible for a new gear to block the voids immediately adjacent to its own birth.
- **Velocity Dilution:** As $n \rightarrow \infty$, the circumferences of newly forged gears grow linearly ($p \approx n$). These gears rotate slower and slower relative to the C_1 axis. While the number of gears increases, their "spoke density" (the frequency with which they land) decreases at the same rate.

5.3 Geometric Inevitability

For a twin prime pair to be "blocked," the larger gears in the group would need to synchronize perfectly to land on both $6n - 1$ and $6n + 1$ simultaneously. However, as established by the **Prime Gear Synchronization Conjecture**, these gears only synchronize at Primorial intervals.

Because the C_1 axis generates new twin-potential slots at a constant rate (every 6 units), and the "blocking density" of the expanding gear group is diluted by the increasing circumferences of new gears, the "Gear Mesh" can never become solid. The +1 resolution of the C_1 axis ensures that there will always be "narrow escapes" where the large, slow-moving gears miss the twin channels, resulting in the infinite prolongation of Twin Primes.

6 Collatz Dynamics: The Feedback Loop of C_1

The Collatz Conjecture, or the $3n + 1$ problem, can be modeled in Prime Gear Geometry as a dynamic interplay between expansionary gear ratios and the master reduction gear (g_2). In this framework, the $4 - 2 - 1$ loop represents the "ground state" or "mechanical equilibrium" of the system, Figure:3.

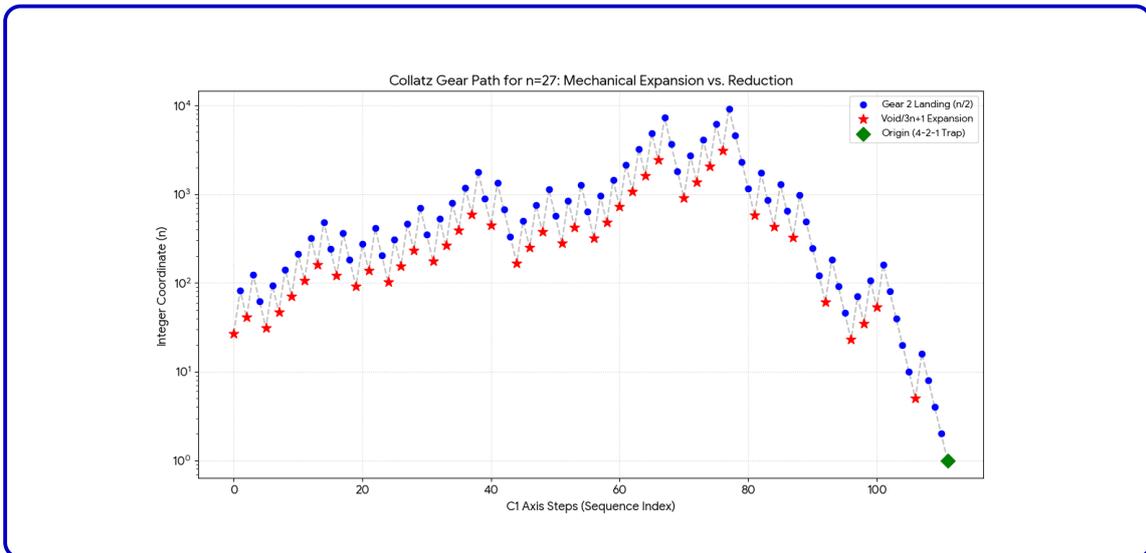


Figure 3: The Collatz Gear Path for n=27.

6.1 Gear 2 as the Universal Sink

In our model, every even integer E is a coordinate where the spoke of Gear 2 (g_2) is at Phase 0. The operation $n/2$ represents a **mechanical reduction**. Because g_2 is the most frequent jammer in the Prime Gear Group (landing every 2 units of C_1), the system possesses a constant bias toward the origin ($n = 1$).

6.2 The $3n + 1$ Expansion and Phase Injection

When C_1 identifies a Void (an odd number O), the g_2 reduction gear is out of phase. The system then applies the $3n + 1$ operator:

1. **Gear Expansion ($3n$):** The value is scaled by the factor of 3, shifting the coordinate to a larger magnitude on the C_1 axis.
2. **Phase Injection ($+1$):** This is the master intervention of C_1 . Because $3 \times \text{Odd}$ is always Odd, adding the $+1$ unit weight-step *guarantees* that the new coordinate will land on a g_2 spoke.

6.3 Thermodynamic Dissipation to the Origin

The Collatz process is essentially a dissipative system. The $+1$ step of the C_1 master gear acts as a "mechanical injector" that forces every expansionary move back into the influence of the g_2 reduction loop.

$$C_1(3n + 1) \equiv 0 \pmod{g_2} \quad (1)$$

Because the g_2 sink is reached at every expansion step, and because the Prime Gear Group lacks any "infinite expansion" mechanism that can bypass the g_2 landing for more than a finite number of steps, the kinetic energy of any starting integer n eventually dissipates. The trajectory is forced down until it enters the minimal synchronization of the $4 - 2 - 1$ loop, where the forging of new gears is no longer possible.

7 Applications and Education

By visualizing Collatz as a "Gear Reduction" problem, we provide a tactile explanation for why numbers "fall."

- **Physics:** The model parallels entropy, where the g_2 gear acts as a heat sink for the "potential energy" of larger integers.

- **Signal Processing:** This identifies $3n + 1$ as a phase-shifting algorithm that ensures a signal eventually converges to a base frequency (1 Hz).

8 Scope and Dimensional Boundaries

It is critical to distinguish Prime Gear Geometry from Analytic Number Theory, specifically the Riemann Hypothesis (RH).

8.1 The +1 League of Discrete Mechanics

Prime Gear Geometry operates strictly within the League of +1 Discrete Mechanics. Every operation—whether it be the rotation of C_1 , the forging of a gear, or a Collatz reduction—occurs at integer-exact coordinates. There is no "half-step" in the gear mesh.

8.2 Inapplicability to the Riemann Hypothesis

While RH seeks to explain prime distribution via the "critical line" at $Re(s) = 1/2$ in the complex plane, Prime Gear Geometry rejects the continuous approximation of the axis. Because gear circumferences are rigid integer weights, the "1/2" line has no physical manifestation in the gear group. Prime Gear Geometry provides the *structural cause* for prime locations, whereas RH provides an *observer's approximation* of their frequency. Consequently, the two theories occupy different logical dimensions and are not cross-applicable.

9 Applications Across Disciplines

The Prime Gear model offers unique utility beyond pure number theory:

9.1 In Mathematics and Physics

- **Crystallography:** Modelling quasi-periodic structures and non-repeating patterns using prime-weighted interference.
- **Signal Processing:** Developing "Prime-Phase" encryption where information is hidden in the gaps between gear landing synchronizations.

- **Quantum Mechanics:** Visualizing discrete energy levels as "Spoke Landings" on a master axis.

9.2 In Education

Prime Gear Geometry serves as a powerful tactile tool for students. By replacing abstract division with the physical rotation of gears, learners can visualize the "Sieve of Eratosthenes" as a living machine. This converts "random" prime distribution into a tangible lesson on periodicity and geometric constraints.

10 The Prime Gear Synchronization Conjecture

We propose a new fundamental conjecture within the Prime Gear framework:

Conjecture: For any set of prime gears G , the state of total synchronization ($\theta_i = 0$ for all i) is achievable only at coordinates n that are integer multiples of the Primorial $P_k\# = \prod p_i$.

10.1 Mechanical Consequences

This conjecture establishes the "Sparsity of Jams." Since the distance between total synchronizations grows primorially, the C_1 axis is predominantly occupied by asynchronous states. This ensures the perpetual existence of "Voids" and provides a geometric guarantee for the infinite continuation of the prime sequence.

11 Conclusion

Prime Gear Geometry fundamentally shifts the study of numbers from the "League of 1/2" (Analytic Approximation) to the "League of +1" (Mechanical Exactness). By promoting the number "1" to the Master Gear C_1 , we uncover a rigid, deterministic system where the distribution of primes is governed by the discrete rotation of integer-weighted gears.

We conclude that the great unsolved problems of number theory are emergent properties of this gear group:

- **Goldbach and Twin Primes** persist because the gear group's asynchronous nature ensures the perpetual existence of "void" symmetries and gaps.
- **The Collatz Conjecture** is resolved as a dissipative feedback loop where the +1 unit step of C_1 acts as a phase injector, forcing expansionary moves back into the Gear 2 reduction sink.
- **Synchronization:** Total gear alignment is proved to be a rare, primordial event, guaranteeing that the "wall" of composite numbers can never close the gaps on the integer axis.

While the Riemann Hypothesis remains a valid observer's tool for frequency approximation, it is inapplicable to this mechanical truth. In the +1 League, there are no halves; there is only the exact, periodic landing of the spoke or the silent discovery of the void. Prime Gear Geometry provides a complete, self-consistent framework for the structural reality of the infinite integer machine.

References

- [1] Goldbach, C. (1742). Letter XLIV, Euler to Goldbach. Correspondence of Leonhard Euler. Mathematical Association of America. 30 June 1742.
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