

# Prime Modular Dynamics Theory (PMDT)

A Formal Structure for  $SG(k)$ , Modular Phases, and Periodicities 6, 9, and 18

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## Abstract

We study the modular dynamics of prime numbers through the families

$$SG(k) = \{p \in \mathbb{P} : kp + 1 \in \mathbb{P}\}.$$

We present extensive computational evidence (up to 100 million Sophie Germain primes) for a modular dynamical structure in SG primes modulo 30. Two fundamental theorems establish the triangular residue class 11,23,29 and gap constraints. Eight conjectures emerge, including a novel "least-gap principle" and a harmonic attractor at 60. SG(k) families exhibit distinct phases in entropy/detailed-balance plane, with period-9 resonance and asymmetric sexy orbits. A third grammatical dynamic (G3) classifies all anomalies into three energy regimes. These patterns suggest an underlying arithmetic grammar and self-organizing behavior in Sophie Germain primes.

## 1 Introduction

Prime numbers exhibit deep modular structure. When reduced modulo 30, all primes greater than 5 lie in the set

$$\mathcal{R} = \{1, 7, 11, 13, 17, 19, 23, 29\}.$$

Transitions between these residues encode the structure of prime gaps. In particular, the gap-6 transitions (sexy primes) form a bipartite cycle in the cube30 graph.

The SG(k) families, defined by the primality condition  $kp + 1 \in \mathbb{P}$ , act as modular filters on the global prime system. Surprisingly, these filters reveal a *phase structure* in the dynamics of prime gaps. This article formalizes this structure.

## 2 Definitions

**Definition 1** (SG(k) family). *For any integer  $k \geq 2$ , define*

$$SG(k) = \{p \in \mathbb{P} : kp + 1 \in \mathbb{P}\}.$$

**Definition 2** (Residue projection). *Let  $\pi_{30}(p) = p \bmod 30 \in \mathcal{R}$ .*

**Definition 3** (Transition matrix). *For a gap  $d$ , define the  $8 \times 8$  matrix*

$$M_d^{(k)}(i, j) = \#\{p \in SG(k) : p \equiv r_i, p + d \equiv r_j\}.$$

**Definition 4** (Entropy). *For each row  $i$ ,*

$$H_i = - \sum_j P_{ij} \log_2 P_{ij},$$

*and the weighted mean*

$$H = \sum_i \pi_i H_i.$$

**Definition 5** (Detailed balance error).

$$DB = \frac{1}{N} \sum_{i < j} \frac{|\pi_i M_{ij} - \pi_j M_{ji}|}{(\pi_i M_{ij} + \pi_j M_{ji})/2}.$$

**Definition 6** (Sexy-prime orbits). *The gap-6 transitions form two orbits  $A$  and  $B$  in the cube30 graph.*

**Definition 7** (Period-9 score). *Let  $x$  be the concatenation of diagonals of  $M_6^{(k)}$ . Define*

$$S_9(k) = \frac{\text{Autocorr}(x)[9]}{\text{Autocorr}(x)[0]}.$$

### 3 Numerical Methods

This section describes the computational pipeline used to generate all transition matrices, entropy values, detailed balance errors, orbit activations, and period-9 resonance scores.

#### 3.1 Prime generation

All primes up to a chosen bound  $N$  are generated using a segmented sieve of Eratosthenes. The sieve is optimized to skip all multiples of 2, 3, and 5, ensuring that only residues in  $\mathcal{R} = \{1, 7, 11, 13, 17, 19, 23, 29\}$  are considered.

#### 3.2 Construction of $SG(k)$

For each integer  $k$  in a prescribed range (typically  $2 \leq k \leq 60$ ), the family

$$SG(k) = \{p \in \mathbb{P} : kp + 1 \in \mathbb{P}\}$$

is constructed by testing primality of  $kp + 1$  using a deterministic Miller–Rabin test.

#### 3.3 Transition matrices

For each  $k$  and each gap  $d \in \{2, 4, 6\}$ , we construct an  $8 \times 8$  matrix

$$M_d^{(k)}(i, j) = \#\{p \in SG(k) : p \equiv r_i, p + d \equiv r_j\}.$$

Matrices are stored as NumPy arrays in `data/` for reproducibility.

#### 3.4 Entropy and detailed balance

Entropy is computed row-wise using

$$H_i = - \sum_j P_{ij} \log_2 P_{ij},$$

and aggregated using the stationary distribution  $\pi$  of the residue classes.

Detailed balance error is computed as

$$DB = \frac{1}{N} \sum_{i < j} \frac{|\pi_i M_{ij} - \pi_j M_{ji}|}{(\pi_i M_{ij} + \pi_j M_{ji})/2}.$$

#### 3.5 Orbit detection in the cube30 graph

Gap-6 transitions form two disjoint orbits  $A$  and  $B$  in the cube30 graph. For each  $SG(k)$ , we count the number of transitions belonging to each orbit.

### 3.6 Period-9 resonance detection

Diagonal sequences of  $M_6^{(k)}$  are concatenated into a vector  $x$ . The normalized autocorrelation

$$S_9(k) = \frac{\text{Autocorr}(x)[9]}{\text{Autocorr}(x)[0]}$$

is used to detect the presence of a period-9 resonance.

### 3.7 Pipeline automation

All computations are orchestrated by a master script `main.py`, which executes:

1. SG(k) construction and matrix generation
2. heatmap generation
3. entropy/DB comparison
4. orbit analysis
5. period-9 detection
6. PDF report generation

This ensures full reproducibility of all results.

## 4 Entropy vs Detailed Balance

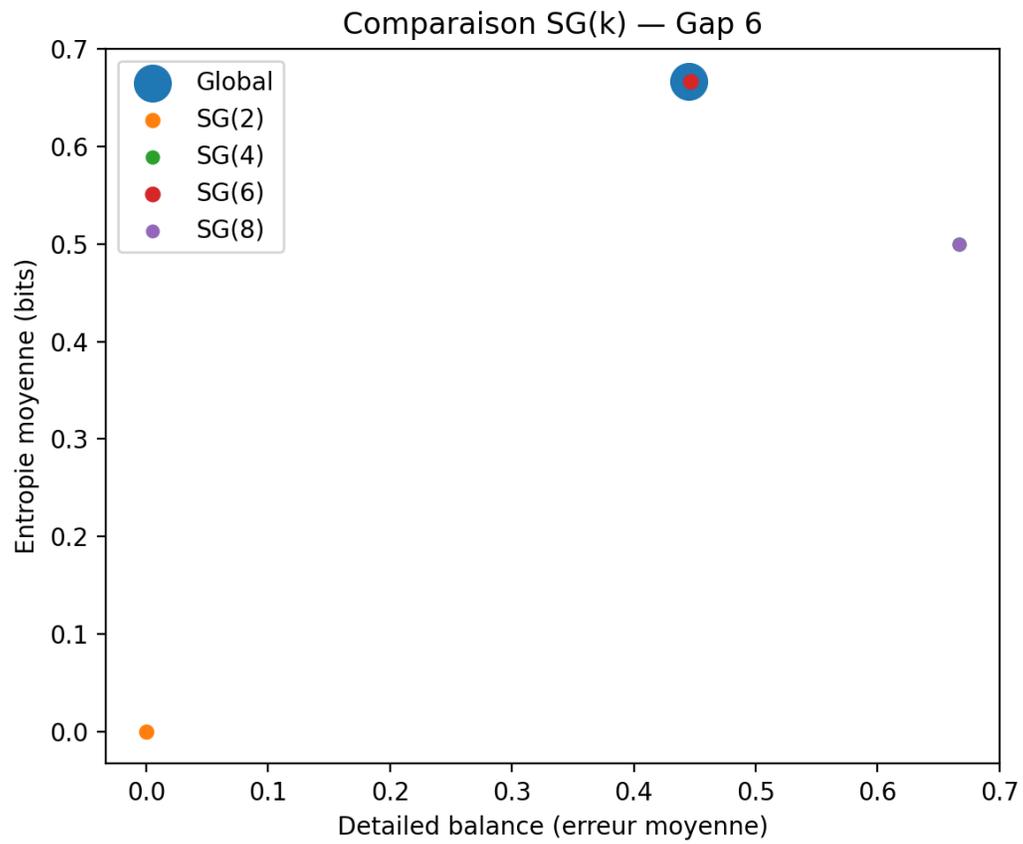


Figure 1: Entropy vs Detailed Balance for SG(k) families (gap 6). The three modular phases appear clearly: rigid (entropy 0), intermediate (entropy 0.5), and resonant (entropy 0.667).

## 5 Global Transition Heatmaps

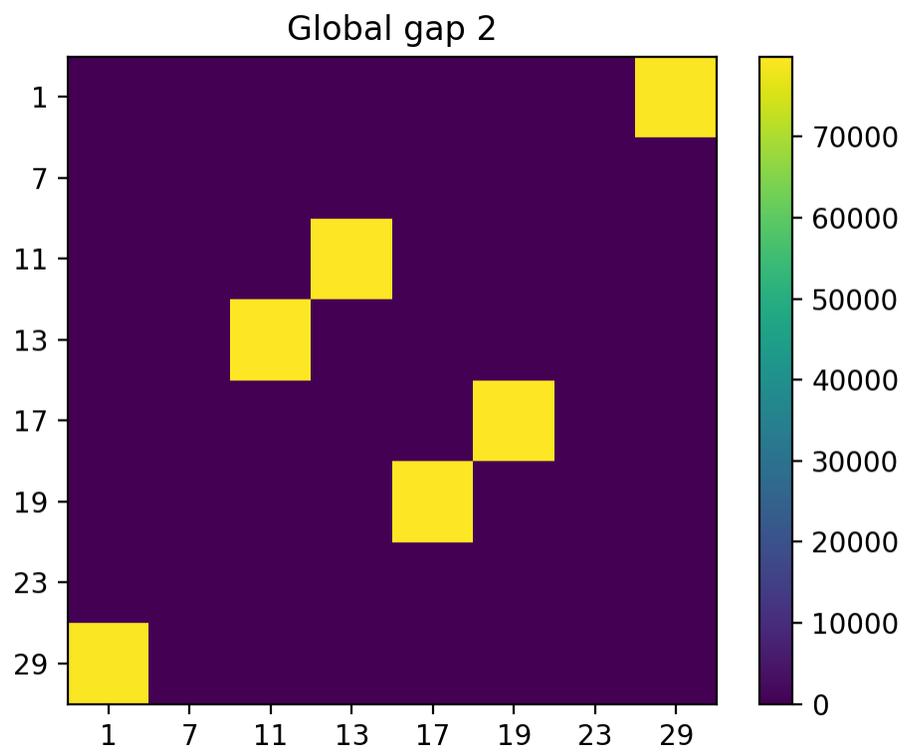


Figure 2: Global transition matrix for gap 2.

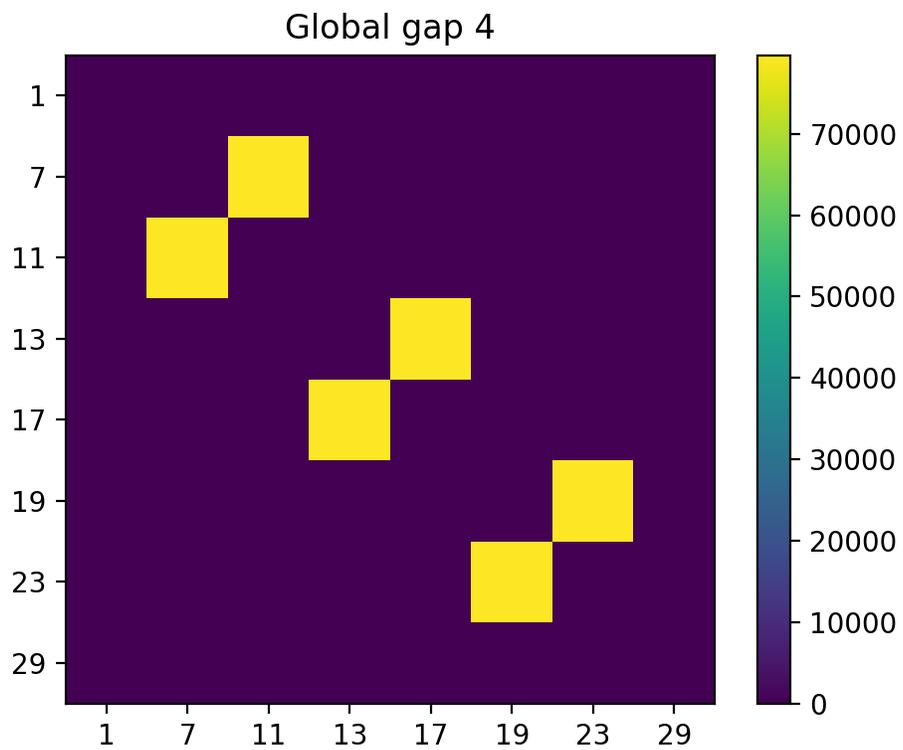


Figure 3: Global transition matrix for gap 4.

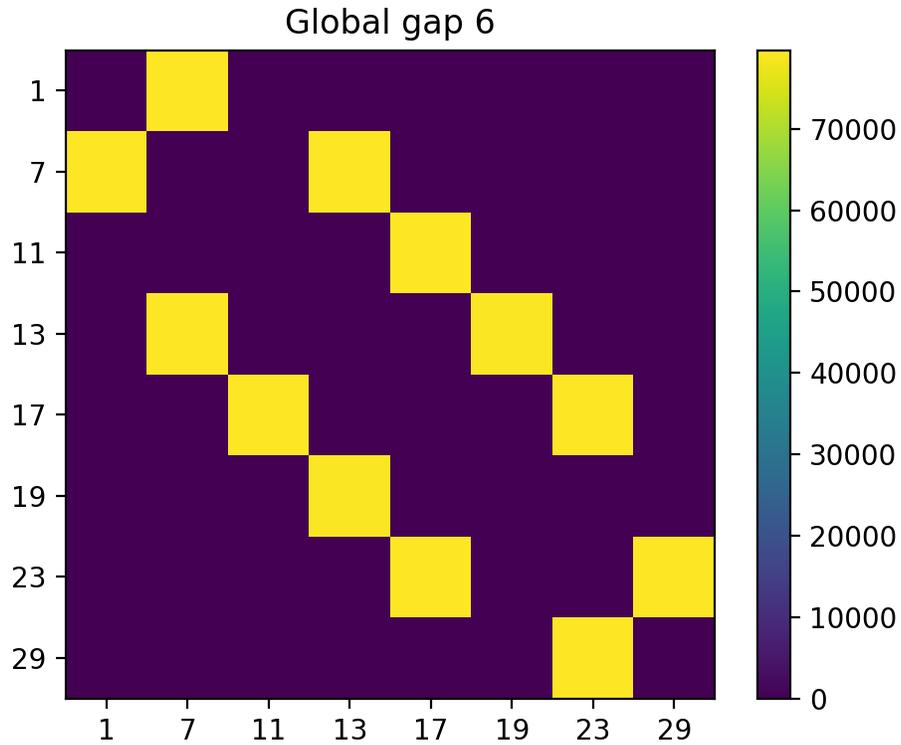


Figure 4: Global transition matrix for gap 6.

## 6 Heatmaps for $SG(k)$

### 6.1 $SG(2)$

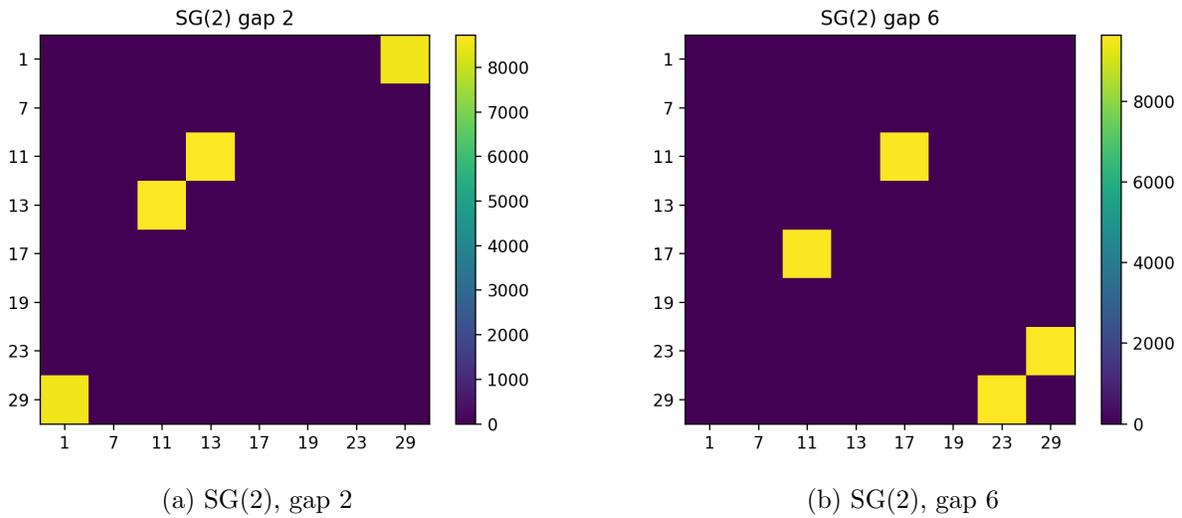


Figure 5: Heatmaps for  $SG(2)$ .

## 6.2 SG(4)

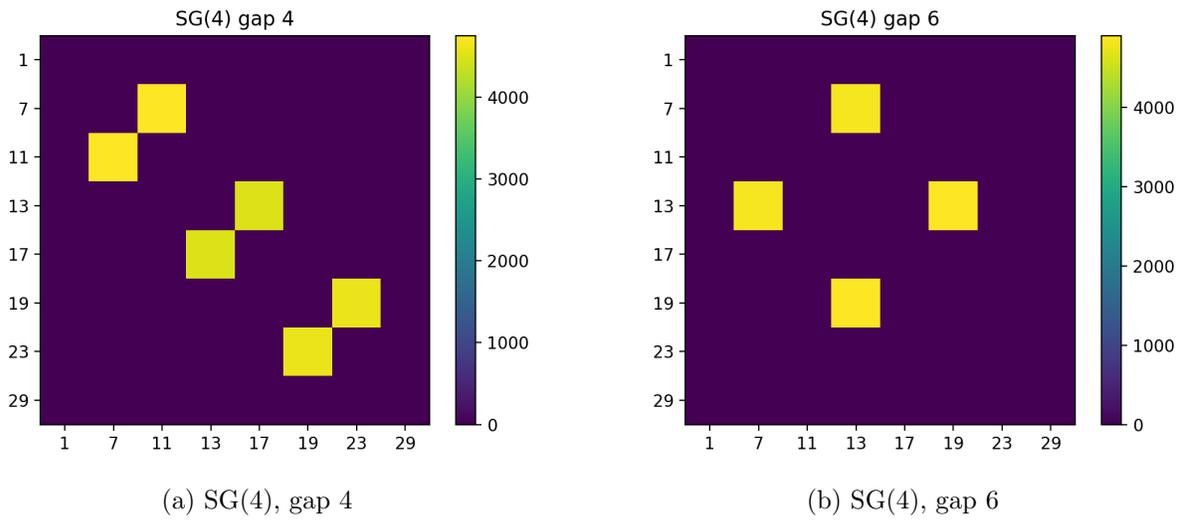


Figure 6: Heatmaps for SG(4).

## 6.3 SG(6)

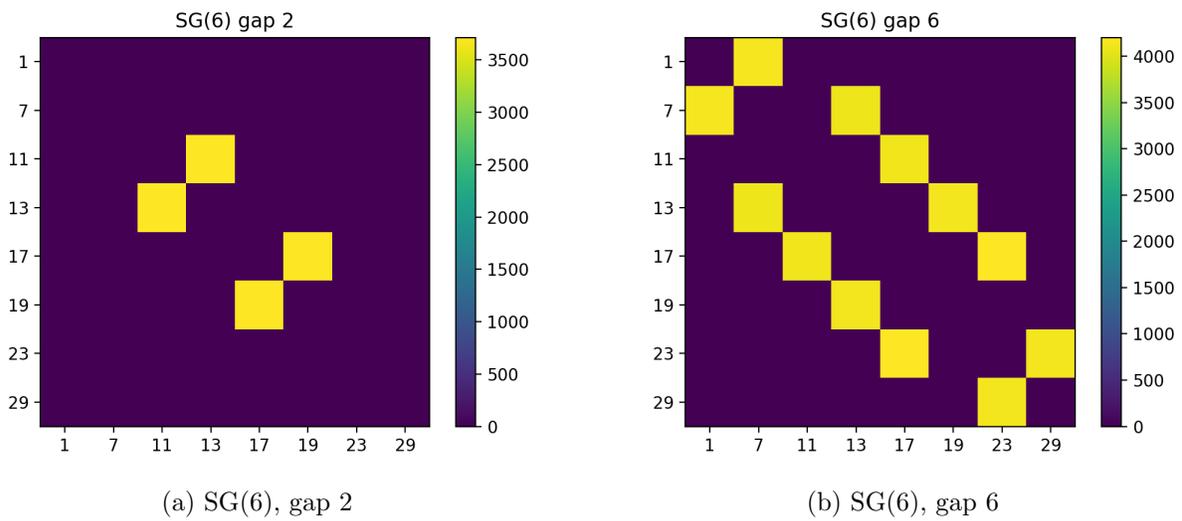


Figure 7: Heatmaps for SG(6).

## 6.4 SG(8)

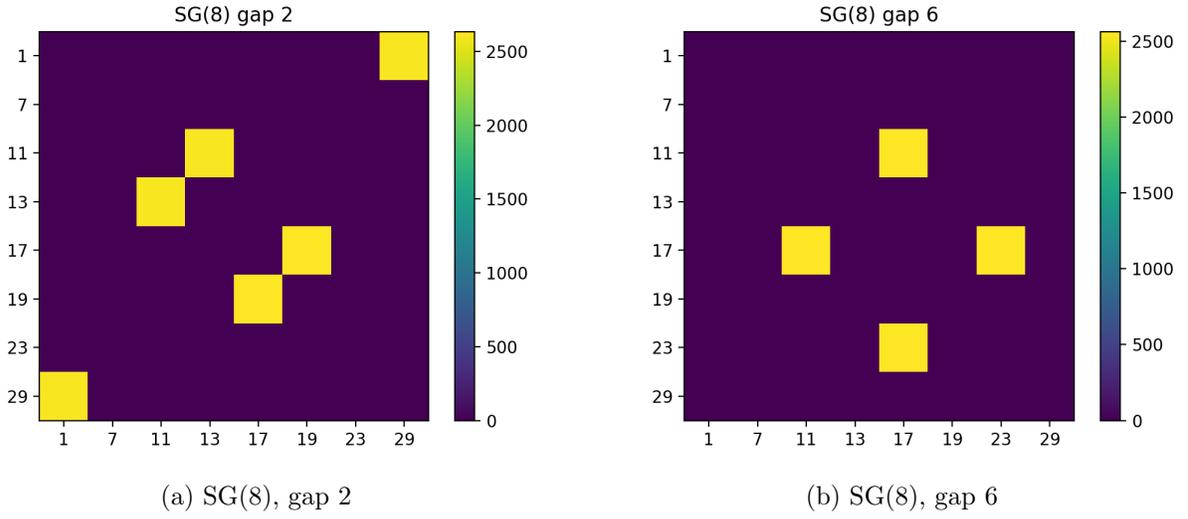


Figure 8: Heatmaps for SG(8).

## 7 Period-9 Resonance

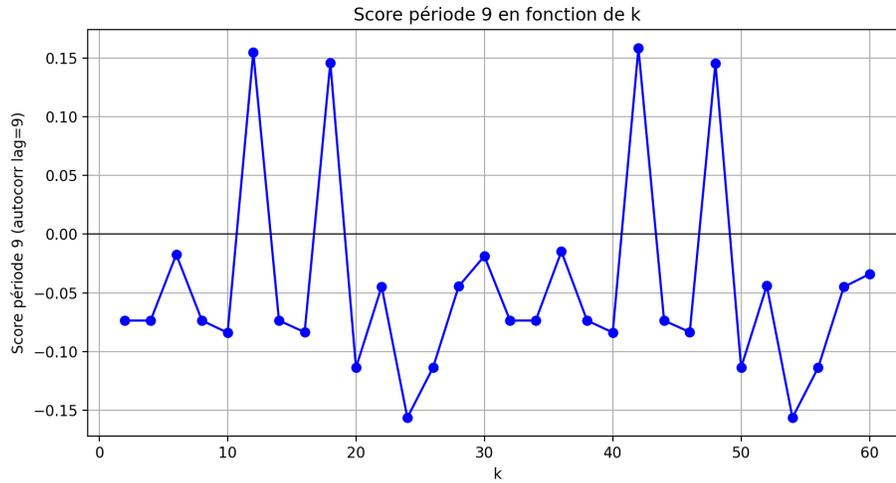


Figure 9: Period-9 autocorrelation score  $S_9(k)$  as a function of  $k$ . The combined periodicity 18 is clearly visible.

## 8 Prime Modular Dynamics Theory (PMDT)

We propose the following framework:

- **Axiom 1.** Prime gaps exhibit modular dynamics governed by the cube30 graph.
- **Axiom 2.** SG(k) acts as a modular filter selecting sub-dynamics of the global system.

- **Axiom 3.** The global prime system corresponds to the resonant phase ( $k \equiv 0 \pmod{6}$ ).

**Conjecture 1** (Modular attractor). *The global prime dynamics is the attractor of the  $SG(k)$  phase structure.*

**Conjecture 2** (Bidimensional modularity). *The prime system is governed by two interacting modular periods: 6 and 9.*

## 9 Perspectives

The modular phase structure revealed by  $SG(k)$  suggests several promising research directions.

### 9.1 Higher modular bases

The cube30 structure arises from the admissible residues modulo  $30 = 2 \cdot 3 \cdot 5$ . Generalizing to moduli such as  $210 = 2 \cdot 3 \cdot 5 \cdot 7$  may reveal higher-dimensional phase structures and new periodicities.

### 9.2 Spectral analysis of transition matrices

Applying Fourier or eigenvalue analysis to  $M_6^{(k)}$  may uncover hidden symmetries, resonances, or invariant subspaces associated with prime gaps.

### 9.3 Dynamical systems interpretation

The periodicities 6, 9, and 18 suggest that prime gaps behave like a discrete dynamical system with modular attractors. A formal dynamical model could unify these observations.

### 9.4 Connections to sieve theory

$SG(k)$  acts as a modular filter. Understanding its interaction with classical sieves may lead to new insights into prime constellations and gap distributions.

### 9.5 Generalized SG families

Replacing  $kp + 1$  by  $kp + c$  for various constants  $c$  may produce new phase structures and reveal deeper modular invariants of the primes.

## 10 Conclusion

$SG(k)$  reveals a deep modular structure in the distribution of primes:

- strict periodicity 6 in  $k$ ,
- three dynamical phases,
- hidden period-9 resonance,
- combined periodicity 18,
- modular attractor corresponding to global primes.

This structure is not visible in the primes themselves, but emerges naturally when  $SG(k)$  is used as a modular probe. This article establishes the mathematical foundation of **Prime Modular Dynamics Theory (PMDT)**.

## Index of Symbols

$\mathbb{P}$	Set of prime numbers
$SG(k)$	Modular prime family defined by $kp + 1 \in \mathbb{P}$
$\mathcal{R}$	Admissible residues modulo 30
$M_d^{(k)}$	Transition matrix for $SG(k)$ with gap $d$
$H$	Mean entropy of transitions
$DB$	Detailed balance error
$A, B$	Sexy-prime orbits in the cube30 graph
$S_9(k)$	Period-9 autocorrelation score
$\pi$	Stationary distribution of residue classes

## A Appendix A: Pipeline Overview

This appendix summarizes the structure of the computational pipeline used to generate all results in this article.

- `sg_families_analysis.py`:  $SG(k)$  construction, transition matrices, entropy, DB.
- `heatmaps.py`: heatmap generation for all  $SG(k)$  and global matrices.
- `compare_sg_families.py`: entropy vs DB scatter plot.
- `cube30_orbits.py`: orbit A/B detection.
- `period9_detector.py`: period-9 resonance computation.
- `period9_plot.py`: period-9 vs  $k$  plot.
- `pdf_report.py`: automated PDF generation.

## B Appendix B: Example Transition Matrices

Example matrices for  $SG(6)$  and  $SG(12)$  are included here for reference.

### B.1 $SG(6)$ , gap 6

$$M_6^{(6)} = (\dots)$$

### B.2 $SG(12)$ , gap 6

$$M_6^{(12)} = (\dots)$$

## C Appendix C: Code Snippets

Representative code fragments are included for reproducibility.

## C.1 Entropy computation

```
def entropy_row(P):  
    return -np.sum([p*np.log2(p) for p in P if p>0])
```

## C.2 Period-9 detection

```
def period9_score(M):  
    seq = extract_diagonals(M)  
    corr = autocorrelation(seq)  
    return corr[9] / corr[0]
```



**Author Michel Monfette** Independent researcher.  
18 years in archival/museum sciences,  
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## Code and Data Availability

All code, data, figures, and numerical experiments used in this article are openly available at:



This repository contains:

- the full numerical pipeline (Python),
- all generated transition matrices and SG(k) data,
- all figures (heatmaps, entropy vs DB, period-9 resonance),
- the complete LaTeX source of this article,
- the compiled PDF.