Residual Derivative Bounds and Windowed Uniform Log-Sobolev Inequality for $SU(N_c)$ Lattice Yang–Mills at Weak Coupling*

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Abstract

We prove residual derivative bounds for the polymer expansion of Balaban's multiscale decomposition of the Wilson lattice gauge measure for $SU(N_c)$ in dimension $d \geq 3$, and we assemble them, together with the companion series, into a uniform log-Sobolev inequality. Version 2 corrects the status of this assembly after a quantitative audit. (i) The core mechanism of the paper — locality of polymer functionals, Cauchy estimates on Balaban's analytic domains, and a volume-independent counting bound for connected polymers containing a fixed link — survives intact and is validated numerically; it yields a pointwise derivative bound on the polymer residual with constants independent of the lattice volume, *conditionally on Balaban's small-field inputs (B1)–(B4)*. (ii) However, the final step of v1's Theorem 3.5, the inequality $k \leq C_{\mathrm{RG}}(1+\beta_k)$, relied on the inverted-sign running-coupling flow of the series erratum; with the correct asymptotic-freedom flow the reduced coupling β_k *decreases* along the cascade, the small-field condition $g_k \leq \gamma_0$ is available only for $k \leq k^*(\beta)$, and the derivative bound holds in the windowed form $C_{\mathrm{res}}(1+\beta)$ for $L_{\mathrm{vol}} \leq e^{C/g^2+O(1)}$. (iii) The assembly of the main theorem inherits two hypotheses identified in the audits of 2602.0052 v2 and 2602.0053 v2: the large-field absorption step requires a power-law penalty exponent — hypothesis (H-P0) — since with the stated polylog floor the suppression factor trivializes ($e^{-c_{\mathrm{sf}}p_0(\gamma_0)} \approx 0.95$ at $\gamma_0 = 0.1$) and the absorption inequality fails at every scale (excess $\geq 10^{4.6}$); and any quantitative use of the conditional fiber LSI via Holley–Stroock carries the penalty $e^{-2\beta n_{\mathrm{plaq}}}$ — hypothesis (H-YGZ) ($\log_{10}\alpha_{\mathrm{blk}} \approx -5559$ at $\gamma_0 = 0.1$, $n_{\mathrm{plaq}} = 64$). (iv) Version 1's Corollary 1.2 and Remark 5.1 claimed that the Dobrushin-type Assumption 6.3 of Paper I is "no longer needed" via the DLR route of the companion [6]; the v2 audit of that companion shows the route *reduces* Assumption 6.3 to an unverified block condition (H-DOB-blk) whose printed bound $c_{ij} \leq \tanh(\beta n_\partial/2)$ trivializes at weak coupling. Accordingly, v1's closing claim is replaced: the uniform LSI of Theorem 1.1 is *windowed and conditional on (H-P0) and (H-YGZ)*, and the mass gap of Corollary 1.2 is additionally conditional on (H-DOB-blk). All quantitative claims are adjudicated in a companion numerical suite (9 deterministic checks).

Contents

*Version 2. Version 1 of this paper carried the title "Unconditional uniform log-Sobolev inequality for $SU(N_c)$ lattice Yang–Mills at weak coupling" and claimed an unconditional, volume-uniform result. After the quantitative audit of the companion series (ai.viXra:2602.0032/0033/0051/0052/0053, all v2), that claim is withdrawn and replaced by the windowed, conditional statement proved here. See the Note added in v2 (Section 6).

1 Introduction

1.1 Main result

The central result of this paper and the companion series [1, 2, 3, 4] is the following. Relative to version 1, the statement is now *windowed* (volume restricted by the running-coupling horizon) and *conditional* on the two audit hypotheses recalled in Section 1.2.

Theorem 1.1 (Windowed conditional uniform LSI)**.** *Fix $d \geq 3$, $G = SU(N_c)$, and an RG block size $L_{\rm RG} \geq 2$. Assume the audit hypotheses* (H-P0) *and* (H-YGZ) *of Section 1.2. There exist $\beta_{\rm wc} < \infty$ and $C = C(d, N_c, L_{\rm RG}, \gamma_0)$ such that for all $\beta \geq \beta_{\rm wc}$ and all finite periodic lattices $\Lambda = (\mathbb{Z}/L_{\rm vol}\mathbb{Z})^d$ inside the volume window*

$$L_{\rm vol} \;\leq\; e^{C/g^2+O(1)}, \qquad \textit{equivalently} \qquad n_{\max} = \lfloor \log_{L_{\rm RG}} L_{\rm vol} \rfloor \;\leq\; k^*(\beta),$$

the Wilson lattice gauge measure μ_β satisfies

$$\mathrm{Ent}_{\mu_\beta}(f^2) \;\leq\; \frac{2}{\alpha_*} \int_{\mathcal{A}} |\nabla f|^2 \, d\mu_\beta \tag{1}$$

for all gauge-invariant Lipschitz $f : \mathcal{A} \to \mathbb{R}$, with $\alpha_* > 0$ depending only on d, N_c, L_{RG}, γ_0, and independent of L_{vol} within the window.

Corollary 1.2 (Windowed conditional mass gap)**.** *Under the hypotheses of Theorem 1.1* and additionally (H-DOB-blk), *the transfer matrix of the Wilson lattice gauge theory has a spectral gap $\Delta_{\mathrm{phys}} > 0$ uniformly in L_{vol} within the window.* (*Version 1 asserted that this is "proved unconditionally in [6]"; the v2 audit of [6] (= ai.viXra:2602.0053 v2) shows that the DLR–LSI route reduces, but does not remove, the Dobrushin-type input; see Remark 5.1.*)

1.2 The audit hypotheses

Version 2 makes explicit the three hypotheses inherited from the series audit; none of them is proved in this paper or its companions.

(H-P0) Power-law penalty. There exist $c_1 > 0$ and $s > 0$, independent of the scale and of L_{vol}, such that the large-field penalty exponent of [4] satisfies $p_0(g) \geq c_1\, g^{-s}$ on the small-field window. *The polylog floor $p_0(g) \geq c_0 |\log g|^{1+\varepsilon_0}$ actually established in the companions is quantitatively insufficient for the absorption step of Section 4, Step 2: with $c_{\mathrm{sf}} = 2/(1+\beta_0)$ the suppression factor is ≥ 0.95 at $\gamma_0 = 0.1$ and the absorption inequality fails at every sampled scale, with excess growing from $10^{4.6}$ ($k = 1$) to $10^{23.2}$ ($k = 32$). Identified in 2602.0052 v2.*

(H-YGZ) Fiber constant without Holley–Stroock collapse. The conditional fast-field measures $\mu_k(\cdot \mid \mathcal{G}_{k+1})$ satisfy a log-Sobolev inequality with constant α_{fiber} bounded below polynomially in g_k (not exponentially in β). *The available route (Bakry–Émery for the Haar/orbit part [2], then Holley–Stroock for the conditional Wilson interaction) yields only $\alpha_{\mathrm{blk}} \geq (N_c/4)e^{-2\beta n_{\mathrm{plaq}}}$, i.e. $\log_{10}\alpha_{\mathrm{blk}} \approx -5559$ at $\gamma_0 = 0.1$, $n_{\mathrm{plaq}} = 64$, which destroys every quantitative step downstream. Identified in 2602.0053 v2.*

(H-DOB-blk) Block Dobrushin condition. The block influence coefficients of the DLR route of [6] satisfy $\delta = \sup_i \sum_j c_{ij} < 1$ at weak coupling. *The printed worst-case bound $c_{ij} \leq \tanh(\beta n_\partial/2)$ gives $\delta < 1$ only for $\beta \lesssim 3.5 \times 10^{-3}$ ($d = 3$, 4^3 blocks) — the opposite of the regime $\beta \geq \beta_{\mathrm{wc}} \gg 1$; an averaged or transport-based criterion is required. Identified in 2602.0053 v2.*

In addition, the *window* is not a hypothesis but a proved constraint: with the correct asymptotic-freedom flow (Appendix A), the small-field condition $g_k \leq \gamma_0$ underlying Balaban's inputs (B1)–(B4) holds only for $k \leq k^*(\beta)$ with $k^*(\beta) = \frac{2N_c}{c_{\mathrm{flow}}}\left(g^{-2} - \gamma_0^{-2}\right) \sim C/g^2$, whence $L_{\mathrm{vol}} \leq e^{C/g^2+O(1)}$. Removing the window would require the strong crossover input (H-XOVER) of ai.viXra:2602.0051 v2.

1.3 The residual derivative hypothesis and its role

The proof of Theorem 1.1 rests on a chain of four companion papers, each contributing a specific analytic or geometric input. The chain is:

(i) Paper I [1] establishes the uniform LSI assuming a cross-scale derivative bound (Assumption 5.4 of [1]).

(ii) Paper II [2] proves a synthetic Ricci curvature bound $\mathrm{Ric}_{\mathcal{B}} \geq N_c/4$ on the orbit space. *(v2 status: the Ricci bound is unconditional and numerically verified in the series suite; its conversion into a conditional fiber LSI with a usable constant inherits (H-YGZ), see Step 1 of Section 4.)*

(iii) Paper III [3] replaces Assumption 5.4 of [1] by two explicit inputs: Hypothesis 3.2 (pointwise derivative bound on the polymer residual) and Hypothesis 4.2 (large-field conditional suppression). *(v2 status: the absorption step combining the two inputs inherits (H-P0) and the flow correction F1.)*

(iv) Paper IV [4] verifies Hypothesis 4.2 for $d \geq 3$ using Balaban's T-operation. *(v2 status: per the series notice of 2602.0052 v2, the suppression is effective only under (H-P0); the verification is of the* form *of Hypothesis 4.2, with a polylog p_0 that is too weak for the assembly.)*

The contribution of the present paper is the derivative bound: v1 claimed it verified Hypothesis 3.2 of [3] exactly as stated; v2 shows that what is actually proved is the *windowed variant* of Section 2.4, with the bare coupling β replacing the running β_k in the final estimate.

1.4 Logical structure

The corrected logical chain is:

$$\underbrace{\text{Paper II (Ricci)}}_{+\text{ (H-YGZ)}} + \underbrace{\text{Paper IV (Hyp 4.2)}}_{+\text{ (H-P0)}} + \underbrace{\text{This paper (Hyp 3.2}^{\text{w}})}_{\text{windowed}} \xrightarrow{\text{Paper III}} \text{Ass. 5.4 of [1]} \xrightarrow{\text{Paper I}} (1).$$

All arrows are valid *within the volume window and conditionally on (H-P0), (H-YGZ)*. Version 1's sentence "All arrows are unconditional once Hypothesis 3.2 is verified" is withdrawn.

1.5 Organization

Section 2 recalls the multiscale decomposition with the *corrected* coupling convention, defines the polymer residual, and re-states Hypothesis 3.2 together with its windowed variant. Section 3 contains the three technical lemmas (locality, single-polymer derivative bounds, polymer counting) — these survive v1 unchanged — and the corrected proof of the derivative bound (Theorem 3.5). Section 4 assembles the chain, now with explicit hypothesis flags at each step. Section 5 updates the series table and the discussion of the mass gap and the Clay problem. Section 6 is the Note added in v2. Appendix A records the corrected flow arithmetic and the window.

2 Setup and conventions

2.1 Lattice, gauge group, and Wilson measure

Let $\Lambda = (\mathbb{Z}/L_{\mathrm{vol}}\mathbb{Z})^d$ be a finite periodic lattice with $d \geq 3$. Denote by $E(\Lambda)$ the set of positively oriented edges (links). The gauge group is $G = SU(N_c)$, and the configuration space is $\mathcal{A} = G^{|E(\Lambda)|}$, equipped with the product Haar measure. The Wilson lattice gauge measure at inverse coupling $\beta > 0$ is

$$d\mu_\beta(U) = \frac{1}{Z_\beta} \exp\Big[-\beta \sum_{p\in P(\Lambda)} \Big(1 - \tfrac{1}{N_c}\,\mathrm{Re}\,\mathrm{tr}\,U_p\Big)\Big] \prod_{e\in E(\Lambda)} dU_e, \tag{2}$$

where the sum is over plaquettes p, U_p is the ordered product of link variables around p, and dU_e is the normalized Haar measure on G.

Coupling convention (corrected in v2). In (2), β is the coefficient of the Wilson action; in the physics normalization $\beta_{\rm phys} = 2N_c/g^2$. We use the reduced inverse coupling $\beta_k := g_k^{-2}$, so that $\beta_{{\rm phys},k} = 2N_c\beta_k$ and $\beta_{{\rm phys},0} = \beta$. Version 1 printed the running-coupling formula as $2N_c\beta_k = \beta + 2b_0k\ln L_{\rm RG}+O(1/\beta)$, i.e. with β_k *increasing* along the cascade ($g_k \to 0$ in the infrared). This is the inverted-sign flow of the series erratum (flag F1, corrected in ai.viXra:2602.0032/0033/0051 v2). The correct one-loop asymptotic-freedom flow in this convention is

$$2N_c\beta_k \;=\; \beta \;-\; 2b_0\,k\ln L_{\rm RG} \;+\; O(1/\beta), \qquad b_0 = \frac{11N_c}{48\pi^2},$$

so that g_k *grows* along the cascade and the small-field condition $g_k \le \gamma_0$ holds precisely for $k \le k^*(\beta)$; see Appendix A. All bounds below are of the form $C(1+\beta)^q$; switching between reduced and physics normalizations only changes C by a factor depending on (N_c, q).

2.2 Multiscale decomposition and filtration

We use the multiscale RG decomposition of [1], based on Balaban's block-spin construction [7–16]. Fix an RG block size $L_{\rm RG} \ge 2$. At each scale $k = 0, 1, \ldots, n_{\max}$, we have:

- A block lattice $\Lambda_k = (\mathbb{Z}/(L_{\rm vol}/L_{\rm RG}^k)\mathbb{Z})^d$.
- A σ-algebra $\mathcal{G}_k$ encoding the "slow" (block) field at scale k, with $\mathcal{G}_0 \supset \mathcal{G}_1 \supset \cdots \supset \mathcal{G}_{n_{\max}}$.
- A space of "fast" directions $\mathcal{E}_k$: left-invariant vector fields on $\mathcal{A}$ that are $\mathcal{G}_k$-measurable but orthogonal to $\mathcal{G}_{k+1}$.

The number of RG steps is $n_{\max} = \lfloor \log_{L_{\rm RG}} L_{\rm vol} \rfloor$. *Correction to v1:* v1 asserted that all constants depend on $n_{\max}$ only through the running coupling g_k, "which is controlled by the RG flow and is independent of $L_{\rm vol}$". With the correct flow this control is available only while $g_k \le \gamma_0$, i.e. for $k \le k^*(\beta)$; the constants of Section 3 are volume-independent, but the *applicability* of the construction requires $n_{\max} \le k^*(\beta)$, which is the volume window of Theorem 1.1.

2.3 Polymer activities and boundary terms

After k steps of Balaban's RG, the effective action contains a polymer expansion: the conditional fast potential at scale k decomposes as

$$V_k(U \mid \mathcal{G}_{k+1}) = S_{k,\rm Wilson}(U) + S_{k,\rm res}(U \mid \mathcal{G}_{k+1}), \tag{3}$$

where $S_{k,\rm Wilson}$ is the Wilson plaquette action at the running coupling g_k, and the residual is

$$S_{k,\rm res}(U \mid \mathcal{G}_{k+1}) = \sum_X R^{(k)}(X; U, \mathcal{G}_{k+1}) + \sum_X B^{(k)}(X; U, \mathcal{G}_{k+1}). \tag{4}$$

Here $R^{(k)}(X)$ are polymer activities indexed by connected polymers X (unions of scale-k blocks), and $B^{(k)}(X)$ are boundary terms arising from the block-spin integration.

Key properties (from Balaban's construction, as recorded in [1], §2.4). In v2 these are stated explicitly as *small-field inputs, available at scale k only under $g_j \le \gamma_0$ for all $j \le k$*, hence only inside the window:

(B1) **Locality:** Each $R^{(k)}(X)$ and $B^{(k)}(X)$ depends only on gauge field variables $\{U_e\}$ with e in the dependence neighborhood $\mathcal{N}(X) \subset \Lambda$, which has diameter $\le C_0\, L_{\rm RG}^k\, {\rm diam}_k(X)$ for a constant $C_0 = C_0(d)$.

(B2) **Analyticity:** Both $R^{(k)}(X)$ and $B^{(k)}(X)$ are analytic in the fast-field variables on the complexified tube $\widetilde{\mathcal{U}}_k^c(X, \tilde{\alpha}_0, \hat{\alpha}_1)$ of [16]. The analyticity radius $\hat{\alpha}_1 > 0$ depends on γ_0 but is uniform in k and L_{vol} (cf. [16] and the inductive scheme of [14]).

(B3) **Exponential decay of activities:** For all U in the analyticity domain,

$$|R^{(k)}(X)| \;\le\; C_R\, e^{-\kappa\, d_k(X)}, \tag{5}$$

where $d_k(X) := |X|$ is the number of scale-k blocks in X, $\kappa > 0$ is a universal decay constant, and $C_R \equiv C_R(\gamma_0)$ (cf. [16], eq. (1.65); [1], Proposition 2.8).

(B4) **Boundary term bound:** On Balaban's analytic domain,

$$|B^{(k)}(X)| \;\le\; C_B \sum_{j=1}^{k} |\Gamma_j^0 \cap X|\, e^{-\kappa\, d_k(X)}, \tag{6}$$

and in particular, using $|\Gamma_j^0 \cap X| \le C_\Gamma |X|$, $|B^{(k)}(X)| \le C'_B\, k\, |X|\, e^{-\kappa d_k(X)}$, with C_B, C'_B depending on γ_0 (cf. [1], §2.4; compare [16], eq. (1.69)).

2.4 Re-statement of Hypothesis 3.2 and its windowed variant

For the reader's convenience, we re-state the hypothesis targeted by this paper.

Hypothesis 2.1 (Hypothesis 3.2 of [3])**.** *There exist $q \ge 0$ and $C_{\text{res}} > 0$, depending on d, N_c, L_{RG} and the RG scheme, but not on L_{vol}, such that for every scale k and every unit fast direction $v \in \mathcal{E}_k$ supported on a single link,*

$$\sup_{U\in\mathcal{A}} \bigl|v\, S_{k,\text{res}}(U \mid \mathcal{G}_{k+1})\bigr| \;\le\; C_{\text{res}}(1+\beta_k)^q, \qquad \beta_k := g_k^{-2}. \tag{7}$$

v2 status. Hypothesis 2.1 *as printed* is not verified in this paper: v1's proof of the final step used the inverted flow (see the proof of Theorem 3.5). What is verified is the windowed variant:

> **(Hyp 3.2$^{\text{w}}$) Windowed residual derivative bound.** There exist $q \ge 0$ and $C_{\text{res}} = C_{\text{res}}(d, N_c, L_{\text{RG}}, \gamma_0)$ such that for every scale $k \le k^*(\beta)$ and every unit fast direction $v \in \mathcal{E}_k$ supported on a single link, $\sup_U |v S_{k,\text{res}}(U \mid \mathcal{G}_{k+1})| \le C_{\text{res}}(1+\beta)^q$ with β the *bare* coupling. Verified in Theorem 3.5 with $q = 1$, conditionally on (B1)–(B4).

Since $\beta_k \le \beta/(2N_c) \cdot 2N_c = \beta$ (reduced units, correct flow), the windowed bound is weaker than (7) exactly where v1's version was stronger; Section 4, Step 2, records what this costs in the assembly.

3 Polymer locality and the residual derivative bound

This section contains the three lemmas at the core of the paper. Lemmas 3.1, 3.2 and 3.4 are unchanged from v1: they are correct as stated, and their mechanics are validated in the companion suite (checks 1–4). Only Theorem 3.5 is restated, since its v1 proof used the inverted flow in the final step.

3.1 Locality of directional derivatives

Lemma 3.1 (Locality)**.** *Let $v \in \mathcal{E}_k$ be a unit fast direction supported on a single link $b \in E(\Lambda)$. Let $F(X)$ be a polymer functional depending only on gauge field variables $\{U_e : e \in \mathcal{N}(X)\}$. Then*

$$vF(X) = 0 \qquad \textit{unless } b \in \mathcal{N}(X). \tag{8}$$

Consequently,

$$v\, S_{k,\mathrm{res}} = \sum_{X:\, b \in \mathcal{N}(X)} vR^{(k)}(X) + \sum_{X:\, b \in \mathcal{N}(X)} vB^{(k)}(X). \tag{9}$$

Proof. By definition, v is a left-invariant vector field on $\mathcal{A}$ acting by differentiation in the fiber direction of the single link b. If $F(X)$ does not depend on U_b (i.e. $b \notin \mathcal{N}(X)$), then $vF(X) = 0$. Equation (9) follows by applying this observation to each term in the decomposition (4). □

We write $\{X \ni b\} := \{X : b \in \mathcal{N}(X)\}$.

3.2 Single-polymer derivative bounds

Lemma 3.2 (Single-polymer derivative bounds)**.** *Assume the Balaban inputs* (B1)–(B4) *hold at scale k with weak-coupling parameter γ_0. Let $v \in \mathcal{E}_k$ be a unit fast direction supported on a single link b. Then for every connected polymer X with $b \in \mathcal{N}(X)$:*

$$\sup_{U \in \mathcal{A}} |vR^{(k)}(X)| \;\le\; \frac{C_R}{\hat{\alpha}_1}\, e^{-\kappa\, d_k(X)}, \tag{10}$$

$$\sup_{U \in \mathcal{A}} |vB^{(k)}(X)| \;\le\; \frac{C'_B\, k\, |X|}{\hat{\alpha}_1}\, e^{-\kappa\, d_k(X)}. \tag{11}$$

Here $\hat{\alpha}_1 > 0$ is Balaban's uniform analyticity radius from (B2) *(depending on γ_0 but uniform in k and L_{vol}), and the constants are as in* (B3)–(B4)*.*

Proof. Polymer activities. By (B2), $R^{(k)}(X)$ is analytic in the fast-field variable U_b on a complex disc of radius $\hat{\alpha}_1 > 0$ around each point $U_b \in G$. By the one-variable Cauchy estimate on this disc and the sup-norm bound (5),

$$|vR^{(k)}(X)| = \left|\frac{\partial}{\partial t}\Big|_{t=0} R^{(k)}(X; e^{tv_b}U_b, \dots)\right| \;\le\; \frac{\sup_{|z| \le \hat{\alpha}_1} |R^{(k)}|}{\hat{\alpha}_1} \;\le\; \frac{C_R}{\hat{\alpha}_1}\, e^{-\kappa d_k(X)}.$$

Boundary terms. By (B2), $B^{(k)}(X)$ is analytic on the same tube, with the sup-norm bound (6); Cauchy's estimate in U_b gives (11). In both cases the constants C_R, C'_B, $\hat{\alpha}_1$, κ depend only on $d, N_c, L_{\mathrm{RG}}, \gamma_0$, not on L_{vol}. □

Remark 3.3 (On the analyticity of boundary terms)*.* The analyticity of $B^{(k)}(X)$ in the fast-field variables, with the same radius $\hat{\alpha}_1$ as for $R^{(k)}(X)$, is part of the inductive output of Balaban's construction ([14], Proposition 4.2, and the inductive analyticity hypotheses of [16]); in [1], §2.4, it is recorded as "the boundary terms $B^{(k)}(X)$ are analytic on the domain $\widetilde{\mathcal{U}}^c_k(X, \tilde{\alpha}_0, \hat{\alpha}_1)$". *v2 note:* these are precisely the small-field inputs; outside the window they are not available, which is one of the two places the window enters (the other being the final step of Theorem 3.5).

3.3 Polymer counting

Lemma 3.4 (Polymer counting, uniform in volume)**.** *Fix a link b at scale k and let $\mathcal{N}$ be the dependence-neighborhood map from* (B1)*. There exists $C_d \geq 1$, depending only on d, such that*

$$\#\{X:\ X \text{ connected},\ b \in \mathcal{N}(X),\ d_k(X) = n\} \ \leq\ C_d^n \qquad \text{for all } n \geq 1. \tag{12}$$

In particular, for any $\kappa > \log C_d$,

$$\sum_{X \ni b} e^{-\kappa d_k(X)} \ \leq\ \sum_{n=1}^{\infty} C_d^n e^{-\kappa n} \ =\ \frac{C_d e^{-\kappa}}{1 - C_d e^{-\kappa}} \ <\ \infty, \tag{13}$$

and this bound is independent of L_{vol}.

Proof. A connected polymer X at scale k is a connected union of scale-k blocks. The condition $b \in \mathcal{N}(X)$ constrains X to lie within distance $C_0 L_{\mathrm{RG}}^k$ of b (by (B1)). The number of connected subsets of the block lattice Λ_k containing a fixed block and having n blocks is at most $(2d)^{2n}$ by a standard lattice-animal bound (cf. [1], Lemma 5.6). Since the link b is adjacent to at most $O(1)$ blocks (a constant depending only on d and C_0), the total count satisfies (12) with $C_d = O((2d)^2)$. Crucially, this bound involves only the local combinatorics of the block lattice and does not depend on L_{vol}. *(Suite check 3 enumerates the counts exactly for $d = 2$, $n \leq 6$ and $d = 3$, $n \leq 4$ on two torus sizes each: the counts $1, 4, 18, 76, 315, 1296$ and $1, 6, 45, 344$ match $n{\cdot}$OEIS A001168/A001931, satisfy* (12)*, and are identical across volumes.)* □

3.4 The residual derivative bound (corrected)

Theorem 3.5 (Windowed residual derivative bound)**.** *Fix $d \geq 3$, $G = SU(N_c)$, $L_{\mathrm{RG}} \geq 2$, and a small-field threshold $\gamma_0 > 0$. Let $\beta \geq \beta_{\mathrm{wc}}$ and let $k \leq k^*(\beta)$, so that the correct flow gives $g_j \leq \gamma_0$ for all $j \leq k$ and Balaban's construction with inputs* (B1)–(B4) *applies. Then there exists $C_{\mathrm{res}} < \infty$, depending only on $(d, N_c, L_{\mathrm{RG}}, \gamma_0)$, such that for every unit fast direction $v \in \mathcal{E}_k$ supported on a single link b, and $\mathcal{G}_{k+1}$-a.e. slow-field configuration,*

$$\sup_{U \in \mathcal{A}} \left| v\, S_{k,\mathrm{res}}(U \mid \mathcal{G}_{k+1}) \right| \ \leq\ C_{\mathrm{res}}\,(1 + \beta), \tag{14}$$

with β the bare coupling. In particular, the windowed variant (Hyp 3.2$^{\mathrm{w}}$) *holds with $q = 1$.*

Proof. By Lemma 3.1, only polymers X with $b \in \mathcal{N}(X)$ contribute:

$$\sup_U |v S_{k,\mathrm{res}}| \ \leq\ \sum_{X \ni b} \sup_U |v R^{(k)}(X)| + \sum_{X \ni b} \sup_U |v B^{(k)}(X)|.$$

By Lemma 3.2 and Lemma 3.4, the first sum is bounded by

$$\sum_{X \ni b} \frac{C_R}{\hat{\alpha}_1} e^{-\kappa d_k(X)} \ \leq\ \frac{C_R}{\hat{\alpha}_1} \cdot \frac{C_d e^{-\kappa}}{1 - C_d e^{-\kappa}} \ =:\ A_1,$$

and, using (11) with $d_k(X) = |X|$, the second by

$$\sum_{X \ni b} \frac{C_B'\, k\, |X|}{\hat{\alpha}_1} e^{-\kappa d_k(X)} \ \leq\ \frac{C_B'\, k}{\hat{\alpha}_1} \sum_{n \geq 1} n\, C_d^n e^{-\kappa n} \ =:\ A_2 \cdot k,$$

the series converging for $\kappa > \log C_d + 1$, say. The constants A_1, A_2 depend only on $(d, N_c, L_{\mathrm{RG}}, \gamma_0)$ — *v2 correction: v1's statement omitted the γ_0-dependence, which enters through $C_R, C'_B, \hat{\alpha}_1$* — and not on L_{vol}.

Final step (corrected). v1 concluded by invoking the printed flow $2N_c\beta_k = \beta + 2b_0 k \ln L_{\mathrm{RG}} + O(1/\beta)$ to get $k \leq C_{\mathrm{RG}}(1 + \beta_k)$. This inequality is an artifact of the inverted sign: under the correct asymptotic-freedom flow, β_k *decreases* in k, and the ratio $k/(1 + \beta_k)$ is unbounded along the cascade (suite check 6: it grows past the window and the small-field hypothesis is lost at $k > k^*$). What the correct flow does give, inside the window, is

$$k \;\leq\; k^*(\beta) \;\leq\; \frac{2N_c}{c_{\mathrm{flow}}}\, g^{-2} \;\leq\; C'_{\mathrm{RG}}\,(1+\beta), \qquad C'_{\mathrm{RG}} = C'_{\mathrm{RG}}(N_c, L_{\mathrm{RG}}),$$

by Appendix A (numerically, $k^*/(1 + \beta_{\mathrm{red},0}) \leq 2N_c/c_{\mathrm{flow}} \approx 62.1$ for $N_c = 3$, $L_{\mathrm{RG}} = 2$, uniformly in g). Therefore

$$\sup_U |vS_{k,\mathrm{res}}| \;\leq\; A_1 + A_2 k \;\leq\; C_{\mathrm{res}}(1+\beta), \qquad C_{\mathrm{res}} := A_1 + A_2 C'_{\mathrm{RG}},$$

which is (14). □

Remark 3.6 (What is and is not volume-uniform)*.* The three ingredients of this section have different statuses. Locality (Lemma 3.1) and counting (Lemma 3.4) are exact, dimension-general, and genuinely volume-independent: they hold for all L_{vol} with no window (suite checks 1, 3). The Cauchy step (Lemma 3.2) is standard, but its inputs (B2)–(B4) are small-field outputs of Balaban's induction, available only while $g_k \leq \gamma_0$. The final step of Theorem 3.5 is where the flow enters: with the correct sign, the bound is in terms of the *bare* $(1 + \beta)$ and requires $k \leq k^*(\beta)$, i.e. $L_{\mathrm{vol}} \leq e^{C/g^2 + O(1)}$. Thus the honest summary is: *the derivative-bound mechanism is a sound interface reduction to (B1)–(B4), volume-uniform inside the window; it is not an unconditional verification of Hypothesis 2.1 as printed.*

4 Assembly: the main theorem, with hypothesis flags

4.1 Chain of implications (annotated)

We trace the chain leading to Theorem 1.1, flagging at each step the audit hypotheses it consumes.

Step 1. *Conditional fiber LSI (Paper II) — flag (H-YGZ).* By [2], Theorem 1.1, the orbit space $\mathcal{B} = \mathcal{A}/\mathcal{G}$ satisfies $\mathrm{Ric}_{\mathcal{B}} \geq N_c/4$ (unconditional; verified numerically in the series suite, 2602.0046). v1 asserted that Bakry–Émery "applied fiber by fiber" yields $\alpha_{\mathrm{fiber}} \geq N_c/4$. This ignores the conditional Wilson interaction on the fiber: perturbing the Haar/orbit reference measure by the conditional potential requires Holley–Stroock and multiplies the constant by $e^{-2\,\mathrm{osc}}$ with $\mathrm{osc} \leq 2\beta n_{\mathrm{plaq}}$, i.e. $\alpha_{\mathrm{blk}} \geq (N_c/4)e^{-2\beta n_{\mathrm{plaq}}}$ (suite check 8: $\log_{10}\alpha_{\mathrm{blk}} \approx -5559$ at $\gamma_0 = 0.1$, $n_{\mathrm{plaq}} = 64$). A fiber constant that is usable downstream is exactly hypothesis (H-P0)'s sibling (H-YGZ), per the 2602.0053 v2 audit.

Step 2. *Integrated cross-scale bounds (Paper III) — flags (H-P0) and F1.* Paper III controls the cross-scale interaction terms by the fiber constant and the two inputs. v1 asserted that in the large-field region the product

$$M_k^2 \cdot \mu_k(Z_k(B) \mid \mathcal{G}_{k+1}) \;\leq\; C_{\mathrm{res}}^2 (1+\beta_k)^{2q} \cdot C_{\mathrm{blk}} \exp(-c\, p_0(g_k))$$

"converges to zero as $g_k \to 0$, since $p_0(g_k) \to \infty$". Both clauses fail: (a) $g_k \to 0$ along the cascade is again the inverted flow; with the correct flow g_k grows and $p_0(g_k)$ *decreases*; (b) even at fixed scale, with the polylog floor and $c_{\mathrm{sf}} = 2/(1+\beta_0)$ the suppression factor is ≈ 0.95 at $\gamma_0 = 0.1$ and the absorption against $L_{\mathrm{RG}}^{-(d-1)k}$ fails at every sampled scale with excess $10^{4.6}$–$10^{23.2}$ (suite check 7; reprise of 2602.0052 v2). With the windowed bound of Theorem 3.5, $(1+\beta_k)^{2q}$ is moreover replaced by $(1+\beta)^{2q}$. The step therefore stands only under (H-P0).

Step 3. *Large-field suppression (Paper IV) — flag (H-P0).* Paper IV verifies the *form* of Hypothesis 4.2 of [3] for $d \geq 3$: $\mu_k(Z_k(B) \mid \mathcal{G}_{k+1}) \leq C_{\mathrm{blk}} \exp(-c\, p_0(g_k))$ with volume-uniform constants. Per the series notice of 2602.0052 v2, the established p_0 is polylog; the suppression is quantitatively effective only under (H-P0).

Step 4. *Residual derivative bound (this paper) — windowed.* Theorem 3.5 verifies the windowed variant (Hyp 3.2$^{\mathrm{w}}$): $\sup_U |vS_{k,\mathrm{res}}| \leq C_{\mathrm{res}}(1+\beta)$ for $k \leq k^*(\beta)$, constants independent of L_{vol}.

Step 5. *Closure: Assumption 5.4 of Paper I.* With Steps 1–4 as amended, the integrated cross-scale bounds of [3] hold *within the window and under (H-P0), (H-YGZ)*; these bounds are the content of Assumption 5.4 of [1].

Step 6. *Uniform LSI (Paper I).* By [1], Theorem 1.1 (conditional on Assumption 5.4): entropy telescoping over the $n_{\max} \leq k^*(\beta)$ scales, the defective-to-tight argument (Rothaus lemma + uniform Poincaré), yield (1) with α_* as stated.

4.2 Proof of Theorem 1.1

Proof of Theorem 1.1. Assume (H-P0) and (H-YGZ), and let $L_{\mathrm{vol}} \leq e^{C/g^2+O(1)}$ so that $n_{\max} \leq k^*(\beta)$. By Steps 1–6: Theorem 3.5 (this paper) supplies the windowed derivative input; [4] supplies the suppression input, effective under (H-P0); [2] plus (H-YGZ) supplies the fiber constant. By [3], Theorem 1.1, Assumption 5.4 of [1] holds on the window; by [1], Theorem 1.1, the log-Sobolev inequality (1) holds with $\alpha_* > 0$ independent of L_{vol} within the window. □

Remark 4.1 (Why the shifted essential supremum suffices). In Step 2, the bound on the large-field contribution involves the conditional expectation $\mathbb{E}[\mathbf{1}_{Z_k}|vV_{<k}|^2 \mid \mathcal{G}_{k+1}]$, which factorizes via the tower property:

$$\mathbb{E}\big[A \cdot \mathbb{E}[\mathbf{1}_{Z_k}|vV_{<k}|^2 \mid \mathcal{G}_{k+1}]\big] \;\leq\; \Big(\operatorname*{ess\,sup}_{\mathcal{G}_{k+1}} \mathbb{E}[\mathbf{1}_{Z_k}|vV_{<k}|^2 \mid \mathcal{G}_{k+1}]\Big) \cdot \mathbb{E}[A]$$

for any non-negative $\mathcal{G}_{k+1}$-measurable A (suite check 5). This structural step is standard and survives v1 unchanged; what v2 changes is the *size* of the two factors (Steps 1–3 above), not the factorization.

Remark 4.2 (The case $d = 2$). The derivative bound (Theorem 3.5) is valid for all $d \geq 2$, since the polymer locality argument is dimension-independent. However, the verification of Hypothesis 4.2 in Paper IV uses properties of the T-operation established only for $d \geq 3$. In $d = 2$, a uniform LSI for the full Wilson measure remains open even in windowed conditional form; partial results for toy models are discussed in [3], §7.

Remark 4.3 (Inherited hypotheses: summary). For citation purposes: every use of this paper's Theorem 1.1 consumes (H-P0) (Steps 2–3), (H-YGZ) (Step 1), and the volume window (Step 4 and $n_{\max} \leq k^*$); Corollary 1.2 additionally consumes (H-DOB-blk). None of these is proved in the series. Any downstream paper citing v1's unconditional statement must be read as conditional/windowed accordingly; this includes the "unconditional mass gap" chain 2602.0053/0054/0055, of which 0053 has been rectified (v2), the present paper is 0055 v2, and 2602.0054 has likewise been rectified (v2) [20]: the retagging of the chain is complete.

5 Discussion

5.1 What is proved

Table 1 updates v1's summary with the post-audit statuses.

Paper	Result	v1 status	v2 (post-audit) status	Restriction
I [1]	Uniform LSI $\Rightarrow$ gap	Cond. Ass. 5.4, 6.3	unchanged; F1 flow corrected	$d \geq 2$
II [2]	$\mathrm{Ric}_{\mathcal{B}} \geq N_c/4$	Unconditional	Ricci unconditional; fiber LSI use $\to$ (H-YGZ)	$d \geq 2$
III [3]	Cross-scale bounds	Cond. Hyps. 3.2, 4.2	+ (H-P0), windowed (F1)	$d \geq 2$
IV [4]	Hyp. 4.2	"Unconditional"	form verified; effective only under (H-P0)	$d \geq 3$
V (this)	Hyp. 3.2	"Unconditional"	windowed variant, cond. (B1)–(B4)	$d \geq 2$
Combined	Uniform LSI (Thm. 1.1)	"Unconditional"	**windowed, cond. (H-P0)+(H-YGZ)**	$d \geq 3$
	Mass gap (Cor. 1.2)	contradictory in v1	**+ (H-DOB-blk)**	$d \geq 3$

Table 1: Summary of the paper series, v1 vs. v2 statuses. The restriction column indicates the minimal dimension. The combined result requires $d \geq 3$ because Paper IV does.

5.2 Mass gap

Proof of Corollary 1.2. The log-Sobolev inequality (1) implies a Poincaré inequality with the same constant: $\mathrm{Var}_{\mu_\beta}(f) \leq \alpha_*^{-1} \int |\nabla f|^2 d\mu_\beta$ for all gauge-invariant f. Two routes to a transfer-matrix gap are on record: (a) [1], Theorem 6.1, which consumes the Dobrushin-type Assumption 6.3 of [1]; (b) the DLR–LSI extension and Stroock–Zegarlinski route of [6], assembled in detail in the companion [20] (v2: conditional and windowed). The v2 audit of [6] (ai.viXra:2602.0053 v2) shows that route (b) does not eliminate the Dobrushin input: it reduces Assumption 6.3 to the block-level condition (H-DOB-blk), which remains unverified, and whose printed worst-case bound trivializes at weak coupling (suite check 9). Under (H-DOB-blk), either route yields $\Delta_{\mathrm{phys}} > 0$ uniformly in L_{vol} within the window. □

Remark 5.1 (Status of Assumption 6.3 — v1's contradiction resolved). Version 1 contained three mutually incompatible statements: Corollary 1.2 ("proved unconditionally in [6]"), Remark 5.1 ("Assumption 6.3 is no longer needed"), and §5.3 (mass gap "pending Assumption 6.3", its verification listed as remaining step (i)). v2 resolves the contradiction in favor of §5.3, as sharpened by the audit: the DLR route *reduces* Assumption 6.3 to (H-DOB-blk) — a genuine improvement of localization, from a global transfer-matrix condition to a block condition — but does not remove it. The printed bound $c_{ij} \leq \tanh(\beta n_\partial/2)$ gives $\delta < 1$ only for $\beta \lesssim 3.5 \times 10^{-3}$ in the representative $d = 3$ geometry, the opposite of the weak-coupling regime; no worst-case oscillation criterion can close this, and an averaged or transport-based criterion is the identified route forward.

Remark 5.2 (Chain notice — updated). The companion 2602.0054 has now been audited and replaced (v2) [20]: its v1 unconditional assembly is withdrawn in favor of a conditional and windowed one (inheriting (H-DOB-blk) and (H-P0), inside the same volume window), the claimed "bypass" of the Dobrushin input is retracted, and its transfer-operator correlation identities are corrected to kernel/symmetrized form (its Remark 2.8; spectrum, hence Δ_{phys}, unaffected). The present paper does not use the row-normalized transfer operator or the trace identities, so it inherits nothing from that correction. With 2602.0051–0055 all at v2, the citation-hygiene part of the series notice of 2602.0052 v2 is discharged; the notice remains in force only through the open hypotheses (H-P0), (H-YGZ), (H-DOB-blk) and the window/(H-XOVER).

5.3 What remains for the Clay Millennium Problem

The Clay problem [17] requires the construction of a continuum $SU(N_c)$ Yang–Mills theory on $\mathbb{R}^4$ satisfying the Wightman (or Osterwalder–Schrader) axioms, together with a strictly positive mass gap. The present series establishes, for $d \geq 3$ at weak coupling, *and now explicitly conditionally*:

(a) A windowed, volume-uniform log-Sobolev inequality on the lattice, conditional on (H-P0) and (H-YGZ) (Theorem 1.1).

(b) A windowed mass gap on the lattice, additionally conditional on (H-DOB-blk) (Corollary 1.2).

The remaining steps are correspondingly longer than v1's list:

(i) Proof of (H-P0): a power-law lower bound on the large-field penalty exponent (the polylog floor is insufficient by suite check 7).

(ii) Proof of (H-YGZ): a fiber LSI constant not degraded by Holley–Stroock (insufficient by ~ 5559 orders, suite check 8).

(iii) Proof of (H-DOB-blk) by an averaged/transport criterion (worst-case criteria provably trivialize, suite check 9).

(iv) Removal of the volume window, requiring the crossover input (H-XOVER) of 2602.0051 v2, i.e. control of the flow beyond the perturbative regime.

(v) Continuum limit $a \to 0$ preserving the gap; Osterwalder–Schrader axioms; and the extension to $d = 4$ with its additional UV renormalization issues.

We emphasize that the present work does not claim to solve the Clay problem, and — correcting v1 — does not claim an unconditional lattice LSI either. It provides a sound, windowed interface reduction of the derivative input to Balaban's small-field theory, and an exact accounting of the hypotheses that remain.

6 Note added in v2

Version 1 of this paper claimed the unconditional verification of Hypothesis 3.2 of [3] and, jointly with the companions, an unconditional volume-uniform LSI and (via [6]) an unconditional mass gap. Following the quantitative audit of the series (ai.viXra:2602.0032/0033/0051/0052/0053, all v2), those claims are withdrawn and replaced as follows.

- The final step of v1's Theorem 3.5 ($k \le C_{\rm RG}(1+\beta_k)$) used the inverted-sign flow (flag F1). Corrected: windowed bound $C_{\rm res}(1+\beta)$ for $k \le k^*(\beta)$, $L_{\rm vol} \le e^{C/g^2+O(1)}$ (Theorem 3.5, Appendix A).

- v1's Theorem 3.5 stated constants "depending only on $(d, N_c, L_{\rm RG})$"; the proof produces A_1, A_2 depending on γ_0 through $C_R, C'_B, \hat\alpha_1$. Corrected throughout.

- Step 2 of the assembly inherits (H-P0) and Step 1 inherits (H-YGZ) (flags F2, F3 of 2602.0052 v2/0053 v2); Step 3's citation of Paper IV as unconditional is downgraded to form-level.

- v1's Corollary 1.2 / Remark 5.1 / §5.3 were mutually contradictory on the status of Assumption 6.3 of [1]; resolved: conditional on (H-DOB-blk) (Remark 5.1).

- v1's reference [5] was a self-citation of the paper itself; removed.

- The interface mechanism of Section 3 (locality + Cauchy + counting) survives unchanged and is validated in the companion suite (checks 1–5): this remains the paper's contribution.

- Added in this replacement: the companion 2602.0054 has since been audited and replaced (v2) [20]; Remarks 4.3 and 5.2 record the completed retagging of the chain, and reference [6] is corrected (v1 had listed 2602.0053 under the title of 2602.0054).

All numerical adjudications above are reproduced by the deterministic suite `verify_2602_0055.py` (9 checks, $\sim$2 s) at `https://github.com/lluiseriksson/THE-ERIKSSON-PROGRAMME`, directory `verification/2602-0055/`.

A The corrected flow and the window

In reduced units $\beta_k = g_k^{-2}$, the one-loop asymptotic-freedom flow reads

$$2N_c\beta_k \;=\; 2N_c\beta_0 \;-\; c_{\rm flow}\, k \;+\; O(1/\beta), \qquad c_{\rm flow} = 2b_0 \ln L_{\rm RG}, \quad b_0 = \frac{11N_c}{48\pi^2},$$

with $2N_c\beta_0 = \beta$ (the bare Wilson coefficient). Thus g_k increases with k: coarse lattices are more strongly coupled, as they must be. The small-field condition $g_k \le \gamma_0$, required by the inputs (B1)–(B4) at every scale $j \le k$, holds precisely for

$$k \;\le\; k^*(\beta) \;:=\; \Big\lfloor \frac{\beta - 2N_c\gamma_0^{-2}}{c_{\rm flow}} \Big\rfloor \;\le\; \frac{2N_c}{c_{\rm flow}}\, g^{-2},$$

so that $k^* \le C'_{\rm RG}(1+\beta_{\rm red,0})$ with $C'_{\rm RG} = 2N_c/c_{\rm flow}$ depending only on $(N_c, L_{\rm RG})$ (≈ 62.1 for $N_c = 3$, $L_{\rm RG} = 2$). Since $n_{\max} = \lfloor \log_{L_{\rm RG}} L_{\rm vol} \rfloor$ scales must fit inside the window, the volume constraint is

$$\log L_{\rm vol} \;\le\; k^*(\beta) \ln L_{\rm RG} \;=\; \frac{C}{g^2} + O(1),$$

numerically $\log L_{\rm vol} \le 32.3/g^2$ at $N_c = 3$, $L_{\rm RG} = 2$, $\gamma_0 = 0.1$, $g = 0.05$ (suite check 6), i.e. $L_{\rm vol} \lesssim 2^{18{,}639}$: the window is astronomically large at weak coupling — the limitation is conceptual (the statement "for all finite $L_{\rm vol}$" is not proved), not practical. By contrast, the printed v1 flow (+ sign) made β_k grow, whence v1's $k \le C_{\rm RG}(1+\beta_k)$; under the correct flow that inequality fails beyond the window ($k/(1+\beta_k)$ grows without bound as β_k falls, and the small-field regime is lost), which is the precise point where v1's unconditional volume-uniformity broke.

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[20] L. Eriksson, *From uniform log-Sobolev inequality to mass gap for lattice Yang–Mills at weak coupling: a conditional and windowed assembly*, ai.viXra:2602.0054 (v2), 2026. Conditional transfer-matrix back end of the chain; inherits (H-DOB-blk)+(H-P0), corrects the transfer-operator identities to kernel/symmetrized form.