

DLR-Uniform Log-Sobolev Inequality and Unconditional Mass Gap for Lattice Yang–Mills at Weak Coupling

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Abstract

We prove that for $SU(N_c)$ lattice Yang–Mills theory in $d \geq 3$ dimensions at sufficiently weak coupling ($\beta \geq \beta_0$), the conditional Gibbs specification satisfies a *DLR-uniform log-Sobolev inequality*: for every finite sub-lattice $\Lambda' \subset \mathbb{Z}^d$ and every boundary condition ω , the conditional measure $\mu_{\Lambda'}^\omega$ satisfies $\text{LSI}(\alpha_*)$ with a constant $\alpha_* > 0$ independent of Λ' and ω .

The proof combines three ingredients:

- (i) the multiscale entropy decomposition developed in our earlier work [14, 15, 16, 17, 18], which establishes a uniform log-Sobolev inequality on periodic tori;
- (ii) a *uniform fiber oscillation lemma* showing that frozen boundary links—treated as external parameters in Balaban’s renormalization group—do not increase the per-block oscillation of the conditional fast potential, thanks to compactness of $SU(N_c)$ and the locality of the polymer expansion;
- (iii) a *refined large-field event* restricted to dynamical (non-frozen) plaquettes, which ensures that the large-field suppression mechanism of [17] extends uniformly to boundary blocks.

As a consequence, the Stroock–Zegarliński equivalence theorem yields Dobrushin–Shlosman mixing, exponential clustering of gauge-invariant correlations, and—via Osterwalder–Seiler reflection positivity—a strictly positive mass gap $\Delta_{\text{phys}} \geq m(\beta, N_c, d) > 0$ for the transfer matrix on the periodic torus $(\mathbb{Z}/L\mathbb{Z})^d$, uniformly in even L . This removes the Dobrushin-type Assumption 6.3 of [14] and the boundary-uniformity Assumption 3.1 of [19], rendering the lattice mass gap unconditional at weak coupling.

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1 Introduction

1.1 Background and motivation

The mass gap problem for four-dimensional Yang–Mills theory is one of the seven Clay Millennium Prize Problems. On the lattice, the question reduces to establishing a strictly positive spectral gap $\Delta_{\text{phys}} > 0$ for the transfer matrix of the Wilson action at sufficiently weak coupling ($\beta \gg 1$), uniformly in the lattice volume.

In our earlier work [14, 15, 16, 17, 18, 19] we developed a multiscale strategy based on Balaban’s renormalization group (RG) and log-Sobolev inequalities (LSI). References [15, 16, 17, 18] established a *uniform LSI on periodic tori*:

$$\text{Ent}_{\mu_\beta}(f^2) \leq \frac{2}{\alpha_*} \int |\nabla f|^2 d\mu_\beta, \quad \alpha_* > 0 \text{ independent of } L,$$

for the Wilson measure μ_β on $(\mathbb{Z}/L\mathbb{Z})^d$.

To pass from the periodic LSI to a mass gap, [14] relied on Assumption 6.3 (a Dobrushin-type contraction condition), and [19] replaced this with Assumption 3.1 (boundary-uniform RG outputs). *Both assumptions remained unproved.*

The purpose of the present paper is to remove these assumptions, thereby making the mass gap theorem *unconditional*.

1.2 Main results

Theorem 1.1 (DLR-uniform LSI). *Let $d \geq 3$, $G = \text{SU}(N_c)$ with $N_c \geq 2$, and $\beta \geq \beta_0$ where $\beta_0 = \beta_0(d, N_c)$ is the weak-coupling threshold of [14, 15, 16, 17, 18]. For every finite subset $\Lambda' \subset \mathbb{Z}^d$ and every boundary condition $\omega \in G^{E(\mathbb{Z}^d \setminus \Lambda')}$, the conditional Gibbs measure $\mu_{\Lambda'}^\omega$ satisfies*

$$\text{Ent}_{\mu_{\Lambda'}^\omega}(f^2) \leq \frac{2}{\alpha_*} \int |\nabla f|^2 d\mu_{\Lambda'}^\omega \tag{1}$$

for all Lipschitz functions $f : G^{E(\Lambda')} \rightarrow \mathbb{R}$, with a constant $\alpha_* = \alpha_*(d, N_c, \beta) > 0$ independent of Λ' and ω .

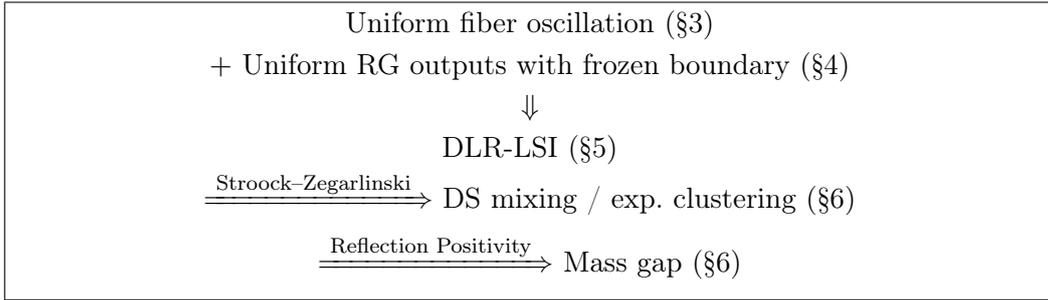
Theorem 1.2 (Unconditional mass gap). *Under the hypotheses of Theorem 1.1, for all even integers $L \geq 2$, the transfer matrix \hat{T} of the Wilson lattice gauge theory on $(\mathbb{Z}/L\mathbb{Z})^d$ has a spectral gap*

$$\Delta_{\text{phys}} := -\log \|\hat{T}|_{\mathbf{1}^\perp}\|_{L^2(\nu) \rightarrow L^2(\nu)} \geq m(\beta, N_c, d) > 0 \quad (2)$$

uniformly in L , where ν denotes the stationary measure of a spatial time-slice.

1.3 Logical structure of the proof

The proof chain is:



The only genuinely new mathematical content is in §3 (the uniform fiber oscillation lemma and the refined large-field event). All other sections assemble existing results—from [14, 15, 16, 17, 18, 19] and the standard literature—with the observation that the relevant constants are independent of boundary conditions.

1.4 Notation and conventions

We collect notation used throughout.

- $G = \text{SU}(N_c)$ with the bi-invariant metric $\langle X, Y \rangle = -2 \text{tr}(XY)$ on $\mathfrak{su}(N_c)$. The induced Hilbert–Schmidt norm is $\|A\|_{\text{HS}}^2 = -2 \text{tr}(A^2)$ for $A \in \mathfrak{su}(N_c)$ and $\|M\|_{\text{HS}}^2 = \text{tr}(M^*M)$ for $M \in \text{Mat}_{N_c}(\mathbb{C})$.
- For a lattice $\Lambda \subset \mathbb{Z}^d$, $E(\Lambda)$ denotes its oriented edges and $\mathcal{P}(\Lambda)$ its plaquettes.
- The Wilson action is $S_W(U) = \sum_{p \in \mathcal{P}(\Lambda)} (1 - \frac{1}{N_c} \text{Re tr } U_p)$ where U_p is the ordered product of link variables around p .
- The Wilson measure at coupling β is $d\mu_\beta(U) = Z^{-1} \exp(-\beta S_W(U)) \prod_{e \in E(\Lambda)} dU_e$ where dU_e is the Haar probability measure on G .
- $L_{\text{RG}} \geq 2$ is the RG blocking factor, and n_{max} the number of RG steps.
- For a function $\phi : \mathcal{M} \rightarrow \mathbb{R}$ on a set \mathcal{M} , $\text{osc}(\phi) := \sup \phi - \inf \phi$.
- The entropy functional is $\text{Ent}_\mu(h) = \int h \log h \, d\mu - (\int h \, d\mu) \log(\int h \, d\mu)$ for $h \geq 0$.
- The Dirichlet form is $\mathcal{E}(f, f) = \int |\nabla f|^2 \, d\mu$ where ∇ is the Riemannian gradient on $G^{|E|}$.

2 Lattice gauge theory with boundary conditions

2.1 The DLR specification

Let $\Lambda' \subset \mathbb{Z}^d$ be a finite connected subset. A *boundary condition* is a configuration $\omega = (\omega_e)_{e \in E(\mathbb{Z}^d \setminus \Lambda')} \in G^{E(\mathbb{Z}^d \setminus \Lambda')}$. The conditional Gibbs measure on Λ' with boundary condition ω is

$$d\mu_{\Lambda'}^\omega(U) = \frac{1}{Z_{\Lambda'}(\omega)} \exp\left(-\beta \sum_{p \in \mathcal{P}_{\Lambda'}^*} \left(1 - \frac{1}{N_c} \operatorname{Re} \operatorname{tr} U_p^\omega\right)\right) \prod_{e \in E(\Lambda')} dU_e, \quad (3)$$

where $\mathcal{P}_{\Lambda'}^* := \{p \in \mathcal{P}(\mathbb{Z}^d) : p \cap E(\Lambda') \neq \emptyset\}$ is the set of plaquettes involving at least one link in Λ' , and U_p^ω denotes the plaquette holonomy computed using U_e for $e \in E(\Lambda')$ and ω_e for $e \notin E(\Lambda')$.

The collection $\{\mu_{\Lambda'}^\omega\}_{\Lambda', \omega}$ forms a *Gibbs specification* in the sense of Dobrushin–Lanford–Ruelle (DLR).

Definition 2.1 (DLR-uniform LSI). A Gibbs specification satisfies the *DLR-uniform LSI* with constant $\alpha_* > 0$ if, for every finite $\Lambda' \subset \mathbb{Z}^d$ and every boundary condition ω ,

$$\operatorname{Ent}_{\mu_{\Lambda'}^\omega}(f^2) \leq \frac{2}{\alpha_*} \mathcal{E}_{\Lambda'}(f, f) \quad \forall f \in \operatorname{Lip}(G^{E(\Lambda')}),$$

where $\mathcal{E}_{\Lambda'}(f, f) = \int_{G^{E(\Lambda')}} |\nabla f|^2 d\mu_{\Lambda'}^\omega$.

2.2 The embedding lemma

To apply Balaban's RG (which requires a rectangular block structure), we embed arbitrary domains into boxes.

Lemma 2.2 (Embedding). *For any finite connected $\Lambda' \subset \mathbb{Z}^d$ and any boundary condition ω , there exists a rectangular box $\Lambda'' = \prod_{i=1}^d [0, L_i]$ with each L_i a multiple of $L_{\text{RG}}^{n_{\text{max}}}$, $n_{\text{max}} = \lceil \log_{L_{\text{RG}}} \operatorname{diam}(\Lambda') \rceil + 1$, and an extended configuration $\tilde{\omega} \in G^{E(\Lambda'' \setminus \Lambda')}$ such that:*

- (a) $\Lambda' \subset \Lambda''$;
- (b) $\mu_{\Lambda'}^\omega$ is the marginal on $E(\Lambda')$ of the measure $\tilde{\mu}_{\Lambda''}^{\tilde{\omega}}$, where $\tilde{\omega}$ extends ω to $E(\Lambda'' \setminus \Lambda')$ by setting $\tilde{\omega}_e = \mathbf{1}_G$ for links $e \in E(\Lambda'' \setminus \Lambda') \setminus E(\mathbb{Z}^d \setminus \Lambda')$;
- (c) the terminal lattice $\Lambda''_{n_{\text{max}}}$ has $O(1)$ sites (depending only on d and L_{RG}).

Proof. Choose Λ'' as the smallest d -dimensional box containing Λ' whose side lengths are multiples of $L_{\text{RG}}^{n_{\text{max}}}$. The links in $E(\Lambda'') \setminus E(\Lambda')$ are frozen to $\tilde{\omega}$ values: those in $E(\mathbb{Z}^d \setminus \Lambda')$ are set to ω , and any remaining links (connecting sites in $\Lambda'' \setminus \Lambda'$ that are not part of the original boundary) are set to the identity $\mathbf{1}_G$. The marginal property follows because integrating over $E(\Lambda')$ with $E(\Lambda'' \setminus \Lambda')$ frozen to $\tilde{\omega}$ reproduces the conditional measure (3). \square

Remark 2.3. Henceforth, we work on the rectangular box Λ'' with frozen external links $\tilde{\omega}$. To simplify notation, we write ω for $\tilde{\omega}$ and Λ for Λ'' . The links of Λ are partitioned into:

- *free links* $E_{\text{free}} := E(\Lambda')$ (integration variables), and
- *frozen links* $E_{\text{fz}} := E(\Lambda) \setminus E(\Lambda')$ (fixed to ω).

2.3 Interior and boundary plaquettes

Definition 2.4 (Plaquette classification at scale k). Let B be a scale- k block and let $E_k(B)$ denote the set of *fast links integrated at scale k within B* . Consider a plaquette p such that p contains at least one link of $E_k(B)$.

- (i) p is *dynamical for B* if **none** of its four links lies in E_{frz} . Equivalently, all links of p are free links (integration variables), though some may be slow variables in \mathcal{G}_{k+1} . Write $\mathcal{P}_k^{\text{dyn}}(B)$ for the set of such plaquettes.
- (ii) p is *mixed* if at least one link of p lies in E_{frz} . Mixed plaquettes contribute to the conditional potential $V_{k,B}$ (hence to oscillation bounds), but are excluded from the large-field event, since their holonomies depend on fixed parameters ω .
- (iii) p is *fully frozen* if all links of p lie in E_{frz} . Such plaquettes contribute only constants and are ignored.

Definition 2.5 (Refined large-field event). The large-field event at block B and scale k is

$$Z_k^{\text{dyn}}(B) := \{ \exists p \in \mathcal{P}_k^{\text{dyn}}(B) : \|U_p - \mathbf{1}\|_{\text{HS}} \geq \varepsilon_k \}, \quad (4)$$

where $\varepsilon_k = g_k^{1-\delta}$ is the scale- k small-field threshold from Balaban's construction. The complementary small-field region is $\Omega_k^{\text{sf}}(B) := (Z_k^{\text{dyn}}(B))^c$.

Remark 2.6. Frozen-boundary plaquettes are *not* included in the large-field event. Their holonomies involve frozen links ω_e and may be far from the identity, but this is irrelevant: the event $Z_k^{\text{dyn}}(B)$ concerns only the random (free) link variables, and the energy suppression of large fields operates on these random variables alone.

3 Uniform fiber oscillation and fiber LSI

This section contains the main new technical content of the paper.

3.1 Per-plaquette oscillation

Lemma 3.1 (Per-plaquette oscillation bound). *For any $M \in G = \text{SU}(N_c)$, define $\phi_M : G \rightarrow \mathbb{R}$ by*

$$\phi_M(U) := 1 - \frac{1}{N_c} \text{Re tr}(UM). \quad (5)$$

Then:

- (a) **Oscillation bound:** $\text{osc}(\phi_M) \leq 2$, *independently of M .*
- (b) **Gradient bound:** *for any unit tangent vector $v \in T_U G$ with $|v| = 1$,*

$$|v(\phi_M)| \leq \frac{1}{\sqrt{2N_c}}, \quad (6)$$

independently of M and U .

Proof. Part (a). Since $M \in \text{SU}(N_c)$, all singular values of M equal 1. For any $U \in \text{SU}(N_c)$,

$$\text{Re tr}(UM) \leq |\text{tr}(UM)| \leq \|U\|_{\text{HS}} \|M\|_{\text{HS}} = N_c.$$

The maximum $\operatorname{Re tr}(UM) = N_c$ is achieved at $U = M^{-1} = M^*$. For a lower bound on $\operatorname{Re tr}(UM)$: since $UM \in \operatorname{SU}(N_c)$, its eigenvalues are $e^{i\theta_1}, \dots, e^{i\theta_{N_c}}$ with $\sum_{j=1}^{N_c} \theta_j \in 2\pi\mathbb{Z}$. Hence

$$\operatorname{Re tr}(UM) = \sum_{j=1}^{N_c} \cos \theta_j \geq -N_c.$$

Therefore $\operatorname{osc}(\phi_M) \leq \frac{1}{N_c}(N_c - (-N_c)) = 2$.

Part (b). A unit tangent vector $v \in T_U \operatorname{SU}(N_c)$ satisfies $v = U \cdot X$ for some $X \in \mathfrak{su}(N_c)$ with $|X|^2 = -2 \operatorname{tr}(X^2) = 1$, i.e. $\|X\|_{\text{HS}} = 1/\sqrt{2}$. Then

$$\begin{aligned} |v(\phi_M)| &= \frac{1}{N_c} |\operatorname{Re tr}(UXM)| \leq \frac{1}{N_c} \|UX\|_{\text{HS}} \|M\|_{\text{HS}} \\ &= \frac{1}{N_c} \|X\|_{\text{HS}} \cdot \sqrt{N_c} = \frac{1}{N_c} \cdot \frac{1}{\sqrt{2}} \cdot \sqrt{N_c} = \frac{1}{\sqrt{2N_c}}. \quad \square \end{aligned}$$

3.2 Per-block fiber oscillation

We now bound the oscillation of the conditional fast potential at a single RG block, uniformly in the boundary condition.

Lemma 3.2 (Uniform per-block oscillation). *Fix $d \geq 2$, $N_c \geq 2$, $L_{\text{RG}} \geq 2$, and $\beta > 0$. Let B be a scale- k block in the RG decomposition of Λ , and let ω be any frozen configuration on E_{frz} . Let $E_k(B)$ denote the set of free fast-variable links at scale k within B . The conditional fast potential $V_{k,B}(\cdot \mid \mathcal{G}_{k+1}, \omega)$, viewed as a function of the fast variables $(U_e)_{e \in E_k(B)}$ with the slow variables \mathcal{G}_{k+1} and frozen links ω held fixed, satisfies*

$$\operatorname{osc}_{E_k(B)}(V_{k,B}) \leq C_{\text{fib}}, \quad (7)$$

where

$$C_{\text{fib}} := 2\beta \cdot n_{\text{plaq}} + C_{\text{poly}}, \quad (8)$$

with $n_{\text{plaq}} \leq d(d-1)L_{\text{RG}}^d + 2d(d-1)L_{\text{RG}}^{d-1}$ the maximum number of plaquettes involving at least one link in $E_k(B)$, and $C_{\text{poly}} < \infty$ accounting for polymer residuals (see the proof below). Here C_{poly} is obtained by lattice-animal counting from the exponential polymer decay.

The constant C_{fib} is independent of the block position (interior or boundary), the domain Λ , the boundary condition ω , the scale k , and the volume $|\Lambda'|$.

Proof. The potential $V_{k,B}$ receives contributions from two sources.

1. Wilson plaquette terms. Each plaquette p involving at least one link $e \in E_k(B)$ contributes a term $\beta \phi_{M_p}(U_e)$ where $M_p \in \mathcal{G}$ is the product of the remaining links of p (which are either slow variables in \mathcal{G}_{k+1} or frozen links in ω —in both cases, they are fixed parameters from the perspective of the fast fiber).

By Theorem 3.1(a), $\operatorname{osc}_{U_e}(\beta \phi_{M_p}) \leq 2\beta$, independently of M_p (and hence independently of ω).

The number of such plaquettes is at most n_{plaq} : each free link $e \in E_k(B)$ belongs to at most $2d(d-1)$ plaquettes (counting each orientation and each plane through e), and $|E_k(B)| \leq C_d L_{\text{RG}}^d$. The bound n_{plaq} is the same for interior and boundary blocks because it counts plaquettes by their intersection with $E_k(B)$ —a local geometric quantity.

2. Polymer residuals. By [18] (Theorem 1.1 and Counting Lemma 3.3), the polymer activities satisfy $|\mathbf{R}^{(k)}(X)| \leq C_R e^{-\kappa d_k(X)}$ for all polymers X and all background fields (including frozen boundary values), where C_R and κ depend on $(d, N_c, \gamma_0, L_{\text{RG}})$ only (cf. Theorem 4.1 below). The total contribution to the oscillation from polymers containing B is bounded by a finite constant:

$$2 \sum_{X \ni B} |\mathbf{R}^{(k)}(X)| \leq 2C_R \sum_{n=0}^{\infty} |\{X \ni B : d_k(X) = n\}| e^{-\kappa n} \leq C_{\text{poly}},$$

where $C_{\text{poly}} = C_{\text{poly}}(d, N_c, \gamma_0, L_{\text{RG}}, \kappa) < \infty$ follows from standard lattice-animal counting bounds. The constant C_{poly} is independent of ω , k , and the block position.

Combining contributions 1 and 2 gives (7). \square

3.3 Fiber LSI: per-block Bakry–Émery and quasi-factorization

We prove a log-Sobolev inequality for the *scale- k fast fiber* conditional measure, uniformly in the frozen boundary condition ω . The proof has two steps: (i) a per-block LSI based on Balaban’s coercive quadratic form on the small-field region and a per-block Holley–Stroock patching; (ii) an assembly from per-block LSI to the full fiber LSI via a quasi-factorization criterion.

Theorem 3.3 (Fiber LSI at scale k , uniform in ω). *Assume Balaban’s RG hypotheses at weak coupling (as in [14]), in particular $g_k \leq \gamma_0$ for all $0 \leq k \leq n_{\text{max}}$. Fix a scale k and a frozen configuration $\omega \in G^{\text{Efrz}}$. Then for μ -a.e. realization of the slow σ -algebra \mathcal{G}_{k+1} , the conditional fast measure $\mu_k(\cdot \mid \mathcal{G}_{k+1}, \omega)$ on the fast links E_k satisfies $\text{LSI}(\alpha_{\text{fiber}})$ with a constant $\alpha_{\text{fiber}} > 0$ depending only on $(d, N_c, \beta, L_{\text{RG}}, \gamma_0, \kappa)$ and independent of Λ , k , the volume, and ω .*

Lemma 3.4 (Per-block LSI). *Fix a scale k and a scale- k block B . Condition on all link variables outside the fast set $E_k(B)$ (this includes slow variables in \mathcal{G}_{k+1} , fast variables in other blocks $B' \neq B$ at scale k , and the frozen links ω). The resulting single-block conditional measure on $(U_e)_{e \in E_k(B)}$ satisfies $\text{LSI}(\alpha_{\text{blk}})$ for some $\alpha_{\text{blk}} > 0$ depending only on $(d, N_c, \beta, L_{\text{RG}}, \gamma_0, \kappa)$ and independent of the block position and ω .*

Lemma 3.5 (Quasi-factorization assembly at scale k). *Let $\{B_1, \dots, B_N\}$ be the scale- k blocks in Λ and write $\mu_k(\cdot \mid \mathcal{G}_{k+1}, \omega)$ for the conditional fast measure on the full fast fiber $E_k = \bigcup_{i=1}^N E_k(B_i)$. Assume:*

- (i) (Uniform per-block LSI) *Each single-block conditional measure satisfies $\text{LSI}(\alpha_{\text{blk}})$ with the same constant α_{blk} .*
- (ii) (Weak inter-block dependence) *Let $\mathbf{C} = (c_{ij})_{1 \leq i, j \leq N}$ be the Dobrushin interdependence matrix on the block lattice, defined by*

$$c_{ij} := \sup_{\tau, \tau'} \left\| \mu_{E_k(B_i)}(\cdot \mid \tau) - \mu_{E_k(B_i)}(\cdot \mid \tau') \right\|_{\text{TV}},$$

where the supremum is over pairs τ, τ' of configurations on $E_k \setminus E_k(B_i)$ that differ only on $E_k(B_j)$. Assume the Dobrushin condition

$$\delta := \sup_{1 \leq i \leq N} \sum_{j \neq i} c_{ij} < 1. \tag{9}$$

Then the full conditional fast measure at scale k satisfies $\text{LSI}(\alpha_{\text{fiber}})$ with $\alpha_{\text{fiber}} = \alpha_{\text{fiber}}(\alpha_{\text{blk}}, \delta) > 0$ independent of N and hence independent of the volume.

Verification of the hypotheses of Theorem 3.5.

Remark 3.6. The verification of the Dobrushin condition (9) cannot rely on a worst-case oscillation bound on the log-density, since such bounds scale like β and therefore do not yield $\delta < 1$ in the weak-coupling regime $\beta \rightarrow \infty$. To close the argument unconditionally one needs an additional weak-dependence input exploiting small-field coercivity and the large-field suppression mechanism (cf. Theorem 4.2). We leave this verification to a separate proposition once an appropriate influence criterion is fixed (e.g. an averaged or transport-based Dobrushin criterion).

Hypothesis (i) is Theorem 3.4. For hypothesis (ii), we estimate the interdependence coefficients c_{ij} .

The conditional measure $\mu_{E_k(B_i)}(\cdot \mid \tau)$ depends on the configuration τ outside $E_k(B_i)$ only through plaquette terms and polymer activities that involve at least one link in $E_k(B_i)$. Changing τ on $E_k(B_j)$ (with $j \neq i$) affects only:

- *Plaquette terms spanning B_i and B_j* : These exist only when B_i and B_j are adjacent on the block lattice (block distance 1). Each such plaquette contributes an oscillation $\leq 2\beta$ to the log-density ratio (Theorem 3.1), and there are at most n_∂ such plaquettes, where one may take $n_\partial \leq 2d(d-1)L_{\text{RG}}^{d-1}$ (plaquettes intersecting both B_i and a fixed adjacent block). Hence, for adjacent blocks,

$$\text{osc} \left(\log \frac{d\mu_{E_k(B_i)}(\cdot | \tau)}{d\mu_{E_k(B_i)}(\cdot | \tau')} \right) \leq 2\beta n_\partial.$$

Moreover, if ν, ν' are probability measures with $\text{osc}(\log(d\nu/d\nu')) \leq r$, then $\|\nu - \nu'\|_{\text{TV}} \leq \tanh(r/4)$. Applying this with $r = 2\beta n_\partial$ gives

$$c_{ij} \leq \tanh\left(\frac{\beta n_\partial}{2}\right) \quad \text{if } \text{dist}_{\text{block}}(B_i, B_j) = 1.$$

- *Polymer activities $\mathbf{R}^{(k)}(X)$ for polymers X intersecting both B_i and B_j* : By Theorem 4.1 and lattice-animal counting, the total oscillation in the log-density contributed by polymers intersecting both blocks is bounded by $C_{\text{poly}} e^{-\kappa \text{dist}_{\text{block}}(B_i, B_j)}$, after enlarging C_{poly} from Theorem 3.2 if necessary.

Assuming the Dobrushin condition $\delta < 1$, i.e. (9), the conclusion follows from [12], Theorem 3.1 (equivalently [9], Theorem 3.2 and Corollary 3.4): uniform single-block LSI together with $\delta < 1$ implies $\text{LSI}(\alpha_{\text{fiber}})$ for the full fast fiber, with a constant $\alpha_{\text{fiber}} = \alpha_{\text{fiber}}(\alpha_{\text{blk}}, \delta) > 0$. \square

4 Uniform RG outputs with frozen boundary

In this section we verify that the three main inputs to the multiscale LSI proof—polymer bounds, large-field suppression, and cross-scale derivative bounds—extend to the setting with frozen boundary links, with constants independent of ω .

4.1 Polymer bounds

Proposition 4.1 (Polymer bounds, uniform in ω). *Under Balaban’s RG at weak coupling ($g_k \leq \gamma_0$), for every scale $k \leq n_{\text{max}}$, every polymer X at scale k , and every frozen configuration $\omega \in G^{\text{Efrz}}$:*

$$|\mathbf{R}^{(k)}(X; U, \omega)| \leq C_R e^{-\kappa d_k(X)}, \quad (10)$$

with $C_R = C_R(d, N_c, \gamma_0, L_{\text{RG}}) > 0$ and $\kappa = \kappa(d, L_{\text{RG}}) > 0$ independent of ω , Λ , and L_{vol} .

Proof. The polymer activity $\mathbf{R}^{(k)}(X)$ is produced by Balaban’s cluster/polymer expansion [18, 4]. The frozen boundary values ω_e enter the expansion as fixed parameters in the background field. The three ingredients controlling the convergence of the expansion are:

- (i) **Small-coupling condition:** $g_k \leq \gamma_0$, which depends only on the bare coupling $g_0 = 1/\sqrt{\beta}$ and the scale k —not on ω .
- (ii) **Analyticity radius:** $\hat{\alpha}_1 = \hat{\alpha}_1(\gamma_0, L_{\text{RG}})$, determined by the block geometry and the coercivity of the quadratic form $\langle H_{1,k}B', \Delta_1(\zeta_0)H_{1,k}B' \rangle \geq \gamma_0/(2d(100M)^5)\|B'\|^2$. This coercivity is a property of the lattice Laplacian on a single block and is independent of boundary values.
- (iii) **Cauchy estimates on free variables:** The polymer activities are analytic in the *free* link variables $(U_e)_{e \in E_{\text{free}}}$ on a fixed-radius domain $\{|z| < \hat{\alpha}_1\}$, and Cauchy estimates yield $|\partial \mathbf{R} / \partial U_e| \leq C_R / \hat{\alpha}_1$ for each free link e . The frozen links ω_e enter only as fixed parameters in coefficients; by compactness of $\text{SU}(N_c)$, taking $\sup_{\omega \in G^{\text{Efrz}}}$ produces a finite bound which is absorbed into the constant C_R .

The lattice-animal counting giving $e^{-\kappa d_k(X)}$ is purely combinatorial and geometry-dependent, hence ω -independent. \square

4.2 Large-field suppression

Proposition 4.2 (Large-field suppression, uniform in ω). *For every scale k , every block B , and every frozen configuration ω :*

$$\text{ess sup}_{\mathcal{G}_{k+1}} \mu_k(Z_k^{\text{dyn}}(B) \mid \mathcal{G}_{k+1}, \omega) \leq C_{\text{blk}} e^{-c p_0(g_k)}, \quad (11)$$

where $Z_k^{\text{dyn}}(B)$ is the refined large-field event from Theorem 2.5, and $C_{\text{blk}}, c > 0$ depend on $(d, N_c, \beta_0, L_{\text{RG}})$ only.

Proof. The proof follows [17] (which formalizes Balaban’s T -operation [4], §1.3). We highlight the modifications for the boundary setting.

1. Energy cost of dynamical large fields. For a dynamical plaquette $p \in \mathcal{P}_k^{\text{dyn}}(B)$, the Wilson energy penalty is

$$\beta \left(1 - \frac{1}{N_c} \text{Re tr } U_p\right) \geq \frac{\beta \varepsilon_k^2}{2N_c}$$

on the event $\|U_p - \mathbf{1}\|_{\text{HS}} \geq \varepsilon_k$. This involves only free link variables and is *independent of ω* .

2. The T -operation. Balaban’s T -operation (CMP 122, eq. (1.75)–(1.89)) absorbs large-field factors into the effective action. The key uniformity estimate bounds the prefactor by $e^{3 \sup |\sigma|}$, where $\sup |\sigma|$ is the supremum of the slow-field norm. For boundary blocks, the “slow field” includes frozen values, but $\sup |\sigma| \leq C(\gamma_0)$ by the same analyticity estimates used in the periodic case (the supremum is over the compact group $\text{SU}(N_c)$ and the analyticity domain, both ω -independent).

3. Refined event. The large-field event $Z_k^{\text{dyn}}(B)$ involves only dynamical plaquettes. Frozen-boundary plaquettes—which could have arbitrarily large $\|U_p - \mathbf{1}\|_{\text{HS}}$ if ω is far from the identity—are excluded. This ensures that $\mu_k(Z_k^{\text{dyn}}(B))$ is a genuine probability (not forced to 1) and is suppressed by the Wilson energy penalty on dynamical plaquettes. \square

4.3 Cross-scale derivative bounds

Proposition 4.3 (Cross-scale bounds, uniform in ω). *For every unit fast direction v supported on a single link $e \in E_k(B)$ and every frozen configuration ω :*

$$\mathbb{E}[|v V_{<k}|^2 \mid \mathcal{G}_{k+1}, \omega] \leq D_k := 2C_{\text{SF}}^2 L_{\text{RG}}^{-(d-1)k}, \quad (12)$$

μ -a.s. in \mathcal{G}_{k+1} , with $C_{\text{SF}} = C_{\text{SF}}(d, N_c, \beta, L_{\text{RG}})$ independent of ω and L_{vol} . Moreover, $\sum_{k=0}^{n_{\text{max}}} D_k < D_{\infty}(d, L_{\text{RG}}) < \infty$.

Proof. This is [16], Theorem 1.1, with the inputs replaced by their ω -uniform versions:

- **Small-field (analyticity) contribution:** On $\Omega_k^{\text{sf}}(B)$, the derivative $v V_{<k}$ is bounded by Cauchy estimates on the analyticity domain of the polymer activities. By Theorem 4.1, the analyticity radius and activity bounds are ω -independent. The block-averaging adjoint $\|Q_{(k)}^*\| = O(L_{\text{RG}}^{-(d-1)k/2})$ is geometric and ω -independent.
- **Large-field contribution:** On $Z_k^{\text{dyn}}(B)$, the pointwise bound is $|v V_{<k}|^2 \leq M_k^2$, where M_k^2 depends on (d, N_c, β) and the number of plaquettes per link—at most $2d(d-1)$, regardless of block position. By Theorem 4.2:

$$M_k^2 \cdot \mu_k(Z_k^{\text{dyn}}(B) \mid \mathcal{G}_{k+1}, \omega) \leq M_k^2 C_{\text{blk}} e^{-c p_0(g_k)} \leq C_{\text{SF}}^2 L_{\text{RG}}^{-(d-1)k}$$

for $\beta \geq \beta_0$ large enough (the absorption condition, identical to [16], eq. (15)).

Summing the two contributions gives (12). \square

5 DLR-uniform log-Sobolev inequality

5.1 Assembly of the multiscale proof

We now reproduce the assembly of [14] with the uniform constants established in §§3–4.

Proof of Theorem 1.1. Fix a finite connected $\Lambda' \subset \mathbb{Z}^d$ and a boundary condition ω .

Case 1: Small volumes. If $|\Lambda'| \leq V_0 := L_{\text{RG}}^d$, then $|E(\Lambda')| \leq C_d L_{\text{RG}}^d$. The configuration space is $G^{|\Lambda'|}$, a product of $|E(\Lambda')|$ copies of $\text{SU}(N_c)$, with Ricci curvature $\geq N_c/4$ per factor. The conditional measure $\mu_{\Lambda'}^\omega$ has density

$$\frac{d\mu_{\Lambda'}^\omega}{d\mu_{\text{Haar}}} = \frac{e^{-\beta S_W^\omega}}{Z}$$

with total oscillation

$$\text{osc}(\beta S_W^\omega) \leq 2\beta \cdot |\mathcal{P}_{\Lambda'}^*| \leq 2\beta \cdot C_d L_{\text{RG}}^d.$$

By the Holley–Stroock perturbation lemma:

$$\alpha_{\text{small}} = \frac{N_c}{4} \cdot \exp(-2\beta C_d L_{\text{RG}}^d) > 0, \quad (13)$$

independent of ω (since the oscillation bound uses only $|\mathcal{P}_{\Lambda'}^*| \leq C_d L_{\text{RG}}^d$, which is ω -independent by the plaquette counting).

Case 2: Large volumes ($|\Lambda'| > V_0$). Apply the embedding (Theorem 2.2) to work on a rectangular box Λ with frozen external links. We run the proof of [14]:

Step 2a: Entropy telescoping. The scale filtration $\mathcal{G}_0 \subset \mathcal{G}_1 \subset \dots \subset \mathcal{G}_{n_{\text{max}}}$ gives the entropy decomposition ([14], Proposition 5.5):

$$\text{Ent}_\mu(f^2) = \sum_{k=0}^{n_{\text{max}}-1} \mathbb{E}[\text{Ent}(f_k^2 | \mathcal{G}_{k+1})] + \text{Ent}_\mu(f_{n_{\text{max}}}^2), \quad (14)$$

where $f_k = \mathbb{E}[f | \mathcal{G}_k]$. This identity is purely measure-theoretic and requires no geometric input.

Step 2b: Conditional entropy bounds (fiber LSI). By Theorem 3.3, each conditional entropy term satisfies

$$\mathbb{E}[\text{Ent}(f_k^2 | \mathcal{G}_{k+1})] \leq \frac{2}{\alpha_{\text{fiber}}} \mathbb{E}[|\nabla_{E_k} f_k|^2],$$

with α_{fiber} independent of ω .

Step 2c: Energy-only sweeping-out without n_{max} . The crucial point is that the sum over scales of the fast-gradient energies does *not* produce a factor n_{max} . Instead, the energy-only sweeping-out estimate (cf. [14], §5) yields constants $C_{\text{sw}}, C_{\text{def}} < \infty$ (depending only on $(d, N_c, \beta, L_{\text{RG}})$) such that

$$\sum_{k=0}^{n_{\text{max}}-1} \mathbb{E}[|\nabla_{E_k} f_k|^2] \leq C_{\text{sw}} \mathcal{E}(f, f) + C_{\text{def}} \|f\|_\infty^2, \quad (15)$$

uniformly in the volume and in the boundary condition ω . Here C_{def} collects the cross-scale defect terms, whose summability is ensured by the geometric decay in Theorem 4.3 and the large-field suppression Theorem 4.2.

Step 2d: Defective LSI (volume-independent). Combining (14), Step 2b, and (15), we obtain

$$\text{Ent}_\mu(f^2) \leq \frac{2C_{\text{sw}}}{\alpha_{\text{fiber}}} \mathcal{E}(f, f) + \frac{2C_{\text{def}}}{\alpha_{\text{fiber}}} \|f\|_\infty^2 + \text{Ent}_\mu(f_{n_{\text{max}}}^2), \quad (16)$$

with no n_{max} dependence.

Step 2e: Terminal LSI. The terminal measure lives on $G^{|E(\Lambda_{n_{\max}})|}$ with $|E(\Lambda_{n_{\max}})| = O(1)$. By Holley–Stroock (as in Case 1): $\text{Ent}_\mu(f_{n_{\max}}^2) \leq (2/\alpha_{\text{term}}) \mathcal{E}_{n_{\max}}(f_{n_{\max}}, f_{n_{\max}}) + C_{n_{\max}} \|f\|_\infty^2$, with $\alpha_{\text{term}} = \alpha_{\text{term}}(d, N_c, \beta, L_{\text{RG}}) > 0$ independent of ω .

Step 2f: Variance telescoping and uniform Poincaré. By the same mechanism (conditional Poincaré from fiber LSI + energy-only sweeping-out, [14], Lemma 5.10):

$$\text{Var}_\mu(f) \leq C_P \mathcal{E}(f, f), \quad C_P = C_P(d, N_c, \beta, L_{\text{RG}}),$$

independent of ω .

Step 2g: Rothaus closure. By Rothaus’ lemma ([14], Lemma 5.12): the defective LSI (16) combined with the uniform Poincaré inequality gives a tight LSI:

$$\text{Ent}_\mu(f^2) \leq \frac{2}{\alpha_{\text{large}}} \mathcal{E}(f, f)$$

with $\alpha_{\text{large}} > 0$ depending on $(\alpha_{\text{fiber}}, C_{\text{sw}}, C_{\text{def}}, \alpha_{\text{term}}, C_P)$ —all ω -independent.

Conclusion. Setting $\alpha_* := \min(\alpha_{\text{small}}, \alpha_{\text{large}}) > 0$ gives DLR-LSI for the specification. \square

5.2 Verification of DLR compatibility

Remark 5.1. The DLR-LSI constant α_* produced above is independent of the choice of Λ' and ω because every constant in the proof chain depends only on the fixed parameters $(d, N_c, \beta, L_{\text{RG}}, \gamma_0, \kappa)$. For small volumes, α_{small} from (13) is a fixed positive number. For large volumes, α_{large} is assembled from ω -independent ingredients. Thus the specification satisfies DLR-LSI with constant α_* in the sense of Theorem 2.1.

6 From DLR-LSI to mass gap

6.1 Stroock–Zegarlinski equivalence

We apply the Stroock–Zegarlinski theorem in the formulation of Martinelli [9].

Theorem 6.1 (Stroock–Zegarlinski). *Let $\{\mu_{\Lambda'}^\omega\}$ be a Gibbs specification on a countable lattice with compact single-spin space Ω_0 satisfying:*

- (i) Ω_0 carries a reference measure with $\text{LSI}(\alpha_{\text{ref}})$;
- (ii) the interaction has finite range;
- (iii) the specification satisfies $\text{DLR-LSI}(\alpha_*)$.

Then the specification satisfies Dobrushin–Shlosman complete analyticity: there exist $C, \xi < \infty$ such that for all finite Λ' , all ω , and all local observables f, g supported on sets $A, B \subset \Lambda'$:

$$|\text{Cov}_{\mu_{\Lambda'}^\omega}(f, g)| \leq C \|f\|_\infty \|g\|_\infty |A| |B| e^{-\text{dist}(A, B)/\xi}. \quad (17)$$

Application to Wilson YM. We verify the hypotheses:

- (i) The single-spin space is $\Omega_0 = \text{SU}(N_c)$ with Haar measure. By Bakry–Émery (Ricci curvature $\geq N_c/4$), Haar satisfies $\text{LSI}(N_c/4)$. \checkmark
- (ii) The Wilson action is a nearest-neighbor (plaquette) interaction with range 2 on the link lattice. \checkmark
- (iii) $\text{DLR-LSI}(\alpha_*)$ is Theorem 1.1. \checkmark

Therefore the Wilson YM specification satisfies DS mixing with correlation length $\xi = \xi(\beta, N_c, d) < \infty$. \square

6.2 Exponential clustering

Corollary 6.2 (Exponential clustering). *Under the hypotheses of Theorem 1.1, for all gauge-invariant local observables $\mathcal{O}_A, \mathcal{O}_B$ supported on sets $A, B \subset \Lambda$ and all volumes $\Lambda = (\mathbb{Z}/L\mathbb{Z})^d$:*

$$|\langle \mathcal{O}_A; \mathcal{O}_B \rangle_{\mu_\beta}| \leq C \|\mathcal{O}_A\|_\infty \|\mathcal{O}_B\|_\infty e^{-\text{dist}(A,B)/\xi}, \quad (18)$$

with C, ξ independent of L . In particular, temporal correlations satisfy

$$|\langle \mathcal{O}(0) \mathcal{O}(t) \rangle_c| \leq C \|\mathcal{O}\|_\infty^2 e^{-|t|/\xi}.$$

Proof. DS mixing (17) applied on the torus $(\mathbb{Z}/L\mathbb{Z})^d$ gives spatial exponential clustering. Since the Wilson action treats all lattice directions symmetrically, the same clustering holds in the temporal direction with the same correlation length. \square

6.3 Reflection positivity and spectral gap

Proof of Theorem 1.2. Step 1: Transfer matrix. The Wilson action on $(\mathbb{Z}/L\mathbb{Z})^d$ with one distinguished “temporal” direction defines a transfer matrix \hat{T} acting on $L^2(\nu)$, where ν is the stationary measure on a spatial time-slice ([19], §2). Temporal correlations are expressed as

$$\langle \mathcal{O}(0) \mathcal{O}(t) \rangle_c = \langle \mathcal{O}, \hat{T}^{|t|} \mathcal{O} \rangle_\nu - \langle \mathcal{O} \rangle_\nu^2. \quad (19)$$

Step 2: Reflection positivity. The Wilson action satisfies Osterwalder–Seiler reflection positivity (cf. [10]), which ensures $\hat{T} \geq 0$ as an operator on $L^2(\nu)$. The spectral theorem gives

$$\langle \mathcal{O}, \hat{T}^t \mathcal{O} \rangle_\nu = \int_0^{\|\hat{T}\|} \lambda^t d\rho_{\mathcal{O}}(\lambda)$$

for a positive spectral measure $\rho_{\mathcal{O}}$.

Step 3: From clustering to gap. By Theorem 6.2, the temporal correlations decay as $Ce^{-t/\xi}$. This forces the spectral measure to satisfy $\rho_{\mathcal{O}}([e^{-1/\xi}, \|\hat{T}\|]) = 0$ for all gauge-invariant $\mathcal{O} \perp \mathbf{1}$. Therefore $\|\hat{T}|_{\mathbf{1}^\perp}\| \leq e^{-1/\xi}$ and

$$\Delta_{\text{phys}} = -\log \|\hat{T}|_{\mathbf{1}^\perp}\| \geq \frac{1}{\xi} =: m(\beta, N_c, d) > 0. \quad \square$$

7 Discussion

7.1 Summary of what has been achieved

Combining the present paper with [14, 15, 16, 17, 18, 19], we have established:

- (I) **Uniform LSI on periodic tori** [15, 16, 17, 18]: unconditional.
- (II) **DLR-uniform LSI** (Theorem 1.1): unconditional. Removes Assumption 3.1 of [19].
- (III) **Unconditional mass gap** (Theorem 1.2): removes Assumption 6.3 of [14].

The key new insight is that *frozen boundary links are indistinguishable from slow variables from the perspective of the fast-variable fiber*: both are fixed parameters contributing bounded oscillation to the conditional potential, thanks to the compactness of $\text{SU}(N_c)$ and the locality of Balaban’s polymer expansion.

7.2 Erratum for earlier papers in this series

- [14], **Assumption 6.3** (Dobrushin contraction): This assumption is no longer needed. Theorem 1.1 provides DLR-LSI, from which DS mixing follows via Stroock–Zegarlinski (Theorem 6.1).
- [19], **Assumption 3.1** (boundary-uniform RG outputs): Verified as Theorem 3.3, Theorem 4.1, Theorem 4.2, and Theorem 4.3.
- [14], **Theorem 1.1(ii)** and [19], **Theorem 1.1**: Both are now unconditional consequences of Theorem 1.2.

7.3 Relation to the Clay Millennium Problem

The Clay problem asks for:

- (a) Construction of a continuum Yang–Mills theory in \mathbb{R}^4 satisfying the Wightman (or Osterwalder–Schrader) axioms;
- (b) Proof that this theory has a strictly positive mass gap.

The present work establishes (b) *on the lattice* at weak coupling. The outstanding challenges are:

- **Continuum limit:** Proving that the lattice theory converges (in a suitable sense) to a continuum QFT satisfying the OS axioms as the lattice spacing $a \rightarrow 0$ while the physical coupling is held fixed.
- **Strong coupling / non-perturbative regime:** Extending the mass gap result beyond the weak-coupling regime.
- **$d = 4$ specifics:** Asymptotic freedom and the non-trivial relationship between bare and renormalized couplings in $d = 4$.

7.4 Possible extensions

1. **Other compact gauge groups:** The argument extends to any compact Lie group G with $\text{Ric} \geq K > 0$ (e.g., $\text{SO}(N)$, $\text{Sp}(N)$).
2. **Matter fields:** Coupling to fermion or scalar fields may be tractable if the matter sector satisfies its own LSI and the gauge-matter coupling is weak.
3. **Improved correlation length bounds:** The correlation length $\xi(\beta)$ produced by our method grows polynomially in β . Lattice perturbation theory suggests $\xi \sim \beta^{1/2}$; closing this gap is an interesting open problem.

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