Archive Erratum and Full Retraction (v3)

Interface Lemmas for the Multiscale Proof of the
Lattice Yang–Mills Mass Gap

Lluis Eriksson • ai.viXra:2602.0052 • July 2026

This version does two things. **First it restores the identity of the record** (§1), which since 2026-07-06 has displayed one paper's metadata while serving a second paper's PDF, leaving this paper's title absent from the public index. **Second, having restored it, it retracts the paper** (§2–§5). Not only its terminal claim: an audit of every numbered statement found that *none of the three interface lemmas is established as printed*, that one auxiliary *construction claim inside a proof* is outright false, and that a second auxiliary proof is invalid over its claimed parameter range. The manuscript follows unmodified so that the claims and their refutations can be read against one another.

Throughout, a distinction is kept that this note's own earlier drafts twice failed to keep: *refuting a printed proof is not refuting the statement it was meant to prove.* Where only the proof fails, that is what is said.

1 Archive erratum: what this record has been serving

Field	**Actual provenance**
Record identifier	`2602.0052`
Title, abstract, category shown	`2602.0036v2`, *Geodesic Convexity...*
PDF served as v2	`2602.0035v2`, *Morse–Bott Spectral Reduction...*
Legitimate content of the record	`2602.0052v1`, *Interface Lemmas...*

Filiation was established from content, not filenames: the file served as v2 carries the cover, subject, theorem numbering (3.1, 4.1) and explicit self-identification ("*version 2 records that...*") of the Morse–Bott manuscript, and is not byte-identical to `2602.0036v2`, itself a genuine separate revision of its own record. Only the metadata came from `2602.0036`; the file came from `2602.0035`. A replacement creates a new version rather than overwriting the old, so *Interface Lemmas*, dated February 2026 and submitted here on 2026-02-12, is the authentic content of this record, not an orphan.

Between 2026-07-06 and the present correction, therefore, the title of this paper did not resolve in the public index.

2 The paper is the last document of its lane still claiming an unconditional gap

The campaign that overwrote this record in July 2026 was, at that moment, retracting the "unconditional" thesis across this lane. Because the write landed on the wrong record, this paper never received its correction.

`2602.0053` v2 is retitled from "... and *Unconditional* Mass Gap..." to "a *Conditional and Windowed* Reduction". `2602.0054` v2 states that "*the inter-block assembly requires the block Dobrushin condition — this is where* (H-DOB-blk) *enters*" and withdraws by name the claim that the route "*bypasses any explicit Dobrushin contraction estimate*". `2602.0055` v2 lists (H-P0), (H-YGZ), (H-DOB-blk) and a volume window, records that "*None of these is proved in the series*", and declares "*the retagging of the chain is complete*". It was not: this record was outside it.

Accordingly **Corollary 7.3** ("Unconditional mass gap", $\Delta_{\text{phys}} \geq m(\beta, N_c, d) > 0$ uniformly in L), **Remark 6.4** ("Dobrushin not needed"), and the closing sentence of the abstract are **withdrawn**. The dating is independently checkable: the bibliography cites [13], [15] and [17] under their pre-July titles, all three since retitled.

3 Two invalid proofs, one of them resting on a false construction

3.1 Lemma A.1: the printed logarithm leaves the Lie algebra

Lemma A.1 constructs $\log U := V \operatorname{diag}(i\theta_1, \ldots, i\theta_{N_c})V^{-1}$ with $\theta_j \in (-\pi, \pi]$, and Remark A.2 emphasises that this "defines $\log U$ for every $U \in SU(N_c)$" with "no exceptional set discarded". Take $U = -I \in SU(2)$, whose eigenvalue arguments are both π:

$$\log(-I) = i\pi I, \qquad \operatorname{tr}\log(-I) = 2\pi i \neq 0, \qquad \text{so} \quad \log(-I) \notin \mathfrak{su}(2).$$

The output leaves the Lie algebra. This is not repaired by shifting one angle by 2π. The block map $Q_k = \exp \circ \operatorname{avg} \circ \log \circ \operatorname{mult}$ needs a logarithm equivariant under conjugation, because Lemma 2.2 asserts $Q_k(g \cdot U) = g_{k+1} \cdot Q_k(U)$. But $-I$ is central: $g(-I)g^{-1} = -I$ for all g, so equivariance would force $\log(-I)$ to commute with all of $SU(2)$, hence to be a traceless multiple of the identity, hence 0 — and $\exp 0 = I \neq -I$.

No global, $\mathfrak{su}(N)$-valued, conjugation-equivariant section of $\exp$ **exists at non-trivial central elements.**

What this does and does not refute. It refutes the construction: the printed map does not take values in $\mathfrak{su}(N)$, and no repair of it can be simultaneously $\mathfrak{su}(N)$-valued, a section of exp, and conjugation-equivariant. It therefore refutes the printed justification that Q_k is a globally defined, $SU(N)$-valued, gauge-covariant map, and with it the printed proof of Lemma 2.2. It does *not* refute the statement of Lemma A.1, which asserts only that $Q_k : \mathcal{A}_k \to \mathcal{A}_{k+1}$ is Borel measurable. A Borel $\mathfrak{su}(N)$-valued logarithm can be selected after branch corrections — dropping equivariance, which measurability does not require — so the measurability claim is plausibly recoverable. **Lemma A.1 is therefore not established; its construction is false.**

Upgrading this to a refutation of the statement would require exhibiting a fine configuration on which the printed formula leaves $SU(N)$. The shape is visible: if a block average runs over $M > 1$ terms and one holonomy equals $-I$ while the others equal I, the principal selection averages to $(i\pi/M)I$, whose exponential has determinant $e^{2\pi i/M} \neq 1$. Whether that holonomy pattern is realisable for the specific paths $\Gamma_{c,x}$ of the block map is a question about its geometry, and is *not* settled here; the claim is not made.

Since Lemma 3.4 conditions on iterates of Q_k, the failure of the global covariance justification reaches the Horizon Transfer chain as well.

3.2 Lemma 6.2: the Holley–Stroock bound is applied in the wrong direction

Lemma 6.2 asserts $\alpha_{\text{blk}} \geq \frac{N_c}{4} e^{-C_{\text{fib}}(\beta_0)}$ with $C_{\text{fib}}(\beta_0) = 2\beta_0 n_{\text{plaq}} + C_{\text{poly}}$, proved from $\operatorname{osc}(W_{k,B}) \leq 2\beta\, n_{\text{plaq}} + C_{\text{poly}}$ by "*bounding the oscillation using $\beta = \beta_0$*". Substituting the smallest β into an upper bound that increases with β does not preserve it. **The printed proof is therefore invalid over the stated range $\beta \geq \beta_0$.**

This does not refute the statement. Holley–Stroock supplies one sufficient lower bound on the log-Sobolev constant; that this particular bound degenerates as β grows says nothing about the optimal constant, which some other argument might still bound uniformly. Nor do three sampled values — $\operatorname{osc}(\beta = 1, 10, 100) = 3.0,\ 30.3,\ 303.1$, measured in `2602.0054` v2 — prove that $\inf_{\beta \geq \beta_0} e^{-\operatorname{osc}(\beta)} = 0$; and the paper supplies only an *upper* bound on osc, which cannot establish that osc grows at all. The correct status is **not established over the unbounded range; the same route does give a uniform bound on a bounded window** $\beta_0 \le \beta \le \beta_1$, with the constant depending on β_1.

Windowing does not repair Lemma 6.3, which additionally needs $\delta_k < \alpha_{\rm blk}/4$ uniformly before applying Yoshida–Giannulis–Zegarlinski — and that inter-block smallness is precisely the content of (H-DOB-blk)/(H-YGZ), unproved in the series. An earlier draft of this note said "Lemma 6.3 stands only in the windowed form"; that was wrong, and is withdrawn.

4 The three interface lemmas are not established as printed

4.1 Lemma C (fiber LSI with boundary)

Lemma 6.3 loses its per-block input (§3.2) and carries the inter-block hypothesis. **Conditional**, not established.

4.2 Lemma A (Horizon Transfer, Lemma 3.11)

Two bridges are missing.

(i) The RCP/RG identification is asserted, not derived. Lemma 3.4 states that the regular conditional probability on a fibre of $Q^{(k)}$ has density proportional to $e^{-S_k^{\rm eff}}$ against an "induced Haar measure on the fibre", and invokes uniqueness of the RCP. A fibre of a nonlinear $Q^{(k)}$ is not a group and carries no canonical Haar measure; a disintegration over it may involve Jacobians, gauge-fixing coordinates and normalisations; and conditioning on $\pi \circ Q^{(k)}$ fixes a gauge *orbit*, not a representative. Uniqueness identifies two kernels only after both are shown to disintegrate the same measure, which is what would have to be proved.

(ii) The horizon component is random but used as if fixed. Y_B (Definition 3.2) is the connected large-field component containing B, and depends on the very fast variables being integrated; the paper does not establish its measurability. Yet Lemma 3.9 writes $\mu_k(L_k(B) \mid \mathcal{G}_{k+1}) = T_k'(Y_B, (\bar{U}, 0))F_B$, whose left-hand side is $\mathcal{G}_{k+1}$-measurable and whose right-hand side is not. The expected form is a sum over branches, $\mu_k(L_k(B) \mid \mathcal{G}_{k+1}) = \sum_{Y \ni B} T_k'(Y, (\bar{U}, 0))F_{B,Y}$, requiring measurability of each branch, disjointness, conditional normalisation, and summability over all $Y \ni B$. The same confusion appears in Proposition 3.10, whose case "$Y_B = \emptyset$" concludes that the conditional probability is 0 — but whether Y_B is empty depends on the fast variables, so conditioning on the slow field cannot make it deterministically zero.

Moreover the per-component bound $T_k(Y)\mathbf{1} \le e^{-cp_0(g_k)}$ does not control a sum over all animals $Y \ni B$ without a size- or distance-dependent estimate and an animal-counting argument. Finally, the statement of Lemma 3.11 declares c to depend only on $(d, N_c, L_{\rm RG}, \gamma_0)$, while its proof takes $c = 2/(1+\beta_0)$ from input (viii); β_0 is missing from the dependency list.

Not established.

4.3 Lemma B (uniform analyticity, Lemma 4.1)

The printed proof argues that "$R^{(k)}$ and $B^{(k)}$ are summands of S_k with disjoint supports; each inherits the analyticity domain". Analyticity of a sum does not give analyticity of the summands; the sentence that follows, citing a direct construction of $B^{(k)}$, is the argument that would have to be made, and is not carried out here. **Pending literal verification against the cited source.**

Its corollaries are worse. Input (v) reads $|B^{(k)}(X)| \le O(1)\sum_{j\le k}|\Gamma_j^0 \cap X|$ — with *no* spatial decay factor, unlike input (iv) for $R^{(k)}$. Yet Corollary 4.2 concludes $|\nabla_v B^{(k)}(X)| \le (C_B k|X|/\hat{\alpha}_1)\, e^{-\kappa d_k(X)}$. A Cauchy estimate converts a sup bound into a derivative bound; it cannot manufacture decay absent from the input. Corollary 4.3 then sums over polymers using exactly that decay. **Corollaries 4.2–4.3 are not established.**

5 A further invalid inference: Lemma 5.6

The proof of Lemma 5.6 argues that since $p_0(g) \geq c_0|\log g|^{1+\epsilon_0}$ and $g_k \to 0$, "$p_0(g_k) \to \infty$ faster than any multiple of k", hence $(1+k)^2 e^{-cp_0(g_k)}$ "decays faster than any geometric sequence". **The displayed assumptions do not imply this.** The input on p_0 is a *lower* bound, so nothing in it prevents p_0 from being much larger — but nothing in it forces p_0 to be larger either, and the conclusion needs the latter.

A model compatible with every printed inequality makes this concrete. Take

$$\beta_k = \beta_0 + Ck, \qquad g_k = (\beta_0 + Ck)^{-1/2}, \qquad p_0(g) = c_0|\log g|^{1+\epsilon_0}.$$

This satisfies the running-coupling bound $\beta_k \leq \beta_0 + Ck$, satisfies $g_k \to 0$, and satisfies $p_0(g) \geq c_0|\log g|^{1+\epsilon_0}$ with equality. In it

$$p_0(g_k) = O\big((\log k)^{1+\epsilon_0}\big),$$

so $e^{-cp_0(g_k)}$ is subexponential — superpolynomially small, but not geometric in k — and the required bound $\leq C_{\mathrm{SF}}^2 L_{\mathrm{RG}}^{-(d-1)k}$ fails. The geometric estimate therefore does not follow from the printed inputs; an additional hypothesis, such as $p_0(g_k) \geq ck$ or the lane's explicit (H-P0), is required. Lemma 5.7 loses its geometric input accordingly.

6 Status

Archive filiation erratum (§1)	Stands
Corollary 7.3, Remark 6.4, closing sentence	**Withdrawn**
Principal-angle log : $SU(N) \to \mathfrak{su}(N)$	**FALSE** as a construction (§3.1)
Definition 2.1 / global block map Q_k	**Not established** as an $SU(N)$-valued gauge-covariant map with the printed logarithm (§3.1)
Lemma A.1 (Borel measurability of Q_k)	Proof invalid, statement **not established**; plausibly recoverable without equivariance
Lemma 2.2 (gauge covariance of Q_k)	Not established globally (§3.1)
Lemma 6.2 (per-block LSI)	Printed proof **invalid**; not established over $\beta \geq \beta_0$; this route does give a uniform bound on a bounded window (§3.2)
Lemma 3.4, 3.9, Prop. 3.10, Lemma 3.11	**Not established** (§4.2)
Lemma 4.1	Pending verification against the source
Corollaries 4.2–4.3	**Not established** (§4.3)
Lemma 5.6	Invalid inference; needs (H-P0)
Lemma 5.7, Lemma 6.3, Theorem 7.1	Conditional / not established

No numbered statement of this paper is claimed here to be false. What is claimed false is one construction inside a proof: the principal-angle map is not an $\mathfrak{su}(N)$-valued logarithm, and no equivariant repair of it exists. Everything else above is a failure of a printed argument. The three interface lemmas may yet hold; so may Lemma 6.2 over the unbounded range; and Borel measurability of a suitably reconstructed block map is not claimed to be false. What this note records is that the printed arguments do not establish them, and that the closure built on top of them does not stand.

Provenance. The mis-filing (§1) was measured on 2026-07-29 by comparing every record's archive metadata against the cover page of its served PDF, the filiation then being fixed by hashes and cover pages after an external reviewer correctly objected that the public evidence alone did not distinguish a mis-filed PDF from a duplicated one. §2 was found by asking, before restoring the record, whether a February claim could re-enter an archive whose July corrections had retracted it. **Sections 3–5 came from a second external audit**, which found that the survivors list of this note's own first draft — Lemmas 3.11, 4.1 and A.1 — did not survive; the same reviewer had already caught an identical over-generous survivors list in the companion retraction ai.viXra:2602.0084 v2. Every point was verified against the unchanged v1 proofs before adoption. That two successive drafts of two different retractions closed their survivors lists too early is recorded here rather than quietly fixed: it is the same failure the retractions are about.

Interface Lemmas for the Multiscale Proof of the Lattice Yang–Mills Mass Gap

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Abstract

We establish three interface lemmas that close the remaining gaps in the proof chain for the mass gap of $\mathrm{SU}(N_c)$ lattice Yang–Mills theory at weak coupling ($\beta \geq \beta_0$) in dimension $d \geq 3$.

Lemma A (Horizon Transfer) establishes a uniform conditional large-field suppression bound $\mu_k(Z_k(B) \mid \mathcal{G}_{k+1}) \leq \exp(-c\, p_0(g_k))$ holding μ_β-a.s., without any admissibility restriction on the background field. The argument identifies the regular conditional probability with Balaban's RG kernel, expresses the large-field activation probability as a ratio controlled by Balaban's localized T-operation, and applies the T-operation small-factor bound.

Lemma B extracts from Balaban's inductive scheme that the boundary terms $\mathbf{B}^{(k)}(X)$ share the same uniform analyticity domain as the polymer activities $\mathbf{R}^{(k)}(X)$, with radius $\hat{\alpha}_1(\gamma) > 0$ independent of k.

Lemma C extends the multiscale LSI to finite volumes with arbitrary frozen boundary conditions ω via tensorization-plus-perturbation, replacing the unverified Dobrushin block condition of [15].

Combined with [9]–[15], these lemmas render the lattice mass gap theorem unconditional.

Contents

1 Introduction

1.1 Purpose and scope

This paper closes three interface gaps between the multiscale log-Sobolev inequality (LSI) framework of [9]–[15] and Balaban's constructive renormalization group [1, 2, 3, 4, 5, 6].

Gap A: The conditional large-field suppression bound $\operatorname{ess\,sup}_{\mathcal{G}_{k+1}} \mu_k(Z_k(B) \mid \mathcal{G}_{k+1}) \le e^{-cp_0(g_k)}$ was not established for all backgrounds. Prior versions attempted to split into "admissible" and "non-admissible" backgrounds; this fails because the essential supremum sees positive-measure sets regardless of their size. We resolve this via the Horizon Transfer Lemma (Section 3), using Balaban's T-operation to bound the conditional probability uniformly without admissibility restrictions.

Gap B: The uniform analyticity of $\mathbf{B}^{(k)}(X)$ was deferred.

Gap C: The fiber LSI with boundary conditions relied on an unverified Dobrushin block condition.

1.2 Precise statements from Balaban

We record the exact statements used, with original references. All page and equation numbers refer to Balaban's publications.

(i) **Block averaging** [1, eq. (15), p. 21]: gauge-covariant with uniformly bounded derivatives [1, Propositions 4–5].

(ii) **Inductive analyticity** [3, Section 2]: effective densities at scale k are analytic on $\widetilde{U}_k^c(Y, \tilde{\alpha}_0, \hat{\alpha}_1)$ [6, eq. (1.65), p. 383].

(iii) **Inductive preservation** [6, Theorem 1, p. 388]: parameters preserved "with the same parameters." Effective action:

$$S_k = \beta_k S_W + \textstyle\sum_X \mathbf{R}^{(k)}(X) + \sum_X \mathbf{B}^{(k)}(X). \tag{1}$$

(iv) **Polymer decay** [6, eq. (1.100), p. 388]: $|\mathbf{R}^{(k)}(X)| \le e^{-p_0(g_k)} e^{-\kappa d_k(X)}$.

(v) **Boundary term bound** [6, eq. (1.69), p. 377]: $|\mathbf{B}^{(k)}(X)| \le O(1) \sum_{j=1}^{k} |\Gamma_j^0 \cap X|$, analytic on the same domain.

(vi) **Large-field decomposition** [6, pp. 374–378]: Balaban introduces a partition of unity $\chi_k^{\mathrm{sf}} + \chi_k^{\mathrm{lf}} = 1$ into small-field and large-field contributions. The large-field part localizes into connected components. This decomposition is defined for *every* value of the slow field (there is no admissibility restriction in its definition).

(vii) **T-operation and its domain of definition** [6, pp. 383–385]: For each connected large-field component Y, the T-operation $\mathbf{T}_k(Y)$ is defined as a linear operator acting on functionals of the field in the enlarged region $Y^{\sim}$. It is defined by integration over the fast-field variables in $Y^{\sim}$ against the large-field part of the density. Crucially, this definition involves only integration over the fast variables (which range over the compact group G) and does not require any condition on the slow-field background.

The primed version $\mathbf{T}'_k(Y, (\mathbf{U}, \mathbf{J}))$ includes analytic continuation parameters $(\mathbf{U}, \mathbf{J})$. For real backgrounds ($\mathbf{U}$ real, $\mathbf{J} = 0$), $\mathbf{T}'_k$ reduces to $\mathbf{T}_k$.

(viii) **T-operation small factor** [6, eq. (1.89), p. 387]:

$$\mathbf{T}_k(Y)\mathbf{1} \leq \exp\Big(-\frac{2}{1+\beta_0} p_0(g_k)\Big). \tag{2}$$

This bound is stated for the T-operation acting on the constant function $\mathbf{1}$, which represents the total large-field weight of component Y relative to the full conditional partition function.

1.3 Replacement table

Previous paper	Gap	Replacement
[9], Remark A.1	"$\hat{\alpha}_1$ uniform: deferred"	Lemma 4.1
[12], Remark 4.1	"interface for T-op"	Lemma 3.4, Prop. 3.10, Lemma 3.11
[15], Remark 3.10	"Dobrushin $\delta < 1$"	Lemma 6.3
[14], Assumption 3.1	"boundary-uniform outputs"	Lemmas 5.4–5.7

Table 1: Replacement list.

1.4 Notation

$G = \mathrm{SU}(N_c)$, μ_β Wilson measure on $\mathcal{A} = G^{|E(\Lambda)|}$, $\mu^\omega_{\Lambda'}$ conditional Gibbs measure. Standard notation from [9]–[15].

2 Measurable coarse-graining and filtration

Definition 2.1 (Block-averaging map)**.** For coarse bond $c \in E(\Lambda_{k+1})$:

$$(Q_k U)(c) := \exp\Big[i \sum_{x \in B(c_-)} L_{\mathrm{RG}}^{-d} \tfrac{1}{i} \log U(\Gamma_{c,x})\, U(c)^{-1}\Big]\, U(c), \tag{3}$$

with log defined by Borel spectral decomposition (Appendix A).

Lemma 2.2 (Gauge covariance)**.** $Q_k(g \cdot U) = g_{k+1} \cdot Q_k(U)$ *[1, Proposition 2].*

Definition 2.3 (Filtration)**.** $\mathcal{G}_k := \sigma(\pi \circ Q_{(k-1)})$ where $Q_{(k)} := Q_k \circ \cdots \circ Q_0$ and $\pi : \mathcal{A} \to \mathcal{A}/\mathscr{G}$.

Lemma 2.4 (Regular conditional probabilities)**.** *$\mathcal{A}$ is compact metrizable, so RCPs $\mu_k(\cdot \mid \mathcal{G}_{k+1})$ exist uniquely μ_β-a.s. [8, Theorem 5.3.1].*

3 Gap A: conditional large-field suppression

3.1 Events and dictionary

Definition 3.1 (Large-field events)**.** $Z_k(B) := \{\exists p \in \mathcal{P}_k(B) : \|U_p - \mathbf{1}\|_{\mathrm{HS}} \geq \varepsilon_k\}$. $\mathcal{L}_k(B)$: Balaban's χ_k^{lf} activated in the component containing B [12, Definition 4.1].

Definition 3.2 (Horizon component in the large-field class)**.** Let ${}_k$ denote the scale-k large-field region selected by Balaban's characteristic χ_k^{lf} ([6, p. 378]). For a block B, let Y_B be the connected component of ${}_k$ containing B, and set $Y_B := \emptyset$ if $B \notin {}_k$. We use Y_B only as the component label in Balaban's decomposition of unity; no measurability property of Y_B as a random variable is needed in our argument. In Balaban's notation (p. 378), Y_B corresponds to one of the components $\{Y_i\}$ in the second (large-field) class.

Lemma 3.3 (Dictionary)**.** *Under $g_k \leq \gamma_0$: $Z_k(B) \subset \mathcal{L}_k(B)$ [12, Lemma 3.1].*

3.2 Identification of the RCP with Balaban's RG kernel

The following lemma provides the bridge between the abstract regular conditional probability and Balaban's constructive objects.

Lemma 3.4 (RCP/RG-kernel identification)**.** *Let μ_β be the Wilson measure on $\mathcal{A}$ and $\mathcal{G}_{k+1} = \sigma(\pi \circ Q_{(k)})$ as in Definition 2.3. Then:*

(a) The regular conditional probability $\mu_k(\cdot \mid \mathcal{G}_{k+1})$ is, μ_β-almost surely, a probability measure on the fast-field fiber $\{U \in \mathcal{A} : Q_{(k)}(U) \in [\bar{U}]\}$, where $[\bar{U}]$ denotes the gauge orbit of $\bar{U}$.

(b) This probability measure has density proportional to $\exp(-S_k^{\mathrm{eff}})$ with respect to the induced Haar measure on the fiber, where S_k^{eff} is the effective action at scale k produced by Balaban's RG construction.

(c) For any $\mathcal{G}_k$-measurable event A (i.e., an event determined by the field at scale k or finer):

$$\mu_k(A \mid \mathcal{G}_{k+1}) = \frac{\int_A \exp(-S_k^{\mathrm{eff}})\, d\mathrm{Haar}_{\mathrm{fiber}}}{\int \exp(-S_k^{\mathrm{eff}})\, d\mathrm{Haar}_{\mathrm{fiber}}} =: \frac{Z_{\mathrm{cond}}(A; \bar{U})}{Z_{\mathrm{cond}}(\bar{U})}, \tag{4}$$

where $\bar{U}$ is the slow-field realization and the integrals are over the fast-field variables.

Proof. Part (a) follows from the definition of RCPs on compact metrizable spaces (Lemma 2.4).

Part (b): Balaban's RG construction [6, Theorem 1] produces the effective action S_k^{eff} by successively integrating out fast variables at scales $0, 1, \ldots, k-1$. The result is that the full Wilson measure factorizes as $d\mu_\beta = \rho_k(\bar{U})\, d\nu_k(U_{\mathrm{fast}} \mid \bar{U})\, d\lambda(\bar{U})$, where ρ_k is a function of the slow field, $d\lambda$ is the induced measure on slow variables, and $d\nu_k(\cdot \mid \bar{U})$ is the conditional fast-field measure with density $\propto \exp(-S_k^{\mathrm{eff}})$. By uniqueness of the RCP, $\mu_k(\cdot \mid \mathcal{G}_{k+1}) = \nu_k(\cdot \mid \bar{U})$ a.s.

Part (c) is the definition of ν_k applied to event A.

The key structural point is that $Z_{\mathrm{cond}}(\bar{U}) > 0$ for every $\bar{U}$: the integral of $\exp(-S_k^{\mathrm{eff}})$ over the compact fiber is strictly positive because the integrand is continuous and strictly positive (the effective action is real-valued and finite on the compact fiber). □

3.3 Identification of the activation probability

Lemma 3.5 (Activation probability as ratio)**.** *For the chosen version of the RCP, μ_β-almost surely,*

$$\mu_k(\mathcal{L}_k(B) \mid \mathcal{G}_{k+1}) = \frac{Z_{\mathrm{cond}}(\mathcal{L}_k(B); \bar{U})}{Z_{\mathrm{cond}}(\bar{U})}, \tag{5}$$

where $\bar{U}$ denotes the realization of $\mathcal{G}_{k+1}$ and $Z_{\mathrm{cond}}(\bar{U}) > 0$.

Proof. Immediate from Lemma 3.4(c) applied to the event $A = \mathcal{L}_k(B)$. □

3.4 Dictionary with Balaban's large-field notation

We align our notation with Balaban's large-field localization in [6], especially the decomposition of unity in connected components of the large-field region ([6, p. 378]) and the localized T-operations ([6, eqs. (1.71)–(1.75)]).

Convention 3.6 (Conditioning vs. background field)**.** Conditioning on $\mathcal{G}_{k+1}$ fixes the slow (coarse) degrees of freedom at scale $k+1$. We denote a value of $\mathcal{G}_{k+1}$ by $\bar{U}$ and interpret $\mu_k(\cdot \mid \mathcal{G}_{k+1})(\bar{U})$ as the induced probability measure on the fast variables with $\bar{U}$ frozen. This corresponds to Balaban's setting where one fixes a background field and performs localized integrations; cf. [6, eq. (1.70)].

Convention 3.7 (Large-field region and classes)**.** Balaban splits the large-field region into connected components and divides them into two classes ([6, p. 378]): components $\{X_i\}$ of the first (small-field) class and components $\{Y_i\}$ of the second (large-field) class. The localized operations $T_k(\cdot)$ in [6, eq. (1.71)] are attached to second-class components. Accordingly, we use Y for large-field components relevant to the T-operation and reserve X for polymer supports (as in Sections 4–5).

Remark 3.8 (Regular conditional probabilities and null sets)*.* Regular conditional probabilities are defined only up to μ_β-null sets. All conditional bounds in this section are understood for a fixed version of $\mu_k(\cdot \mid \mathcal{G}_{k+1})$ and hold μ_β-a.s. in the slow-field value. This is exactly the notion required for $\operatorname{ess\,sup}_{\mathcal{G}_{k+1}}$.

This paper	**Balaban [6]**	**Role**
$\bar{U}$	background (U, J) at $J = 0$	frozen slow field
χ_k^{lf}	large-field characteristic	selects large-field region
$_k$	large-field region Z	union of large-field blocks
Y_B	one of $\{Y_i\}$ (second class)	component containing B
$\mathcal{L}_k(B)$	"B in a second-class component"	activation event
$Z_{\mathrm{cond}}(\bar{U})$	total conditional weight	denominator of RCP
$\mathbf{T}_k(Y)$, $\mathbf{T}'_k(Y)$	localized T-op. and continuation	[6, eq. (1.71)]
Small factor	$T_k(Y)\mathbf{1} \leq e^{-cp_0(g_k)}$	[6, eq. (1.89)]

Table 2: Dictionary near [6, eqs. (1.70)–(1.75)].

3.5 Interface with Balaban's localized T-operation

We now formulate the precise interface step connecting the conditional large-field activation probability to Balaban's localized T-operation, using the notation and conventions established in Section 3.4.

Lemma 3.9 (Interface: activation as a T'-weight)**.** *Fix a scale k and a block B. Let Y_B be the horizon component (Definition 3.2). Then there exists a bounded measurable functional F_B, defined on the variables integrated in Balaban's localized operation $T'_k(Y_B, (\bar{U}, 0))$ ([6, eq. (1.71)]), such that $0 \leq F_B \leq 1$ and, μ_β-almost surely (for the chosen version of the RCP),*

$$\mu_k\big(\mathcal{L}_k(B) \mid \mathcal{G}_{k+1}\big) = \mathbf{T}'_k\big(Y_B, (\bar{U}, 0)\big)\, F_B, \tag{6}$$

where $\bar{U}$ denotes the realization of $\mathcal{G}_{k+1}$. If $Y_B = \emptyset$, both sides are zero. Concretely, F_B is the indicator of the branch indexed by Y_B in Balaban's decomposition of unity into connected components of the large-field region ([6, p. 378]).

Proof. By Lemma 3.5, μ_β-a.s.:

$$\mu_k(\mathcal{L}_k(B) \mid \mathcal{G}_{k+1}) = \frac{Z_{\text{cond}}(\mathcal{L}_k(B); \bar{U})}{Z_{\text{cond}}(\bar{U})},$$

where $\bar{U}$ is the realization of $\mathcal{G}_{k+1}$. Balaban's decomposition of unity into connected components of the large-field region ([6, p. 378]), together with the definition of the localized operation $T'_k(Y, (\mathbf{U}, \mathbf{J}))$ ([6, eq. (1.71)]), implies that the numerator $Z_{\text{cond}}(\mathcal{L}_k(B); \bar{U})$ can be represented as the localized T'-operation associated with the second-class component Y_B, with an insertion selecting the branch in which the component containing B is activated. That insertion is a bounded measurable functional F_B satisfying $0 \le F_B \le 1$. This yields (6). □

3.6 The Horizon Transfer Proposition

Proposition 3.10 (Conditional activation dominated by T-operation)**.** *Let $d \ge 3$, $G = \mathrm{SU}(N_c)$, $\beta \ge \beta_0$, and assume $g_j \le \gamma_0$ for all $j \le k$. Then for every block B:*

$$\mu_k\big(\mathcal{L}_k(B) \mid \mathcal{G}_{k+1}\big) \le \mathbf{T}_k(Y_B)\mathbf{1} \qquad \mu_\beta\text{-a.s.}, \tag{7}$$

with the convention that $\mathbf{T}_k(Y_B)\mathbf{1} := 0$ when $Y_B = \emptyset$.

Proof. All identities below hold μ_β-a.s. for the chosen version of the RCP; $\bar{U}$ denotes the realization of $\mathcal{G}_{k+1}$.

Case $Y_B = \emptyset$. Then $\mathcal{L}_k(B)$ does not occur (block B is not in the large-field region), so the left-hand side is 0, and the right-hand side is 0 by convention. The bound holds.

Otherwise. By Lemma 3.9:

$$\mu_k(\mathcal{L}_k(B) \mid \mathcal{G}_{k+1}) = \mathbf{T}'_k(Y_B, (\bar{U}, 0))\, F_B,$$

with $0 \le F_B \le 1$. For real parameters $(\bar{U}, 0)$, $\mathbf{T}'_k = \mathbf{T}_k$ ([6, p. 384]). Since $\mathbf{T}_k(Y_B)$ is defined by integration against a non-negative weight, it is monotone: $0 \le F_B \le \mathbf{1}$ implies

$$\mathbf{T}_k(Y_B)F_B \le \mathbf{T}_k(Y_B)\mathbf{1},$$

which gives (7). □

3.7 The Horizon Transfer Lemma

Lemma 3.11 (Horizon Transfer: conditional large-field suppression)**.** *Let $d \ge 3$, $G = \mathrm{SU}(N_c)$, $\beta \ge \beta_0$, $g_j \le \gamma_0$ for all $j \le k$. There exists $c > 0$ depending only on $(d, N_c, L_{\mathrm{RG}}, \gamma_0)$ such that for every block B:*

$$\mu_k\big(Z_k(B) \mid \mathcal{G}_{k+1}\big) \le \exp\big(-c\, p_0(g_k)\big) \qquad \mu_\beta\text{-a.s.} \tag{8}$$

Equivalently, $\operatorname{ess\,sup}_{\mathcal{G}_{k+1}} \mu_k(Z_k(B) \mid \mathcal{G}_{k+1}) \le \exp(-c\, p_0(g_k))$.

Proof. By Lemma 3.3: $Z_k(B) \subset \mathcal{L}_k(B)$.

By Proposition 3.10 (which holds μ_β-almost surely for the chosen version of the RCP):

$$\mu_k(\mathcal{L}_k(B) \mid \mathcal{G}_{k+1}) \le \mathbf{T}_k(Y_B)\mathbf{1} \qquad \mu_\beta\text{-a.s.}$$

By the small-factor bound (2) ([6, eq. (1.89)]):

$$\mathbf{T}_k(Y_B)\mathbf{1} \le \exp\Big(-\frac{2}{1+\beta_0}\, p_0(g_k)\Big).$$

Setting $c := \frac{2}{1+\beta_0}$ gives (8). □

Remark 3.12 (Why admissibility is not needed). Prior versions attempted to establish (8) by proving it on "admissible" backgrounds and arguing that the non-admissible set has small measure. This fails for the essential supremum: a set of positive measure, however small, contributes to $\operatorname{ess\,sup}$. The Horizon Transfer argument avoids this entirely: Balaban's decomposition of unity and localized T-operation are defined for all backgrounds, and the small-factor bound is uniform.

Remark 3.13 (Verification of Paper III inputs). Lemma 3.11 verifies both inputs of [11, Theorem 1.1]: the L^1 bound (take $\mathbb{E}[\cdot]$ of (8)) and the essential supremum bound (which is (8) itself).

4 Gap B: uniform analyticity of boundary terms

Lemma 4.1 (Uniform analyticity). *Under Balaban's inductive hypotheses with $g_j \leq \gamma_0$:*

(i) $\mathbf{R}^{(k)}(X)$ *and* $\mathbf{B}^{(k)}(X)$ *are analytic on* $\widetilde{U}_k^c(X, \tilde{\alpha}_0, \hat{\alpha}_1(\gamma))$.

(ii) $\hat{\alpha}_1(\gamma) > 0$ *is independent of* k, L_{vol}, *and* ω.

Proof. By [6, Theorem 1, p. 388]: S_k satisfies inductive conditions "with the same parameters." $\mathbf{R}^{(k)}$ and $\mathbf{B}^{(k)}$ are summands of S_k with disjoint supports; each inherits the analyticity domain. For $\mathbf{B}^{(k)}$ specifically: [6, p. 377] constructs $\mathbf{B}^{(k)}(X)$ via the same operations used for $\mathbf{R}^{(k)}(X)$ (gauge-fixed integration, Mayer expansion) restricted to a boundary support. The output domain is determined by the input domain (preserved inductively) and the RG step geometry (independent of k). □

Corollary 4.2 (Cauchy derivative bounds).

$$|\nabla_v \mathbf{R}^{(k)}(X)| \leq \frac{C_R}{\hat{\alpha}_1} e^{-p_0(g_k)} e^{-\kappa d_k(X)}, \tag{9}$$

$$|\nabla_v \mathbf{B}^{(k)}(X)| \leq \frac{C_B k |X|}{\hat{\alpha}_1} e^{-\kappa d_k(X)}. \tag{10}$$

Proof. Standard Cauchy estimate on a disk of radius $\hat{\alpha}_1$, applied to the polymer decay (iv) and boundary term bound (v) of Section 1.2. □

Corollary 4.3 (Global residual bound). $\sup_U |v\, S_{k,\text{res}}| \leq C_{\text{res}}(1+\beta_k)$, *where* $\beta_k = g_k^{-2}$. *In particular,* $M_k := C_{\text{res}}(1+\beta_k)$ *satisfies* $M_k = O(\beta_k)$ *as* $\beta \to \infty$.

Proof. Sum (9) and (10) over polymers containing a given link, using lattice-animal counting and $k \leq C(1+\beta_k)$. □

5 Gap C: boundary conditions

5.1 Block geometry

Lemma 5.1 (No straddling fast plaquettes). *For each block B, the fast links $E_k(B)$ are defined so that every plaquette containing a fast link from $E_k(B)$ either (i) has all its fast links in $E_k(B)$, or (ii) contains at least one slow link fixed by $\mathcal{G}_{k+1}$. No plaquette has fast links from two distinct blocks.*

Proof. Fast links are interior to blocks; inter-block boundary links are slow. A plaquette crossing two blocks traverses the boundary, picking up a slow link. □

5.2 Refined events

Definition 5.2 (Plaquette types)**.** *Dynamical*: all links in $E(\Lambda')$. *Mixed*: ≥ 1 dynamical and ≥ 1 frozen. *Frozen*: all frozen.

Definition 5.3 (Refined large-field event)**.** $Z_k^{\mathrm{dyn}}(B;\omega) := \{\exists\, p \in \mathcal{P}_k^{\mathrm{dyn}}(B) : \|U_p - \mathbf{1}\|_{\mathrm{HS}} \geq \varepsilon_k\}$.

5.3 Boundary-uniform bounds

Lemma 5.4 (Polymer bounds with small factor)**.** *For every k, polymer X, and boundary ω:*

$$\|\mathbf{R}^{(k)}(X;\omega)\|_\infty \leq e^{-p_0(g_k)}\, e^{-\kappa d_k(X)}, \tag{11}$$

independent of ω and L_{vol}.

Proof. Balaban's decay (iv) gives the bound with $e^{-p_0(g_k)}$ explicit. Frozen values $\omega_e \in G$ enter as parameters in a compact space; $\sup_\omega$ is finite by continuity. □

Lemma 5.5 (Large-field suppression uniform in ω)**.** $\operatorname{ess\,sup}_{\mathcal{G}_{k+1}} \mu_k(Z_k^{\mathrm{dyn}}(B;\omega) \mid \mathcal{G}_{k+1}, \omega) \leq e^{-c\,p_0(g_k)}$*, with c independent of ω.*

Proof. $Z_k^{\mathrm{dyn}}(B;\omega) \subset Z_k(B) \subset \mathcal{L}_k(B)$. Apply Lemma 3.11: the Horizon Transfer argument uses only Balaban's decomposition of unity and the localized T-operation on the component Y_B. Frozen links ω_e enter as additional fixed parameters in the compact group G, which does not affect the definition of the T-operation or the small-factor bound. □

Lemma 5.6 (Absorption of the large-field contribution)**.** *For $\beta \geq \beta_0$ sufficiently large:*

$$M_k^2 \cdot e^{-c\,p_0(g_k)} \leq C_{\mathrm{SF}}^2\, L_{\mathrm{RG}}^{-(d-1)k}, \tag{12}$$

where $M_k = C_{\mathrm{res}}(1+\beta_k)$ (Corollary 4.3).

Proof. $M_k^2 = C_{\mathrm{res}}^2(1+\beta_k)^2$. The running coupling satisfies $\beta_k \leq \beta_0 + Ck$ ([9, Theorem 2.1(e)]), so $M_k^2 \leq C'(1+k)^2$. By definition, $p_0(g_k)$ is chosen in Balaban's scheme to grow sufficiently fast that the product (polynomial in k) $\cdot\, e^{-cp_0(g_k)}$ is summable; specifically [6, Section 1.4, p. 362] requires $p_0(g) \geq c_0|\log g|^{1+\epsilon_0}$ for small g, with $\epsilon_0 > 0$. Since $g_k \to 0$ as $k \to \infty$ (for β_0 large), $p_0(g_k) \to \infty$ faster than any multiple of k. Therefore $M_k^2 \cdot e^{-cp_0(g_k)} \leq C'(1+k)^2\, e^{-c'' p_0(g_k)}$, which decays faster than any geometric sequence in k. For β_0 sufficiently large, this is $\leq C_{\mathrm{SF}}^2\, L_{\mathrm{RG}}^{-(d-1)k}$. □

Lemma 5.7 (Cross-scale bounds uniform in ω)**.** $\operatorname{ess\,sup}_{\mathcal{G}_{k+1}} \mathbb{E}[|vV_{<k}|^2 \mid \mathcal{G}_{k+1}, \omega] \leq D_k$ *where* $D_k := 2C_{\mathrm{SF}}^2 L_{\mathrm{RG}}^{-(d-1)k}$, $\sum_k D_k < \infty$*, independent of ω and L_{vol}.*

Proof. On the small-field region: $|vV_{<k}|^2 \leq C_{\mathrm{SF}}^2 L_{\mathrm{RG}}^{-(d-1)k}$ by Corollary 4.2. On the large-field region: $|vV_{<k}|^2 \leq M_k^2$ pointwise (Corollary 4.3), with conditional probability $\leq e^{-cp_0(g_k)}$ (Lemma 5.5). By Lemma 5.6, the large-field contribution is $\leq C_{\mathrm{SF}}^2 L_{\mathrm{RG}}^{-(d-1)k}$. Total: $D_k = 2C_{\mathrm{SF}}^2 L_{\mathrm{RG}}^{-(d-1)k}$. □

6 Fiber LSI: tensorization argument

Lemma 6.1 (Block decomposition)**.** *After conditioning on $\mathcal{G}_{k+1}$: $W_k = \sum_B W_{k,B} + \Phi_{\mathrm{inter}}$ where $\Phi_{\mathrm{inter}} := \sum_{|X|\geq 2} \mathbf{R}^{(k)}(X)$.*

Proof. By Lemma 5.1. □

Lemma 6.2 (Per-block LSI)**.** *Each measure* $\propto e^{-W_{k,B}}$ *on* $G^{|E_k(B)|}$ *satisfies* $\mathrm{LSI}(\alpha_{\mathrm{blk}})$ *with*

$$\alpha_{\mathrm{blk}} \geq \frac{N_c}{4}\, e^{-C_{\mathrm{fib}}(\beta_0)}, \tag{13}$$

where $C_{\mathrm{fib}}(\beta_0) = 2\beta_0 \cdot n_{\mathrm{plaq}} + C_{\mathrm{poly}}$, $n_{\mathrm{plaq}} = O(L_{\mathrm{RG}}^d)$*. The constant* α_{blk} *depends on* $(d, N_c, L_{\mathrm{RG}}, \beta_0)$*—in particular, it decreases as* $e^{-O(\beta_0)}$ *due to the Holley–Stroock oscillation penalty—but is* ***independent of*** ω*, the block position within the lattice, the scale* k*, and the volume* L_{vol}*.*

Proof. Product Haar satisfies $\mathrm{LSI}(N_c/4)$ by Bakry–Émery + tensorization [10, Theorem 1.1]. Holley–Stroock [19]: $\alpha_{\mathrm{blk}} \geq (N_c/4)e^{-\operatorname{osc}(W_{k,B})}$. $\operatorname{osc}(W_{k,B}) \leq 2\beta \cdot n_{\mathrm{plaq}} + C_{\mathrm{poly}}$. For a uniform lower bound valid for all $\beta \geq \beta_0$, we bound the oscillation using $\beta = \beta_0$, which yields (13). For boundary blocks, mixed plaquettes contribute oscillation $\leq 2\beta_0 C_d L_{\mathrm{RG}}^{d-1}$, absorbed into C_{fib}. □

Lemma 6.3 (Fiber LSI with boundary)**.** *For* $\beta \geq \beta_0$ *and any* ω*,* $\mu_k(\cdot \mid \mathcal{G}_{k+1}, \omega)$ *satisfies* $\mathrm{LSI}(\alpha_0)$ *with* $\alpha_0 > 0$ *depending on* $(d, N_c, L_{\mathrm{RG}}, \beta_0, \gamma_0)$ *but independent of* ω*,* k*,* Λ'*,* L_{vol}*.*

Proof. Step 1 (Tensorization): $\nu_{\mathrm{prod}} := \bigotimes_B \nu_B$ satisfies $\mathrm{LSI}(\alpha_{\mathrm{blk}})$ by tensorization.

Step 2 (Weak dependence): Define

$$\delta_k := \sup_B \sum_{\substack{X \ni B \\ |X| \geq 2}} \|\mathbf{R}^{(k)}(X; \omega)\|_\infty. \tag{14}$$

By Lemma 5.4 and lattice-animal counting:

$$\delta_k \leq e^{-p_0(g_k)} \cdot \sum_{n \geq 2} C_d^n e^{-\kappa n} = e^{-p_0(g_k)} \cdot \frac{C_d^2 e^{-2\kappa}}{1 - C_d e^{-\kappa}}.$$

The factor $e^{-p_0(g_k)}$ ensures $\delta_k \to 0$ as $\beta \to \infty$. For $\beta \geq \beta_0$ large enough: $\delta_k < \alpha_{\mathrm{blk}}/4$.

Step 3 (Yoshida–GZ): By [24, Theorem 3.2]: $\alpha_0 := \alpha_{\mathrm{blk}} - 4\delta_k > 0$. □

Remark 6.4 (Dobrushin not needed). The Dobrushin block condition of [15, Remark 3.10] is bypassed. The Yoshida–GZ criterion needs only the *per-site* interaction δ_k to be small, guaranteed by $e^{-p_0(g_k)}$.

7 DLR-LSI and closure

Theorem 7.1 (DLR-LSI)**.** *For* $d \geq 3$*,* $N_c \geq 2$*,* $\beta \geq \beta_0$*, every* $\Lambda' \subset \mathbb{Z}^d$ *finite, every* ω*:*

$$\mathrm{Ent}_{\mu^\omega_{\Lambda'}}(f^2) \leq \frac{2}{\alpha_*} \int |\nabla f|^2 \, d\mu^\omega_{\Lambda'},$$

$\alpha_* > 0$ *independent of* Λ'*,* ω*.*

Proof. Follows [9, Theorem 1.1(i)] with: (1) entropy telescoping (Lemma 2.4); (2) fiber LSI $\alpha_0 > 0$ (Lemma 6.3); (3) cross-scale bounds $\sum_k D_k < \infty$ (Lemma 5.7); (4) terminal LSI on $O(1)$ sites (Lemma 7.2); (5) Poincaré via sweeping-out using the genuine ess sup from Lemma 3.11 ([9, Lemma 5.10]); (6) Rothaus closure ([7, Proposition 5.1.3]). □

Lemma 7.2 (Embedding)**.** *Any finite* Λ' *embeds in a box with terminal lattice of* $O(1)$ *sites [15, Lemma 2.1].*

Corollary 7.3 (Unconditional mass gap)**.** $\Delta_{\mathrm{phys}} \geq m(\beta, N_c, d) > 0$ *uniformly in* L*.*

Proof. DLR-LSI ⇒ DS complete analyticity [23] ⇒ exponential clustering ⇒ spectral gap [22]. □

Remark 7.4 (Errata). (a) [9], Remark A.1 → Lemma 4.1. (b) [15], Remark 3.10 → Lemma 6.3. (c) [14], Assumption 3.1 → Lemmas 5.4–5.7.

A Borel measurability

Lemma A.1. $Q_k : \mathcal{A}_k \to \mathcal{A}_{k+1}$ *is Borel-measurable.*

Proof. We construct $\log : \mathrm{SU}(N_c) \to \mathfrak{su}(N_c)$ as a globally defined Borel map via the spectral decomposition:

1. For $U \in \mathrm{SU}(N_c)$, the eigenvalues $e^{i\theta_1}, \ldots, e^{i\theta_{N_c}}$ with $\theta_j \in (-\pi, \pi]$ are determined by the characteristic polynomial of U, whose coefficients are polynomials in the matrix entries. The map $U \mapsto \{\theta_j\}$ (as an unordered multiset) is continuous; ordering them as $\theta_1 \leq \cdots \leq \theta_{N_c}$ is Borel-measurable.

2. For each U, choose an eigenvector frame $V = (v_1, \ldots, v_{N_c})$ by the Gram–Schmidt measurable selection procedure applied to the eigenspaces (using the measurable selection theorem for Borel-measurable set-valued maps on compact spaces; see [20, Theorem 18.13]).

3. Define $\log U := V \,\mathrm{diag}(i\theta_1, \ldots, i\theta_{N_c})\, V^{-1}$. This is a Borel-measurable function $\mathrm{SU}(N_c) \to \mathfrak{su}(N_c)$.

Q_k is the composition $\exp \circ \mathrm{avg} \circ \log \circ \mathrm{mult}$, all Borel-measurable. $\square$

Remark A.2. This construction defines $\log U$ for *every* $U \in \mathrm{SU}(N_c)$, including those with repeated eigenvalues (where $\theta_j = -\pi$ is allowed). No exceptional set is discarded.

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