Uniform Coercivity, Pointwise Large-Field Suppression, and Conditional Closure of the Lattice Yang–Mills Mass Gap at Weak Coupling in $d = 4$

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February 2026. Version 2: July 2026.

Abstract

We address the remaining interface gaps in the programme [E26I]–[E26VIII] toward a uniform log-Sobolev inequality (LSI) and transfer-matrix spectral gap for lattice SU(N_c) Yang–Mills in $d = 4$ at weak coupling. Four gaps are treated: (G1) the pointwise-in-background validity of Balaban's T-operation small-factor bound — stated in v2 as the explicit hypothesis (H-PTW), since the detailed audit appendices announced in v1 were absent from the document (their references rendered as "??"); (G2) a uniform small-field coercivity estimate for the effective action; (G3) uniform analyticity of boundary terms; (G4) a quantitative bootstrap of all constants. **The central correction of v2:** the sign of the one-step coupling drift in v1's Theorem 5.2 ($g_{k+1}^{-2} = g_k^{-2} + 2b_0 \ln L_{\mathrm{RG}}$, coupling weakening toward the infrared) is inverted relative to asymptotic freedom, and contradicts the companions' own use of $n_{\max} \sim 1/(2b_0 g^2 \ln 2)$ (ai.viXra:2602.0032 §8, 2602.0033, 2602.0041 §7.3), a formula meaningful only if the coupling *grows* along blocking and exits the weak regime. With the corrected sign the monotone bootstrap of v1's Theorem 5.3 reverses: the inductive conditions are guaranteed only up to the finite horizon $k^*(\beta) = (g_0^{-2} - \gamma_0^{-2})/(2b_0 \ln L_{\mathrm{RG}}) + O(1)$, and since the multiscale construction uses $\log_2 L$ scales, the conclusion holds *on the volume window* $\log_2 L \le k^*(\beta)$, i.e. $L \le e^{C/g^2 + O(1)}$ with $C = 1/(2b_0) = 24\pi^2/(11N_c)$ — exactly the window of the companion papers ai.viXra:2602.0032/0033 (v2). Full volume-uniformity would additionally require a strong-coupling handoff beyond the crossover scale (Osterwalder–Seiler regime), stated as hypothesis (H-XOVER) and not established here; accordingly the "unconditional closure" of v1 is retitled to *conditional closure*. Version 2 also repairs the dangling "??" references, supplies the missing proof of Lemma 3.1, rewrites the proof of Lemma 7.4 in the series' fundamental-trace convention (its statement $W''(0) = 1/(2N_c)$ is correct; the v1 proof mixed normalized and fundamental traces), corrects $\sum_{k\ge 0}(k+1)2^{-3k} = 64/49$ (v1: 8/49), and fills in the companion identifiers. All corrections are verified in a companion numerical suite.

1 Introduction

1.1 Context and objective

On $\Lambda = (\mathbb{Z}/L\mathbb{Z})^d$ with Wilson action

$$A(U) := \sum_{p \in \mathcal{P}(\Lambda)} \Big(1 - \frac{1}{N_c} \mathrm{Re}\,\mathrm{Tr}\, U_p\Big), \qquad S_\beta = \beta A(U), \qquad \beta = \frac{2N_c}{g^2}, \tag{1}$$

the mass gap is $\Delta_{\mathrm{phys}}(\beta, L) = -\log \|\widehat{T}|_{1^\perp}\|$. The series [E26I]–[E26VIII] establishes the route Uniform LSI $\Rightarrow$ DLR-LSI $\Rightarrow$ SZ mixing $\Rightarrow$ RP $\Rightarrow$ $\Delta_{\mathrm{phys}} > 0$. This paper treats the four interface gaps (G1)–(G4); v2 states honestly which are closed and which become explicit hypotheses.

1.2 The four gaps

(G1) pointwise large-field suppression (v2: hypothesis (H-PTW), §3); (G2) fiber-level control without exponential collapse (§2); (G3) uniform analyticity of boundary terms (§4); (G4) positivity of α_* and simultaneous compatibility of constants (§5), now on the volume window of Theorem 1.1.

1.3 Main result

Theorem 1.1 (Mass gap, lattice, $d = 4$, weak coupling; windowed and conditional)**.** *For $G = \mathrm{SU}(N_c)$, $d = 4$, there exists $\beta_0 = \beta_0(N_c) < \infty$ such that for all $\beta \geq \beta_0$ and all even $L \geq 2$* with

$$\log_2 L \;\leq\; k^*(\beta) := \frac{g_0^{-2} - \gamma_0^{-2}}{2b_0 \ln L_{\mathrm{RG}}} + O(1) \qquad \textit{(equivalently } L \leq e^{C/g^2 + O(1)},\ C = \tfrac{1}{2b_0} = \tfrac{24\pi^2}{11N_c}\textit{)},$$

and conditionally on (H-PTW) *(§3) and the companion inputs [E26I]–[E26VIII]:*

$$\mathrm{Ent}_{\mu_\beta}(f^2) \leq \frac{2}{\alpha_*} \sum_{e \in E(\Lambda)} \int |\nabla_e f|^2 \, d\mu_\beta, \qquad \alpha_* = \alpha_*(N_c, \beta_0) > 0, \tag{2}$$

and

$$\Delta_{\mathrm{phys}}(\beta, L) \;\geq\; c(N_c, \beta_0) \;>\; 0. \tag{3}$$

Removing the volume window would additionally require hypothesis (H-XOVER) *below.* (*v1 asserted* (2)–(3) *unconditionally and uniformly in all L; see Remark 5.4 for why the corrected coupling flow forces the window.*)

(H-XOVER) Strong-coupling handoff. *Beyond the crossover scale $k^*(\beta)$ the effective theories are at $O(1)$ or strong coupling; volume-uniformity for all L requires establishing the conditional inequalities of the multiscale decomposition at those scales by strong-coupling methods (Osterwalder–Seiler cluster expansion) and matching them to the weak-coupling regime. This is not carried out in this paper or, to our knowledge, in the companions.*

1.4 Notation and conventions

As in v1: $\langle X, Y\rangle = -2\,\mathrm{Tr}(XY)$ with Tr the *fundamental* (unnormalized) trace ($\mathrm{Tr}\,\mathbf{1} = N_c$) — v2 note: this convention is used consistently, repairing the mixed usage in v1's §7.2; $\beta = 2N_c/g^2$; $p_0(g) \geq c_0|\log g|^{1+\epsilon_0}$; $\varepsilon_k = c_* g_k^{1-\delta}$; $R^{(k)}, B^{(k)}, T_k$ as in [Bal89b].

2 Small-field coercivity

As in v1 (this section survives the audit unchanged in substance). Setup: conditional fiber measure

$$d\mu_k^{\bar{U}}(U^{\text{fast}}) = \frac{1}{Z_k(\bar{U})} \exp\bigl(-S_{\text{eff},k}(U_{\text{fast}}, \bar{U})\bigr) \prod_{e\in E_k(B)} dU_e. \tag{4}$$

Definition 2.1. *$\Omega_k^{\text{sf}}(B;\bar{U})$: all plaquettes satisfy $\|U_p - 1\|_{\text{HS}} < \varepsilon_k$.*

Definition 2.2. *Exponential chart around a minimiser $U_e^{(0)}(\bar{U})$.*

Assumption 2.3 (Balaban one-step inputs (published))**.** *(B1) one-step RG map; (B2) polymer bounds $|R^{(k)}(X)| \le e^{-p_0(g_k)|X|}e^{-\kappa d_k(X)}$; (B3) analyticity with radius $\hat{\alpha}(\gamma)$ independent of k; (B4) T-operation small-factor bound $T_k(Y)1 \le \exp(-c_{\text{sf}}p_0(g_k)|Y|)$. (v2: the phrase "Pointwise-in-background validity: ??" of v1 is replaced by the explicit hypothesis (H-PTW) of §3; the running-coupling control is derived in §5/§7 with the corrected sign, on the finite horizon.)*

Remark 2.4. As in v1: coercivity is used only in the inductive regime; large-field configurations are handled by the T-operation mechanism.

Proposition 2.5 (Riemannian Hessian bound)**.** *In $\Omega_k^{\text{sf}}(B;\bar{U})$, under the inductive hypotheses at scale k and $g_k \le \gamma_0$:*

$$\text{Hess}_{\text{Riem}}(V_{k,B})(A,A) \;\ge\; \frac{\beta_k\gamma_{\text{Bal}}}{4}\|A\|^2, \tag{5}$$

with the proof as in v1 (quadratic part from (IC6); cubic and curvature corrections $O(g_k^{1-\delta}) + O(\beta_k^{-1})$ relative; polymer contribution $O(\beta_k^{-1})$ relative; thresholds (T4)–(T5)).

Remark 2.6. As in v1: (5) is a convexity input for the sweeping-out estimates, not a global fiber LSI.

Proposition 2.7 (Conditional LSI, Holley–Stroock, honest version)**.** *For every $\beta \ge \beta_0$ and every $\bar{U}$, the fiber measure satisfies* LSI(α_0) *with*

$$\alpha_0 := \alpha_{\text{Haar}} \exp\bigl(-2\,\text{osc}(V_{k,B})\bigr) > 0, \tag{6}$$

L-independent, possibly exponentially small in β (as in v1, Remark 2.8).

Remark 2.8. Exponentially small α_0 is acceptable for the existence statement $\Delta_{\text{phys}} > 0$.

3 Large-field suppression: the (H-PTW) hypothesis

v1 claimed to resolve (G1) by "auditing Balaban's proof", with the detailed audit in appendices whose references rendered as "??" — *the announced audit appendices are absent from the v1 document.* v2 therefore states the required property as an explicit hypothesis, keeping v1's three-point supporting sketch as a remark.

Lemma 3.1 (RCP as fibre integral)**.** *There is a version of the regular conditional probability given for every $\bar{U}$ by the ratio of fibre integrals*

$$\mu_k(A \mid \mathcal{G}_{k+1})(\bar{U}) = \frac{\int_A e^{-S_{\mathrm{eff},k}(U_{\mathrm{fast}},\bar{U})} \prod dU_e}{\int e^{-S_{\mathrm{eff},k}(U_{\mathrm{fast}},\bar{U})} \prod dU_e}, \tag{7}$$

well-defined and continuous in $\bar{U}$.

Proof. (Supplied in v2; v1's proof reference rendered as "??".) The integrand is continuous and strictly positive on the compact fibre $G^{|E_k(B)|}$, so both integrals are finite, the denominator is bounded below by $e^{-\sup S_{\mathrm{eff},k}} > 0$, and continuity in $\bar{U}$ follows from dominated convergence (the action is continuous in $\bar{U}$ on compacts). The resulting kernel integrates correctly against $\mathcal{G}_{k+1}$-measurable functions by Fubini, hence is a version of the conditional probability. □

Corollary 3.2. *Pointwise bounds in $\bar{U}$ imply essential-supremum bounds.*

Theorem 3.3 (Uniform large-field suppression; conditional on (H-PTW))**.** *Assume* (H-PTW) *below. For $\beta \geq \beta_0$, $0 \leq k \leq n_{\max}$, every block B and every $\bar{U}$:*

$$\mu_k\big(Z_k(B) \mid \mathcal{G}_{k+1} = \bar{U}\big) \ \leq\ C_{\mathrm{blk}}\, e^{-c_{\mathrm{lf}}\, p_0(g_k)}. \tag{8}$$

(H-PTW) Pointwise small factor. *For every real background $\bar{U}$, every connected large-field component Y, and every k: $T_k(Y)1(\bar{U}) \leq \exp(-c_{\mathrm{sf}} p_0(g_k)|Y|)$, i.e. Balaban's small-factor estimate [Bal89b, eq. (1.89)] holds pointwise in $\bar{U}$ and not only for admissible backgrounds.*

Proposition 3.4 (Supporting sketch for (H-PTW); from v1)**.** *Three observations support (H-PTW): (i) the T-operation is a ratio of strictly positive fibre integrals (Lemma 3.1), defined for every real $\bar{U}$, with the $O(\beta_k)$ offset cancelling in the ratio; (ii) the estimates in Balaban's derivation of (1.89) use only uniform inductive bounds whose constants do not reference the particular $\bar{U}$; (iii) the admissibility/analyticity-domain restrictions concern the complexified continuation used for the* next *RG step, not the real-valued ratio. (v2: this is v1's Proposition 3.4 sketch; the "detailed audit" it deferred to was not contained in the v1 document, so we do not claim (i)–(iii) constitute a proof. Discharging (H-PTW) requires writing out that audit.)*

Given (8)'s ingredients, the lattice-animal summation is as in v1:

$$\mu_k(Z_k(B) \mid \bar{U}) \leq \sum_{n\geq 1} C_d^n e^{-c_{\mathrm{sf}} p_0(g_k) n} \leq 2C_d e^{-c_{\mathrm{sf}} p_0(g_k)} \tag{9}$$

under (T6) ($C_d e^{-c_{\mathrm{sf}} p_0} \leq \frac{1}{2}$; counting bound verified by brute force in the suite).

4 Uniform analyticity of boundary terms

Proposition 4.1 (Analyticity of $B^{(k)}$)**.** *As in v1: $B^{(k)}(X)$ is analytic with radius $\hat{\alpha}_1(\gamma)$ independent of k, with*

$$\sup_{|z|\leq\hat{\alpha}_1} |B^{(k)}(X; U+z)| \leq C_B(k+1)|X| e^{-\kappa d_k(X)}, \qquad |\nabla B^{(k)}(X)| \leq \frac{C_B(k+1)|X|}{\hat{\alpha}_1} e^{-\kappa d_k(X)}. \tag{10}$$

The proof (inductive conditions, cluster-expansion localisation, sup-norm bound from [Bal89b] eq. (1.69), Cauchy estimate) is as in v1. Note $(k+1) \leq n_{\max}+1$, which on the window of Theorem 1.1 is $O(k^*(\beta))$, a β-dependent but L-independent constant.

5 Quantitative bootstrap (corrected)

5.1 Control of the running coupling: corrected sign and finite horizon

Definition 5.1 (Wilson projection and running coupling). *As in v1, with the v2-corrected normalization of Lemma 7.4: writing* $S_k^{\mathrm{loc}} = \beta_k A(U) + S_k^{\mathrm{loc,irr}}$,

$$\beta_k := \frac{1}{W''(0)} \frac{d^2}{dt^2}\Big|_{t=0} S_k^{\mathrm{loc}}\big(U_p = \exp(tT)\big) = 2N_c \frac{d^2}{dt^2}\Big|_{t=0} S_k^{\mathrm{loc}}, \qquad g_k^{-2} := \frac{\beta_k}{2N_c}. \tag{11}$$

Theorem 5.2 (One-step coupling renormalization; sign corrected in v2). *There exist* $\gamma_{\mathrm{run}} > 0$, $C_{\mathrm{run}} < \infty$ *such that if (IC1)–(IC6) hold at scale* k *with* $g_k \le \gamma_{\mathrm{run}}$*:*

$$g_{k+1}^{-2} = g_k^{-2} \;-\; 2b_0 \ln L_{\mathrm{RG}} + \delta_k, \qquad |\delta_k| \le C_{\mathrm{run}} g_k^2, \qquad b_0 = \frac{11N_c}{48\pi^2}: \tag{12}$$

the coupling grows *toward the infrared (asymptotic freedom). (v1 stated* (12) *with* $+2b_0 \ln L_{\mathrm{RG}}$*; Remark 5.4.)*

Theorem 5.3 (Bootstrap with finite horizon; corrected in v2). *Let* γ_0 *satisfy (T1)–(T7). If* $g_0 \le \gamma_0/\sqrt{2}$ *(say), then* g_k *is (weakly) increasing and*

$$g_k \le \gamma_0 \qquad \text{for all } 0 \le k \le k^*(\beta), \qquad k^*(\beta) \ge \frac{g_0^{-2} - \gamma_0^{-2}}{2b_0 \ln L_{\mathrm{RG}} + C_{\mathrm{run}}\gamma_0^2} \;=\; \frac{C}{g^2}\big(1+O(1)\big), \;\; C = \frac{1}{2b_0},$$

by iterating (12) *downward from* g_0^{-2}*; under (T3) (*$C_{\mathrm{run}}\gamma_0^2 \le b_0 \ln L_{\mathrm{RG}}$*) the guaranteed horizon is at least half the one-loop value (numerically exhibited in the suite, including the noisy recursion). (v1's Theorem 5.3 claimed the monotone bound* $g_{k+1} \le g_k \le \gamma_0$ for all k *— true only with the inverted sign, under which the coupling never exits the weak regime and the "global small-coupling hypothesis" appears closable unconditionally. With the correct sign the property is a finite-horizon statement, and since the multiscale construction uses* $n_{\max} = \log_2 L$ *scales, the constructions of §§2–4 are validated only for* $\log_2 L \le k^*(\beta)$*: the volume window of Theorem 1.1.)*

Remark 5.4 (The sign story; new in v2). Three independent witnesses fix the sign: (i) asymptotic freedom — the effective coupling of a nonabelian gauge theory grows at coarser scales; v1's Table 1 ("β_k increasing") describes infrared freedom instead; (ii) the companions' own usage: $n_{\max} \sim 1/(2b_0 g^2 \ln 2)$ (2602.0032 §8, 2602.0033, 2602.0041 §7.3) is the exit time of the growing coupling, meaningless under v1's sign (the suite exhibits that with the + sign the equation $g_k = \gamma_0$ has no solution); (iii) the corrected flow reproduces exactly the volume window $L \le e^{C/g^2 + O(1)}$, $C = 24\pi^2/(11N_c)$, of the conditional companions 2602.0032/0033 (v2) — the series becomes mutually consistent. The windowed Theorem 1.1 is therefore not a loss peculiar to this paper: it is the honest common boundary of the programme, and passing it requires (H-XOVER).

Definition 5.5 (Master threshold). $\gamma_0 > 0$ *largest satisfying (T1)–(T7) as in v1 (with (T3) now read as "remainder subdominant to the drift", guaranteeing the horizon of Theorem 5.3);* $\beta_0 := 2N_c/\gamma_0^2$*. Existence: suite, Block 7.*

Remark 5.6. As in v1: the weak-dependence threshold δ_* is a fixed positive number; $\delta_k \le C_{\mathrm{geom}} e^{-p_0(g_k)} \le \delta_*$ under (T7).

Proposition 5.7 (Simultaneous compatibility, on the window)**.** *For $\beta \geq \beta_0$ and $0 \leq k \leq \min(n_{\max}, k^*(\beta))$: (a) $g_k \leq \gamma_0$ (Theorem 5.3); (b) the Hessian bound (5); (c) the large-field bound (8) under (H-PTW); (d) $\delta_k \leq \delta_*$; (e) $\sum_k D_k \leq D_\infty < \infty$ with $D_k \leq C_D(k+1)L_{\mathrm{RG}}^{-(d-1)k}$ and*

$$\sum_{k\geq 0}(k+1)2^{-3k} = \left(\frac{8}{7}\right)^2 = \frac{64}{49} \tag{13}$$

(v1 printed 8/49*; the value, verified in the suite, is* 64/49 *— finiteness, which is all that is used, is unaffected); (f) $\alpha_* = (C_E/2 + C_BC_P)^{-1} > 0$, L-independent (Rothaus; the identity $(C_E/2 + C_BC_P)^{-1} = 2/(C_E + 2C_BC_P)$ is machine-checked). The non-circular order of determination is as in v1's §5.2.*

6 Assembly: proof of Theorem 1.1

As in v1's §6, with the v2 statuses: Step 1 (uniform LSI on the window) uses Theorem 5.3 (finite horizon), Proposition 2.7, the cross-scale bounds of [E26III]–[E26V], Theorem 3.3 (under (H-PTW)), and Proposition 5.7. Step 2 (DLR-LSI) uses [E26VI]–[E26VII] with the pointwise (in $\bar{U}$) large-field bound. Step 3 (Stroock–Zegarlinski) and Step 4 (reflection positivity $\Rightarrow$ gap) are as in v1. Every step is confined to the volume window; beyond it, (H-XOVER) would be needed.

7 Coupling constant bootstrap: complete proof (corrected)

7.1 Inductive conditions at a single scale

Definition 7.1. *(IC1)–(IC6) as in v1.*

Assumption 7.2 (One-step RG map (published input))**.** *As in v1.*

Remark 7.3. As in v1: strictly one-step.

7.2 Normalizations and the Wilson direction

Lemma 7.4 (Second variation of the Wilson density; proof rewritten in v2)**.** *Let $T \in \mathfrak{su}(N_c)$ with $\|T\|^2 = -2\,\mathrm{Tr}(T^2) = 1$ (*Tr *fundamental) and $U(t) = \exp(tT)$. Then*

$$W(U(t)) = \frac{t^2}{4N_c} + O(t^4), \qquad \frac{d^2}{dt^2}\Big|_{t=0} W(U(t)) = \frac{1}{2N_c}. \tag{14}$$

Proof. (v2; v1's proof mixed the normalized trace "tr" with the fundamental Tr, writing "$\mathrm{Tr}(T^2) = N_c\mathrm{tr}(T^2) = -N_c/2$" alongside "$\mathrm{Re}\,\mathrm{Tr} = N_c - t^2/4$", which are mutually inconsistent; the statement (14) is nevertheless correct in the fundamental convention, as verified numerically for SU(2/3/4) in the suite.) With $\|T\|^2 = -2\,\mathrm{Tr}(T^2) = 1$: $\mathrm{Tr}(T^2) = -\frac{1}{2}$. Then $\mathrm{Re}\,\mathrm{Tr}\exp(tT) = N_c + \frac{t^2}{2}\mathrm{Re}\,\mathrm{Tr}(T^2) + O(t^4) = N_c - \frac{t^2}{4} + O(t^4)$, so $W = 1 - \frac{1}{N_c}\mathrm{Re}\,\mathrm{Tr} = \frac{t^2}{4N_c} + O(t^4)$. □

Definition 7.5 (Wilson projection)**.** *As in (11), with $W''(0) = 1/(2N_c)$; well-definedness and consistency as in Lemma 7.6.*

Lemma 7.6 (Well-definedness and consistency)**.** *As in v1 (translation covariance; conjugation invariance; the extraction recovers β_k when $S_k^{\mathrm{loc,irr}}$ has vanishing plaquette-second-variation). Verified numerically on a one-plaquette action $\beta A + c\,W^2$ (the W^2 term has vanishing second variation), 12 cases, exact recovery; suite.*

7.3 Base case

Lemma 7.7. *As in v1: the Wilson action satisfies (IC1)–(IC6) at $k = 0$ for $\beta \geq \beta_0$.*

7.4 Vacuum polarization structure

Lemma 7.8 (Unique marginal operator)**.** *As in v1 (Ward–Takahashi + hypercubic symmetry): $\Pi_{\mu\nu}^{(k)}(p) = c_k(\delta_{\mu\nu}\hat{\Delta}(p) - \hat{d}_\mu \hat{d}_\nu) + \Pi_{\mu\nu}^{(k)\prime}(p)$.*

Remark 7.9. As in v1: $b_k(g_k) := c_k(g_k)/(2N_c)$ is the drift of g^{-2}.

Lemma 7.10 (Cauchy bound on remainder)**.** *As in v1: $|b_k(g_k) - b_k(0)| \leq C_{\mathrm{an}} g_k^2$.*

Lemma 7.11 (One-loop value; sign corrected in v2)**.** $b_k(0) = -2b_0 \ln L_{\mathrm{RG}}$*: integrating out ultraviolet fluctuations between scales k and $k+1$* lowers g^{-2} *(asymptotic freedom); equivalently the shift of β is $-4N_c b_0 \ln L_{\mathrm{RG}}$. The magnitude is the universal one-loop coefficient [26, 27, 28, 29].* (*v1 stated $+2b_0 \ln L_{\mathrm{RG}}$, citing a normalization-compatibility argument (§7.8) which compared to Balaban's recursion with an index convention running toward the ultraviolet; when both recursions are oriented fine→coarse, the sign is $-$. Remark 5.4 gives the independent witnesses.*)

7.5 Proof of Theorem 5.2 and normalization compatibility

As in v1's §7.7 with Lemma 7.11's sign, and §7.8 rewritten accordingly: Balaban's recursion, oriented fine→coarse, has drift $-2b_0 \ln L_{\mathrm{RG}}$ in g^{-2}; the v1 orientation (expressing the finer coupling in terms of the coarser) accounts for the + sign it quoted.

Note added in v2

Prepared with the companion suite `verify_2602_0051.py` (repository `THE-ERIKSSON-PROGRAMME`, `verification/2602-0051/`; 10 checks, all passing, ∼3 s, NumPy/SciPy). Changes: (1) title retagged "Unconditional" → "Conditional" Closure; Theorem 1.1 restated on the volume window $L \leq e^{C/g^2 + O(1)}$, $C = 24\pi^2/(11N_c)$, conditional on (H-PTW), with (H-XOVER) identified as what full volume-uniformity would additionally require; (2) the sign of the one-step coupling drift corrected (Theorem 5.2, Lemma 7.11, Table 1's "β_k increasing"): v1's sign is anti-asymptotic-freedom and inconsistent with the companions' $n_{\max}$; the corrected flow reverses the monotone bootstrap of Theorem 5.3 into a finite-horizon statement and aligns this paper's conclusion exactly with the windows of ai.viXra:2602.0032/0033 (v2) (Remark 5.5′=Remark 5.4; suite Blocks 3a–3c); (3) the dangling "??" references of v1 ((B4), Lemma 3.1, Proposition 3.4 — the announced audit appendices absent from the document) repaired: Lemma 3.1 now has a proof, and the pointwise small-factor property is the explicit hypothesis (H-PTW) with v1's three-point sketch retained as supporting (not proving) material; (4) Lemma 7.4's proof rewritten in the fundamental-trace convention (statement correct; v1's proof mixed conventions; verified numerically); (5) $\sum(k+1)2^{-3k} = 64/49$ (v1: 8/49); (6) companion identifiers filled in: [21]=ai.viXra:2602.0055, [22]=2602.0054, [23]=2602.0053, [24]=2602.0052; the v1 self-citation [25] removed; [17]–[20] retain their classic-viXra identifiers. All v1 numbered statements keep their numbers and positions; (H-PTW)/(H-XOVER) are unnumbered tagged hypotheses.

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