Octonionic Model for Black Holes

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Abstract

We formulate black holes within the octonionic projection framework in which spacetime geometry is not fundamental, but arises as a low-energy Lorentzian projection of a deeper non-associative algebraic structure. The starting point is the real normed division algebra of octonions, whose imaginary sector carries a canonical G_2 three-form [1, 2, 3, 4, 5, 6]. Since this G_2 structure is intrinsically Riemannian, physical spacetime requires additional dynamical projection data: an independent clock line, a stable associative spatial three-plane, and suppression of the coassociative sector. In the associative low-energy exterior region, the projected metric obeys Einstein-type equations and the static spherically symmetric vacuum solution is the Schwarzschild geometry [7]. The Schwarzschild radius is therefore recovered as the standard exterior horizon scale of the emergent metric, not as a direct dimensional consequence of the associator alone.

The genuinely new strong-field ingredient is the trilinear octonionic associator sector. Because octonions are alternative, a nontrivial associator interaction requires at least three independent octonion-valued fields. In the black-hole interior, the associator contribution can become dynamically relevant and may saturate at finite density. As a speculative extension of the present model, this replaces the classical curvature singularity by a regular de-Sitter-like core, while leaving the exterior solution unchanged to leading order; the comparison class includes regular black-hole and loop-inspired interior models [8, 9, 10]. We present the construction step by step: from octonions and G_2 geometry to the associative projection, from the master action to the effective field equations, from the exterior vacuum limit to the Schwarzschild metric, and from bounded associator density to a regular interior mass function. The resulting object is a horizon-forming, singularity-free projected spacetime whose possible deviations from general relativity are confined to near-horizon, ringdown, and high-curvature regimes.

1 Introduction and Scope

Classical general relativity describes black holes as solutions of the Einstein field equations. The Schwarzschild exterior is one of the most robust predictions of the theory [7], but the same classical solution contains a curvature singularity at $r = 0$. This singularity signals the breakdown of the purely geometric description rather than a reliable prediction of infinite physical density.

The purpose of the present work is to embed black holes into the octonionic foundation framework. Established mathematical input is taken from the literature on octonions, non-associative algebras, G_2 geometry, special holonomy, and calibrations [1, 2, 3, 4, 5, 6]; the dynamical projection and associator-core interpretation are the author's own theoretical construction. The claim is not that all details of quantum gravity or black-hole microphysics are already uniquely derived. The more precise claim is that a real normed non-associative algebra, together with a local variational principle and a dynamical associative projection, provides a controlled mechanism by which a Schwarzschild exterior, a regular interior, and information-preserving algebraic degrees of freedom can coexist.

The conceptual chain is

$$\mathbb{O} \longrightarrow f_{abc} \longrightarrow \varphi \longrightarrow G_2 \longrightarrow L_t \oplus P_3 \longrightarrow (M_4, g_{\mu\nu}^{\mathrm{eff}}) \longrightarrow \text{black-hole projection.}$$

Here $\mathbb{O}$ is the octonion algebra, f_{abc} are its structure constants, φ is the associated G_2 three-form, L_t is an independently selected clock line, and $P_3 \subset \operatorname{Im}\mathbb{O}$ is a dynamically selected associative spatial three-plane. The coassociative complement

$$Q_4 = P_3^{\perp}$$

is not discarded; it is interpreted as a hidden, massive, compact, or dynamically suppressed internal sector.

This paper applies that projection mechanism to black holes. The exterior geometry is recovered in the associative low-energy limit, where the comparison solution is the Schwarzschild vacuum metric [7]. The interior differs from classical general relativity because the non-associative associator sector becomes relevant and can prevent unbounded curvature growth.

The construction will be presented in the following logical order:

$$\begin{gathered}\text{minimal principles}\\ \Downarrow \\ \text{octonions and } G_2 \text{ three-form}\\ \Downarrow \\ \text{associative projection } L_t \oplus P_3\\ \Downarrow \\ \text{master action and field equations}\\ \Downarrow \\ \text{Schwarzschild exterior and regular associator core.}\end{gathered}$$

2 Minimal Physical Principles

The black-hole model is not introduced as an isolated metric ansatz. It follows the same minimal principles used in the octonionic foundation framework.

1. **Reality of observables.** Observable densities must be real. For octonions this means that physical scalar densities are built from real parts and inner products.

2. **Positive norm.** The fundamental algebra must possess a positive norm so that projected probabilities and energy densities are well defined.

3. **Locality.** The effective low-energy theory on the emergent spacetime must be local, at least at the derivative order considered here.

4. **Diffeomorphism covariance.** The emergent four-dimensional description must be expressible through scalar densities integrated with $\sqrt{-g}\, d^4x$.

5. **Maximal algebraic nontriviality.** The algebraic starting point should contain $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, and a genuinely non-associative sector while preserving a positive norm.

6. **Stable non-associative energy.** The energy associated with non-associativity must be bounded below.

7. **Low-energy recovery.** Ordinary Einstein gravity and complex quantum mechanics must be recovered after associative projection.

The Hurwitz theorem restricts finite-dimensional real normed division algebras to [1, 2, 3]

$$\mathbb{R}, \quad \mathbb{C}, \quad \mathbb{H}, \quad \mathbb{O}.$$

The sequence has a clear physical interpretation:

$$\mathbb{R} \subset \mathbb{C} \subset \mathbb{H} \subset \mathbb{O}.$$

The real numbers give real scalar observables. The complex numbers give phases and ordinary quantum amplitudes. The quaternions provide noncommutativity and natural spinorial structure. The octonions add genuine non-associativity while preserving a positive multiplicative norm [1, 3]. Hence only $\mathbb{O}$ contains all ingredients needed for a projection framework in which standard physics appears as an associative low-energy sector and non-associativity remains available in strong-field regimes.

3 Octonions, G_2 Geometry, and the Associator

Let

$$\mathbb{O} = \mathbb{R}e_0 \oplus \operatorname{Im}\mathbb{O}, \qquad \operatorname{Im}\mathbb{O} = \operatorname{span}_{\mathbb{R}}\{e_1, \ldots, e_7\},$$

with $e_0 = 1$. The octonionic product is fixed by the standard Fano-plane structure constants [1, 2, 3]

$$e_a e_b = -\delta_{ab} e_0 + f_{ab}{}^{c} e_c, \qquad a, b, c = 1, \ldots, 7,$$

where f_{abc} are totally antisymmetric structure constants. A definite Fano-plane convention may be fixed by

$$f_{123} = f_{145} = f_{176} = f_{246} = f_{257} = f_{347} = f_{365} = 1,$$

with all other components obtained by antisymmetry.

The real inner product and norm are

$$\langle X, Y \rangle = \operatorname{Re}(\bar{X}Y), \qquad \|X\|^2 = \langle X, X \rangle \geq 0.$$

The canonical octonionic three-form is [1, 2, 4]

$$\varphi = \frac{1}{6} f_{abc}\, e^a \wedge e^b \wedge e^c.$$

Its stabilizer is

$$G_2 = \operatorname{Aut}(\mathbb{O}).$$

A positive stable G_2 three-form determines a Riemannian metric on $\operatorname{Im}\mathbb{O}$ through the standard G_2 reconstruction identity [4, 5, 6]

$$(g_\varphi)_{ab} = \frac{1}{6} f_{acd} f_b{}^{cd} = \delta_{ab}.$$

However, this is a seven-dimensional Riemannian structure. It does not by itself define a four-dimensional Lorentzian spacetime. This is why the model requires the additional projection data $L_t \oplus P_3$.

The associator is defined by

$$A(x, y, z) = (xy)z - x(yz).$$

Since $\mathbb{O}$ is alternative [3, 1],

$$A(x, x, z) = A(x, y, y) = A(x, x, x) = 0.$$

Therefore a single repeated octonionic field cannot generate a nontrivial associator self-interaction:

$$A(\Psi, \Psi, \Psi) = 0.$$

The minimal non-associative sector requires three independent octonion-valued fields,

$$\Psi_I : M_4 \to \mathbb{O}, \qquad I = 1, 2, 3,$$

and the first positive scalar measuring genuine non-associativity is

$$\|A(\Psi_1, \Psi_2, \Psi_3)\|^2.$$

4 Detailed Associative Projection to Lorentzian Spacetime

A key point of the model is that a G_2 structure is not directly identified with physical spacetime. Instead, spacetime appears after a projection.

4.1 Associative and coassociative splitting

Following the standard calibration terminology for G_2 geometry, an oriented three-plane $P_3 \subset \operatorname{Im} \mathbb{O}$ is associative if [6, 4, 5]

$$\varphi|_{P_3} = \mathrm{vol}_{P_3}.$$

Then

$$\mathbb{R} \oplus P_3 \simeq \mathbb{H},$$

so the corresponding subalgebra is associative. Its orthogonal complement

$$Q_4 = P_3^{\perp}$$

is coassociative in the calibrated G_2 sense [6, 5]. Thus the imaginary octonions split as

$$\mathrm{Im}\,\mathbb{O} = P_3 \oplus Q_4.$$

4.2 Clock line and Lorentzian signature

Because the standard G_2 metric is Riemannian [4, 5], the present model introduces as an own dynamical assumption that a Lorentzian metric requires an independent clock line

$$L_t = \mathrm{span}(u).$$

Let τ_μ be the corresponding clock one-form and let $h_{\mu\nu}$ be the spatial metric induced by g_φ on P_3. More explicitly, the clock data are assumed to satisfy

$$\tau_\mu u^\mu = 1, \qquad h_{\mu\nu} u^\nu = 0, \qquad h_{\mu\nu} > 0 \quad \text{on } P_3.$$

The effective Lorentzian metric is then

$$g^{\mathrm{eff}}_{\mu\nu} = -\tau_\mu \tau_\nu + h_{\mu\nu}.$$

Under these normalization and orthogonality assumptions, $g^{\mathrm{eff}}_{\mu\nu}$ has signature $(-+++)$ on the projected four-dimensional spacetime. This makes explicit where the Lorentzian sign enters: the three spatial directions come from an associative G_2 plane, while the negative direction is supplied by the clock field.

4.3 Low-energy stability condition

A low-energy associative sector is selected by a potential of the schematic form

$$V_{\mathrm{eff}} = V_{\mathrm{loc}} + \lambda \|A(\Psi_1, \Psi_2, \Psi_3)\|^2 + \frac{M_Q^2}{2} \sum_I \|\Pi_Q \Psi_I\|^2 + \frac{M_{\mathrm{mis}}^2}{2} \|\varphi|_{P_3} - \mathrm{vol}_{P_3}\|^2.$$

Here Π_Q projects onto the coassociative sector. The four terms have distinct roles:

V_{loc} :	selects the local vacuum and mass spectrum,
$\lambda \|A\|^2$:	suppresses genuinely non-associative excitations,
$M_Q^2 \|\Pi_Q \Psi\|^2$:	gaps coassociative modes,
$M_{\mathrm{mis}}^2 \|\varphi|_{P_3} - \mathrm{vol}_{P_3}\|^2$:	selects calibrated associative planes.

At energies

$$E \ll M_Q, \qquad E \ll M_{\mathrm{mis}}, \qquad \epsilon_A(E) \ll 1,$$

the effective dynamics is confined to

$$L_t \oplus P_3,$$

and ordinary four-dimensional Lorentzian physics is recovered.

5 Master Action and Coupling Dimensions

The effective projected action is taken in the form. Its Einstein-Hilbert part is the established low-energy gravitational structure, whereas the octonionic multiplet, associator potential, and projection-dependent nonminimal term are model-specific theoretical assumptions:

$$S[g, \Psi_I] = \int_{M_4} d^4x\sqrt{-g}\left[\frac{1}{2\kappa}(R - 2\Lambda) - \sum_{I=1}^{3} \frac{\alpha_I}{2} g^{\mu\nu}\langle \nabla_\mu \Psi_I, \nabla_\nu \Psi_I\rangle - V_{\rm loc}(\Psi_1, \Psi_2, \Psi_3) - \lambda\|A(\Psi_1, \Psi_2, \Psi_3)\|^2$$

The role of each term is:

$\frac{1}{2\kappa}R$:	leading metric dynamics after Lorentzian projection,
Λ :	vacuum curvature contribution,
$\langle \nabla\Psi, \nabla\Psi\rangle$:	propagation of the octonionic multiplet,
$V_{\rm loc}$:	local vacuum selection,
$\lambda\|A\|^2$:	minimal positive non-associative invariant,
$\xi R\langle\Psi, \Psi\rangle$:	leading nonminimal curvature feedback.

In natural units $c = \hbar = 1$, canonical normalization gives

$$[\Psi_I] = 1, \qquad [A(\Psi_1, \Psi_2, \Psi_3)] = 3, \qquad [\|A\|^2] = 6,$$

and therefore

$$[\lambda] = -2.$$

It is useful to write

$$\lambda = \frac{\eta_A}{M_*^2},$$

where M_* is the non-associative scale and η_A is dimensionless. The dimensionless expansion parameter is

$$\epsilon_A(E) \sim \eta_A \left(\frac{E}{M_*}\right)^2.$$

Low-energy consistency requires

$$\epsilon_A(E) \ll 1$$

in all currently tested regimes.

The gravitational coupling is

$$\kappa = \frac{8\pi G}{c^4}$$

in SI units, or $\kappa = 8\pi G$ in natural units. The nonminimal curvature term modifies the effective gravitational coupling:

$$\frac{1}{2\kappa_{\text{eff}}} = \frac{1}{2\kappa} - \xi \sum_{I=1}^{3} \langle \Psi_I, \Psi_I \rangle.$$

Consistency with precision weak-field tests of general relativity demands [14]

$$\left| 2\kappa\xi \sum_I \langle \Psi_I, \Psi_I \rangle \right| \ll 1$$

in solar-system and laboratory environments.

6 Projected Field Equations

Variation with respect to the emergent metric gives the projected Einstein-type equations; the variational structure follows the standard general-relativistic form, while the stress-tensor components below are the model-specific octonionic additions:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T^{\text{eff}}_{\mu\nu},$$

where

$$T^{\text{eff}}_{\mu\nu} = T^{\text{kin}}_{\mu\nu} + T^{\text{loc}}_{\mu\nu} + T^{\text{assoc}}_{\mu\nu} + T^{\xi}_{\mu\nu}.$$

For the kinetic term one obtains

$$T^{\text{kin}}_{\mu\nu} = \sum_I \alpha_I \left[\langle \nabla_\mu \Psi_I, \nabla_\nu \Psi_I \rangle - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \langle \nabla_\rho \Psi_I, \nabla_\sigma \Psi_I \rangle \right].$$

The local potential gives

$$T^{\text{loc}}_{\mu\nu} = -g_{\mu\nu} V_{\text{loc}}.$$

If the associator norm contains no additional explicit metric dependence beyond the projected scalar density, the leading contribution is

$$T^{\text{assoc}}_{\mu\nu} = -g_{\mu\nu} \lambda \| A(\Psi_1, \Psi_2, \Psi_3) \|^2.$$

More generally, additional metric-dependence terms can arise from the projection maps, covariant derivatives, or internal contractions.

For

$$F = \sum_I \langle \Psi_I, \Psi_I \rangle,$$

the nonminimal term contributes schematically

$$T^{\xi}_{\mu\nu} = 2\xi \left[F G_{\mu\nu} + g_{\mu\nu} \Box F - \nabla_\mu \nabla_\nu F \right].$$

The term proportional to $FG_{\mu\nu}$ may be shifted to the left-hand side and interpreted as the effective coupling κ_{eff}.

The variation with respect to the fields gives

$$\alpha_I \Box_g \Psi_I - \frac{\partial V_{\text{loc}}}{\partial \Psi_I} - 2\lambda D_I^\dagger A(\Psi_1, \Psi_2, \Psi_3) - 2\xi R \Psi_I = 0, \qquad I = 1, 2, 3.$$

The adjoint maps $D_I^\dagger$ are defined by

$$\langle A, A(\Psi_1, \dots, \delta\Psi_I, \dots, \Psi_3) \rangle = \left\langle D_I^\dagger A, \delta\Psi_I \right\rangle.$$

This is the mathematically correct trilinear replacement for any schematic single-field associator equation.

7 Step-by-Step Recovery of the Schwarzschild Exterior

Consider a static, spherically symmetric compact object. The exterior derivation below follows the classical Schwarzschild vacuum argument [7], reinterpreted here as the associative projection limit of the octonionic model. The assumptions defining the exterior region are:

$$r > R_{\text{core}}, \qquad A(\Psi_1, \Psi_2, \Psi_3) \approx 0, \qquad \nabla_\mu \Psi_I \approx 0, \qquad V_{\text{loc}} \approx 0.$$

Then

$$T^{\text{eff}}_{\mu\nu} \approx 0.$$

For $\Lambda = 0$ on black-hole scales, the projected metric equations reduce to

$$G_{\mu\nu} = 0.$$

The most general static, spherically symmetric line element can be written as

$$ds^2 = -e^{2\Phi(r)} c^2 dt^2 + e^{2\Lambda_r(r)} dr^2 + r^2 d\Omega^2.$$

The vacuum equations imply [7]

$$e^{-2\Lambda_r(r)} = 1 - \frac{2GM}{c^2 r},$$

and

$$\Phi'(r) = \frac{GM/c^2}{r^2\left(1 - \frac{2GM}{c^2 r}\right)}.$$

Asymptotic flatness fixes the integration constant and gives

$$e^{2\Phi(r)} = 1 - \frac{2GM}{c^2 r}.$$

Therefore

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

The horizon scale is

$$r_s = \frac{2GM}{c^2}.$$

In the present framework this result is interpreted as a low-energy projection result. It is not derived from a direct algebraic scaling of the associator alone. Rather, it follows because the associative exterior projection reproduces Einstein-type vacuum dynamics.

8 Interior Associator Saturation

The difference from classical general relativity appears in the interior. The idea that black-hole singularities may be replaced by regular cores has established precedents in regular black-hole and loop-inspired scenarios [8, 9, 10]; the specific associator-saturation mechanism below is the author's own speculative extension. In the Schwarzschild solution the curvature invariants diverge as $r \to 0$. In the octonionic model, the interior probes the non-associative sector. The relevant invariant is

$$\mathcal{A}^2(r) = \|A(\Psi_1, \Psi_2, \Psi_3)\|^2.$$

A regular black-hole configuration requires a bounded associator sector,

$$\mathcal{A}^2(r) \leq \mathcal{A}^2_{\max}.$$

This bound may originate from nonlinear G_2 constraints, a bounded effective potential, or the dynamical return toward stable associative sectors.

The corresponding energy density is written as

$$\rho_A(r) = \lambda\mathcal{A}^2(r) + \rho_{\mathrm{loc}}(r) + \rho_{\mathrm{kin}}(r),$$

where the first term is the dominant non-associative contribution in the simplest strong-field approximation. If kinetic and local terms also remain finite, then

$$\rho_A(r) < \infty.$$

In the saturated core,

$$\mathcal{A}^2(r) \to \mathcal{A}^2_{\max},$$

so that

$$\rho_A(r) \to \rho_0 \sim \lambda \mathcal{A}^2_{\max}.$$

A finite central density produces a de-Sitter-like interior. The effective metric function near $r = 0$ becomes

$$f(r) \simeq 1 - \frac{r^2}{\ell_A^2},$$

with

$$\ell_A^{-2} = \frac{8\pi G}{3c^4}\rho_0 \sim \frac{8\pi G}{3c^4}\lambda \mathcal{A}^2_{\max},$$

when ρ_0 denotes an energy density. If instead $\rho_0^{\text{mass}} = \rho_0/c^2$ denotes a mass density, the equivalent expression is

$$\ell_A^{-2} = \frac{8\pi G}{3c^2}\rho_0^{\text{mass}}.$$

In natural units this is equivalently

$$\ell_A^{-2} \sim \frac{\kappa}{3}\lambda \mathcal{A}^2_{\max},$$

up to numerical factors depending on the precise stress-tensor normalization.

Thus the classical singularity is replaced by a finite-curvature associator core.

9 Effective Regular Metric and Mass Function

A useful projected effective metric is of the same static regular-black-hole form used in comparison models [8, 9, 10]:

$$ds^2 = -f(r)c^2dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2,$$

with

$$f(r) = 1 - \frac{2Gm(r)}{c^2 r}.$$

This one-function form, equivalently $g_{tt}g_{rr} = -c^2$ in the present coordinates, is not the most general static spherically symmetric metric. It assumes an effective radial equation of state

$$T^t{}_t = T^r{}_r, \qquad \text{equivalently} \qquad p_r = -\rho$$

for the regular core sector, as in many de-Sitter-core regular-black-hole models. Without this additional assumption, the general projected ansatz would contain two independent functions,

$$ds^2 = -e^{2\Phi(r)}c^2dt^2 + \left(1 - \frac{2Gm(r)}{c^2r}\right)^{-1} dr^2 + r^2d\Omega^2.$$

The mass function is defined by

$$m(r) = \frac{4\pi}{c^2} \int_0^r \rho_A(r')r'^2dr'$$

if ρ_A denotes an energy density. In units where ρ_A is a mass density, the factor c^{-2} is omitted:

$$m(r) = 4\pi \int_0^r \rho_A^{\text{mass}}(r')r'^2dr'.$$

This distinction should be fixed consistently in a final numerical implementation.

For large r,

$$m(r) \to M, \qquad f(r) \to 1 - \frac{2GM}{c^2r},$$

so the Schwarzschild exterior is recovered.

For small r, finite associator energy density implies

$$m(r) \simeq \frac{4\pi}{3c^2}\rho_0 r^3.$$

Then

$$f(r) \simeq 1 - \frac{8\pi G\rho_0}{3c^4}r^2$$

if ρ_0 is an energy density, or

$$f(r) \simeq 1 - \frac{8\pi G\rho_0^{\text{mass}}}{3c^2}r^2$$

if ρ_0^{mass} is a mass density. In relativistic notation this is the standard de-Sitter-core form

$$f(r) = 1 - \frac{\Lambda_{\text{eff}}}{3}r^2,$$

with

$$\Lambda_{\text{eff}} = \frac{8\pi G}{c^4}\rho_0.$$

10 Curvature Invariants of the Regular Core

For the de-Sitter-like core

$$f(r) = 1 - \frac{r^2}{\ell_A^2},$$

the effective cosmological constant is

$$\Lambda_{\text{eff}} = \frac{3}{\ell_A^2}.$$

The curvature invariants are finite:

$$R = 4\Lambda_{\text{eff}} = \frac{12}{\ell_A^2},$$

$$R_{\mu\nu}R^{\mu\nu} = 4\Lambda_{\text{eff}}^2 = \frac{36}{\ell_A^4},$$

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{8}{3}\Lambda_{\text{eff}}^2 = \frac{24}{\ell_A^4}.$$

Thus the regularity condition is not merely qualitative. It is explicitly controlled by the finite associator length scale ℓ_A.

11 Matching Between Exterior and Interior

A complete regular black-hole model must match the de-Sitter-like core to the Schwarzschild exterior, as is also required in phenomenological regular black-hole constructions [8]. This can be done by choosing a smooth density profile such as

$$\rho_A(r) = \rho_0 \exp\left(-\frac{r^3}{r_0^3}\right)$$

or, more generally,

$$\rho_A(r) = \frac{\rho_0}{1 + (r/r_0)^n}, \qquad n > 3.$$

The condition $n > 3$ ensures finite total mass. The total mass is

$$M = \frac{4\pi}{c^2}\int_0^\infty \rho_A(r) r^2 dr.$$

The transition scale r_0 is fixed by the requirement that this integral produce the observed mass M.

A physically acceptable profile must satisfy:

$$\rho_A(0) = \rho_0 < \infty,$$

$$m(r) \sim r^3 \quad (r \to 0),$$

$$m(r) \to M \quad (r \to \infty),$$

$$f(r) \to 1 - \frac{2GM}{c^2 r} \quad (r \to \infty).$$

These conditions guarantee a regular core and the correct exterior limit.

12 Horizon as a Projected Phase Boundary

The horizon is defined by

$$f(r_h) = 0.$$

To leading order,

$$r_h \simeq r_s = \frac{2GM}{c^2}.$$

If the interior correction is small near the outer horizon, one may write

$$f(r) = 1 - \frac{r_s}{r} + \delta f_A(r),$$

with

$$|\delta f_A(r_h)| \ll 1.$$

The corrected horizon is

$$r_h = r_s + \delta r_h.$$

Expanding $f(r_h) = 0$ around r_s gives

$$0 = f(r_s) + f'_{\mathrm{GR}}(r_s)\delta r_h + \delta f_A(r_s),$$

where

$$f'_{\mathrm{GR}}(r_s) = \frac{1}{r_s}.$$

Therefore

$$\delta r_h \simeq -r_s\, \delta f_A(r_s).$$

This shows explicitly that the horizon shift is small if the associator correction at the outer horizon is small.

In the octonionic interpretation, the horizon is not a fundamental singular surface. This is a model-specific interpretation, distinct from but phenomenologically constrained by observed horizon-scale images of M87* and Sgr A* [11, 12]. It is the locus where the exterior associative spacetime projection changes causal character. The deeper octonionic degrees of freedom remain well defined across this projected boundary.

The physical regimes are

$$r > r_h : \quad \text{weak associator sector and classical spacetime projection,}$$

$$r \lesssim r_h : \quad \text{strong associator sector and nonlinear algebraic dynamics.}$$

Thus the horizon is a causal boundary of the emergent metric, while the underlying algebraic state remains regular.

13 Geodesic Completeness

If the effective metric is smoothly extendable through the core and all curvature invariants are bounded, the usual curvature-singularity obstruction to extending timelike and null geodesics is removed. This statement follows the logic of regular-core black-hole models [8, 9, 10], while the sufficient local regularity conditions below are formulated in the present associator language:

$$\rho_A(r) < \infty, \qquad p_A(r) < \infty, \qquad \mathcal{A}^2(r) \leq \mathcal{A}^2_{\text{max}}.$$

For a radial timelike geodesic in the effective metric,

$$ds^2 = -f(r)c^2dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2,$$

the conserved energy per unit mass is

$$E = f(r)c^2\frac{dt}{d\tau}.$$

The normalization condition gives

$$-c^2 = -f(r)c^2\left(\frac{dt}{d\tau}\right)^2 + f(r)^{-1}\left(\frac{dr}{d\tau}\right)^2.$$

Using the conserved energy,

$$\left(\frac{dr}{d\tau}\right)^2 = \frac{E^2}{c^2} - c^2f(r).$$

Near the regular core,

$$f(r) \simeq 1 - \frac{r^2}{\ell_A^2},$$

so the right-hand side remains finite. Therefore the radial velocity remains finite and the proper time evolution does not end at a curvature divergence. The same local conclusion holds for null geodesics with an affine parameter. Full global geodesic completeness, however, requires an explicit smooth extension of the matched interior–exterior geometry and must be checked for the complete solution, not merely for the local de-Sitter-core expansion.

14 Information Conservation

Within the present speculative model, information is not destroyed at a classical singularity because no such singular endpoint exists in the fundamental algebraic description. The information is encoded in the octonionic field configuration and in the associator structure:

$$\Psi_I(x), \qquad A(\Psi_1, \Psi_2, \Psi_3).$$

A possible tensorial structure for an information-sensitive current is

$$J^{\mu}_{\text{info}} \sim \epsilon^{\mu\nu\rho\sigma} \left\langle A(\Psi_1, \Psi_2, \nabla_\nu \Psi_3), \nabla_\rho \Psi_I \right\rangle u_\sigma.$$

At the present level this must not be interpreted as a derived Noether current. A conservation law of the form

$$\nabla_\mu J^{\mu}_{\text{info}} = 0$$

can only be asserted after the exact symmetry of the full octonionic effective action has been identified. The expression above is therefore a schematic covariant candidate, not yet a proven conserved current.

The physical interpretation is:

$$\text{classical singularity absent} \quad \Rightarrow \quad \text{no endpoint where algebraic information is destroyed.}$$

Hawking radiation may still appear thermal at leading semiclassical order, but the underlying algebraic evolution can remain information-preserving; this is a model claim that requires a dedicated derivation of the quantum state and backreaction.

15 Thermodynamic Interpretation

At leading order the projected horizon entropy is the standard Bekenstein–Hawking expression, written here as the standard semiclassical benchmark for the projected horizon [15, 16].

$$S_{\text{BH}} = \frac{k_B c^3 A_h}{4G\hbar}, \qquad A_h = 4\pi r_h^2.$$

The microscopic interpretation differs from a purely geometric one. The entropy counts the number of octonionic configurations that project to the same exterior black-hole geometry:

$$S_{\text{BH}} \sim \log N_{\text{proj}}.$$

Associator fluctuations may induce subleading corrections,

$$S = S_{\text{BH}} + \Delta S_A,$$

with

$$\Delta S_A \sim \alpha_A \log S_{\rm BH} + O(S_{\rm BH}^{-1}).$$

The coefficient α_A is model-dependent and must be computed from the spectrum of projected associator modes.

The Hawking temperature remains, at leading order, the standard Schwarzschild value [16].

$$T_H = \frac{\hbar c^3}{8\pi G M k_B},$$

provided the outer horizon is Schwarzschild-like. A small horizon correction

$$r_h = r_s + \delta r_h$$

induces

$$\frac{\delta T_H}{T_H} \simeq -\frac{\delta r_h}{r_s}$$

to leading order.

16 Observable Signatures

The model is designed to agree with general relativity in the weak-field exterior region, in accordance with precision tests of general relativity [14]. Deviations are expected only when associator corrections are non-negligible. The relevant small parameters are

$$\epsilon_A(E) \sim \eta_A \left(\frac{E}{M_*}\right)^2, \qquad \frac{\ell_A^2}{r_s^2}, \qquad \kappa\xi F.$$

16.1 Photon Ring and Shadow

The Schwarzschild photon-sphere radius is the standard value for the Schwarzschild exterior [7]:

$$r_{\rm ph}^{\rm GR} = \frac{3GM}{c^2} = \frac{3}{2} r_s.$$

For a general metric function

$$ds^2 = -f(r)c^2dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2,$$

the photon sphere satisfies

$$r f'(r) - 2f(r) = 0.$$

Let

$$f(r) = 1 - \frac{r_s}{r} + \delta f_A(r).$$

Writing

$$r_{\rm ph} = r_{\rm ph}^{\rm GR} + \delta r_{\rm ph},$$

one obtains to first order

$$\delta r_{\rm ph} \simeq \frac{r_{\rm ph}^2}{3r_s} \left[r_{\rm ph}\delta f_A'(r_{\rm ph}) - 2\delta f_A(r_{\rm ph})\right]_{r_{\rm ph}=3r_s/2}.$$

Equivalently,

$$r_{\rm ph} = r_{\rm ph}^{\rm GR}(1+\delta_{\rm ph}), \qquad |\delta_{\rm ph}| \ll 1.$$

The sign and magnitude of $\delta_{\rm ph}$ are not universal in the present effective formulation. A robust test is a correlated pattern among photon-ring radius, near-horizon redshift, and ringdown data, with horizon-scale imaging and black-hole spectroscopy providing the natural observational comparison classes [11, 12, 13].

16.2 Ringdown

Quasinormal modes are sensitive to the exterior geometry and, at subleading order, to boundary conditions associated with the regular interior [13]. One may parameterize

$$\omega_{\rm QNM} = \omega_{\rm GR}(1+\delta_{\rm QNM}),$$

with the phenomenological scaling estimate

$$\delta_{\rm QNM} = O\left(\frac{\ell_A^2}{r_s^2}\right)$$

for a compact core scale $\ell_A \ll r_s$. This scaling is dimensional and model-motivated rather than derived from a full perturbation calculation. The precise coefficient and even the leading power require solving the perturbation equations for the matched regular geometry. For astrophysical black holes the correction is expected to be small, but it may become relevant in precision ringdown spectroscopy or for small primordial black holes.

16.3 Absence of Strong Hard-Surface Echoes

In the simplest speculative version of the present model, the octonionic black hole does not replace the horizon by a hard reflective surface. The horizon is a phase boundary of the projected spacetime, not a material shell. Therefore the simplest version of the model predicts no strong late-time gravitational-wave echoes of the type expected from reflective exotic compact objects. Weak dispersive effects are not excluded.

16.4 Nonthermal Information Leakage

Since the fundamental octonionic dynamics is assumed to be norm-preserving, the present model suggests that Hawking radiation should not be exactly thermal at the deepest level. This is a theoretical expectation of the model, not an experimentally established statement. The expected nonthermal correction is entropy-suppressed,

$$\Delta_{\mathrm{nonthermal}} \sim \frac{1}{S_{\mathrm{BH}}}.$$

This is negligible for astrophysical black holes but may be more relevant for microscopic or primordial black holes.

17 Consistency with Low-Energy Tests

The model remains consistent with standard tests only if the exterior associative projection satisfies the weak-field bounds summarized in precision tests of general relativity [14]:

$$A(\Psi_1, \Psi_2, \Psi_3) \approx 0, \qquad \nabla_\mu \Psi_I \approx 0, \qquad \delta g^{(A)}_{\mu\nu} \ll 1$$

in weak-field environments.

In post-Newtonian language, associator-induced corrections must satisfy limits of the same order as current constraints on post-Newtonian deviations [14]:

$$|\delta\gamma_A| \ll 10^{-5}, \qquad |\delta\beta_A| \ll 10^{-4}.$$

These conditions are naturally met if M_* is sufficiently high, if associator excitations are gapped, or if the coassociative sector is dynamically suppressed.

For exterior Schwarzschild vacuum,

$$R_{\mu\nu} = 0, \qquad R = 0,$$

so a nonminimal coupling proportional only to $R\langle\Psi, \Psi\rangle$ produces no exterior Ricci-scalar correction. Corrections may still appear through the Weyl curvature, projection-dependent terms, or nonzero near-horizon associator profiles.

18 Relation to Other Regular Black-Hole Models

The phenomenology resembles several regular black-hole approaches, including Bardeen-type metrics, de-Sitter-core black holes, Planck-star scenarios, and loop-inspired interiors [8, 9, 10]. The distinctive feature of the present model is the origin of the regularization mechanism. The de-Sitter-like core is not added as an ad hoc matter distribution. It arises from the bounded non-associative associator sector:

$$\mathcal{A}^2 \le \mathcal{A}^2_{\rm max}.$$

Thus the singularity is avoided because the underlying algebraic structure does not allow an unlimited growth of the effective non-associative energy density.

19 Main Result

The complete black-hole chain is

$$\mathbb{O} \to A(\Psi_1, \Psi_2, \Psi_3) \to T^{\rm assoc}_{\mu\nu} \to G_{\mu\nu} = \kappa T^{\rm eff}_{\mu\nu} \to g^{\rm BH}_{\mu\nu}.$$

In the exterior,

$$T^{\rm eff}_{\mu\nu} = 0, \qquad g_{\mu\nu} \simeq g^{\rm Schwarzschild}_{\mu\nu}.$$

In the interior,

$$\mathcal{A}^2 \to \mathcal{A}^2_{\rm max}, \qquad g_{\mu\nu} \to g^{\text{de Sitter core}}_{\mu\nu}.$$

Therefore the central physical statement is

The Schwarzschild exterior is recovered, while the classical singularity is replaced by a finite associato

20 Conclusion

The octonionic black-hole model is a strong-field application of the broader dynamical projection framework. The exterior solution agrees with general relativity because the low-energy associative sector reproduces Einstein-type vacuum dynamics and hence the Schwarzschild exterior [7]. The interior differs because the trilinear non-associative associator sector becomes dominant and saturates at finite density.

The Schwarzschild radius remains

$$r_s = \frac{2GM}{c^2},$$

but its interpretation changes. It is the horizon scale of an emergent Lorentzian projection, not evidence for a fundamental geometric singularity. The underlying octonionic degrees of freedom remain regular beyond the projected horizon.

The main new element is the application of associator saturation to black-hole interiors. This is not an established result of the cited literature, but the specific theoretical proposal of the present work, to be compared phenomenologically with regular-core models and observational constraints [8, 9, 10, 11, 12, 13, 14]. This connects horizon formation, singularity avoidance, possible geodesic extension, information-preservation mechanisms, and possible near-horizon deviations within the same variational octonionic mechanism used to derive the low-energy emergence of spacetime, gravity, and quantum mechanics.

A Useful G_2 and Octonionic Identities

With

$$e_a e_b = -\delta_{ab} + f_{ab}{}^c e_c,$$

the basic G_2 identities include [1, 2, 4, 5, 6]

$$f_{amn} f_b{}^{mn} = 6\delta_{ab},$$

$$f_{abm} f_{cd}{}^m = \delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc} + \psi_{abcd},$$

where $\psi = *_\varphi \varphi$ is the dual G_2 four-form. These identities are used in reconstructing the internal metric and in identifying associative and coassociative subspaces.

B Dimensional Summary

In natural units,

$$[d^4x] = -4, \qquad [\mathcal{L}] = 4, \qquad [\partial_\mu] = 1, \qquad [R] = 2.$$

The kinetic term gives

$$[\Psi] = 1.$$

The trilinear associator gives

$$[A(\Psi_1, \Psi_2, \Psi_3)] = 3, \qquad [\|A\|^2] = 6.$$

Therefore

$$[\lambda] = -2, \qquad \lambda = \frac{\eta_A}{M_*^2}.$$

The dimensionless suppression factor is

$$\epsilon_A(E) \sim \eta_A \left(\frac{E}{M_*}\right)^2.$$

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