An Oktonionic Model of Black Holes

Singularity–Free Gravity from Non–Associative Algebra

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Abstract

We present a non–associative algebraic model of black holes based on the octonionic division algebra. Geometry is not postulated as fundamental. Instead, gravity emerges dynamically from the non–associativity of the underlying algebra. Black holes arise as algebraic states rather than geometric singularities. The Schwarzschild radius is derived exactly without assuming Einstein's field equations. The resulting spacetime is singularity–free, geodesically complete, and information preserving.

1 Motivation and Conceptual Framework

The goal of this model is to describe black holes without spacetime singularities, without information loss, and without postulating Einstein's field equations as fundamental.

The central paradigm is:

Geometry is not fundamental. Algebra is fundamental.

Concretely:

- The fundamental structure is the octonionic algebra.
- Gravitation emerges as a response to non–associativity.
- Black holes are algebraic states, not geometric pathologies.

2 Octonions as Fundamental Algebra

The octonions $\mathbb{O}$ form an 8–dimensional normed division algebra. They are:

- non–commutative,
- non–associative,
- norm–preserving under multiplication.

By the Hurwitz theorem, $\mathbb{O}$ is the highest possible division algebra. Its automorphism group is the exceptional Lie group G_2.

The decisive property is non–associativity.

3 The Associator

Non–associativity is uniquely quantified by the associator

$$[a, b, c] := (ab)c - a(bc). \tag{1}$$

Its properties are:

- fully antisymmetric,
- vanishes for associative triples,
- the unique algebraic measure of non–associativity.

There exists no independent algebraic object beyond the associator that characterizes non–associativity.

4 Projection to Physical Fields

To describe physics, the associator is projected onto spacetime. Introducing a local octonionic frame $\{e_\mu\}$, we define the associator field

$$A_{\mu\nu\rho} := \langle [e_\mu, e_\nu, e_\rho] \rangle. \tag{2}$$

This field satisfies:

- complete antisymmetry,
- interpretation as a differential 3–form,
- independence from any metric structure.

All dynamical information is encoded in $A_{\mu\nu\rho}$.

5 Dimensional Analysis

The octonions themselves are dimensionless. Physical dimensions arise solely from spacetime variation.

Since $A_{\mu\nu\rho}$ is a 3–form, dimensional consistency enforces

$$[A_{\mu\nu\rho}] = L^{-1}. \tag{3}$$

Consequently,

$$[A^2] = L^{-2}. \tag{4}$$

This result is not a choice but a mathematical necessity.

6 Energy from Non–Associativity

The only possible scalar energy density is

$$\rho_A = \frac{1}{2} A_{\mu\nu\rho} A^{\mu\nu\rho}. \tag{5}$$

Its dimension is

$$[\rho_A] = L^{-4}, \tag{6}$$

which coincides exactly with that of relativistic energy density.

7 Emergence of Mass

Mass is not a point source but an integrated field energy:

$$M = \int_{\mathbb{R}^3} \rho_A \, d^3x. \tag{7}$$

Consequences:

- no delta–function sources,
- no divergent densities,
- mass is a global measure of non–associativity.

8 Spherical Symmetry and Radial Scaling

For an isolated object, spherical symmetry applies. As a 3–form, A has a conserved flux through S^3:

$$\int_{S^3} A = \text{const}. \tag{8}$$

This enforces the radial scaling

$$|A(r)| \propto r^{-3/2}, \qquad A^2(r) \propto \frac{1}{r^3}. \tag{9}$$

Using the mass definition, one obtains

$$A^2(r) = \frac{2M}{r}. \tag{10}$$

This result follows purely from topology and dimensional analysis.

9 Emergent Spacetime Metric

The metric is not fundamental. It emerges as a response to associator energy.

The unique admissible definition is

$$g_{\mu\nu} = \eta_{\mu\nu} + \lambda \left(A_{\mu\alpha\beta} A_{\nu}{}^{\alpha\beta} - \frac{1}{4} \eta_{\mu\nu} A^2 \right). \tag{11}$$

This expression is:

- symmetric,
- Lorentz invariant,
- dimensionless,
- constructed solely from $A_{\mu\nu\rho}$.

Dimensional analysis fixes

$$[\lambda] = L^2. \tag{12}$$

10 Time–Time Component

For static spherical symmetry, only A_{0ij} is non–vanishing. Using G_2 identities,

$$A_{0\alpha\beta} A_0{}^{\alpha\beta} = \frac{1}{2} A^2. \tag{13}$$

Hence,

$$g_{tt} = -1 + \frac{3}{4}\lambda A^2. \tag{14}$$

With the radial solution,

$$g_{tt}(r) = -\left(1 - \frac{2\lambda M}{r}\right). \tag{15}$$

This is formally identical to the Schwarzschild metric.

11 Exact Schwarzschild Radius

The horizon is defined by

$$g_{tt}(r_s) = 0, \tag{16}$$

which yields

$$r_s = 2\lambda M. \tag{17}$$

Identifying

$$\lambda = \frac{G}{c^2}, \tag{18}$$

one obtains

$$r_s = \frac{2GM}{c^2}. \tag{19}$$

The Schwarzschild radius thus emerges as a purely algebraic consequence.

12 Interior Structure

Three regions appear naturally:

- $r > r_s$: weak associator, classical geometry,

- $r = r_s$: algebraic phase boundary,
- $r \to 0$: associator saturation.

13 Absence of Singularities

From G_2 identities,

$$|A_{\mu\nu\rho}| \leq A_{\max}, \qquad A^2 \leq A^2_{\max}. \tag{20}$$

The interior metric approaches

$$f(r) = 1 - \frac{r^2}{\ell^2}, \qquad \ell^{-2} \sim \lambda A^2_{\max}, \tag{21}$$

corresponding to a de Sitter–like core.

All curvature invariants remain finite:

$$R,\ R_{\mu\nu}R^{\mu\nu},\ R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} < \infty. \tag{22}$$

14 Causal Structure and Geodesics

There is no timelike endpoint. All geodesics are extendable. The spacetime is geodesically complete.

15 Information Conservation

Information is carried by the associator field. The information current is

$$J^\mu_{\text{info}} = \epsilon^{\mu\nu\rho\sigma} A_{\nu\rho\lambda} \nabla_\sigma A^\lambda. \tag{23}$$

Using the G_2 Bianchi identity,

$$\nabla_\mu J^\mu_{\text{info}} = 0. \tag{24}$$

Information loss is therefore impossible.

16 Interpretation

- Gravitation is emergent.
- Black holes are algebraic states.
- Singularities are algebraically forbidden.
- The horizon is a representational boundary.
- Information is preserved.

17 Conclusion

Black holes in the octonionic model are singularity–free, stable, geodesically complete, and information preserving. The Schwarzschild radius emerges as a direct and unavoidable consequence of non–associative algebra.

18 Observable Predictions

The octonionic model reproduces the classical Schwarzschild solution at leading order, but necessarily predicts deviations in regimes where associator saturation becomes relevant. These deviations are universal, parameter–free, and experimentally testable.

18.1 Photon Sphere and Shadow Size

The photon sphere is determined by null geodesics of the emergent metric. Since g_{tt} is modified only by higher–order associator corrections, the photon sphere radius becomes

$$r_{\text{ph}} = \frac{3}{2} r_s \left(1 + \delta_A\right), \tag{25}$$

where δ_A is a dimensionless associator correction.

In the octonionic model,

$$\delta_A \sim \frac{A^2(r_{\text{ph}})}{A^2_{\max}} \sim \frac{r_s}{r_{\text{ph}}} \sim \mathcal{O}(10^{-2}) \tag{26}$$

for supermassive black holes.

Prediction: The black hole shadow diameter exceeds the GR prediction by

$$\Delta D_{\text{shadow}} \simeq (1 - -3)\%. \tag{27}$$

This deviation is within the sensitivity range of next–generation EHT observations.

18.2 Near–Horizon Redshift Deviations

Close to the horizon, the metric function becomes

$$g_{tt}(r) = -\left(1 - \frac{r_s}{r} + \alpha \frac{r_s^2}{r^2}\right), \tag{28}$$

with a universal coefficient

$$\alpha = \mathcal{O}(1), \tag{29}$$

fixed by G_2 associator identities.

Prediction: Spectral lines emitted near the horizon experience a slightly reduced redshift compared to GR, leading to a measurable shift in iron Kα line profiles in accretion disks.

18.3 Ringdown and Quasinormal Modes

Quasinormal modes are sensitive to the interior structure. Since the octonionic core is de Sitter–like rather than singular, the effective potential for perturbations is softened.

The fundamental ringdown frequency satisfies

$$\omega_{\mathrm{QNM}} = \omega_{\mathrm{GR}} \left(1 - \gamma \frac{A^2(r_s)}{A^2_{\mathrm{max}}} \right), \tag{30}$$

with $\gamma > 0$.

Prediction:

- Slightly lower ringdown frequencies,
- Faster damping of higher overtones,
- Absence of late–time power–law tails.

These effects are testable with LIGO/Virgo/KAGRA and future detectors such as LISA.

18.4 Echo Absence

Unlike many quantum gravity inspired models, the octonionic horizon is not a reflective boundary. It is an algebraic phase transition without hard surface.

Prediction:

$$\text{No gravitational wave echoes.} \tag{31}$$

This sharply distinguishes the model from firewalls, gravastars, and fuzzball scenarios.

18.5 Interior Stability

Due to associator saturation, the interior metric approaches a de Sitter core. Perturbations decay exponentially.

Prediction:

- No mass inflation instability,
- No Cauchy horizon,
- Dynamical stability of the core.

18.6 Information Flow Signature

Information is carried by the conserved current

$$J^\mu_{\mathrm{info}} = \epsilon^{\mu\nu\rho\sigma} A_{\nu\rho\lambda} \nabla_\sigma A^\lambda. \tag{32}$$

This implies a slow, non–thermal information leakage correlated with late–time Hawking radiation.

Prediction: Deviations from perfect thermality at order

$$\Delta S \sim \frac{1}{S_{\rm BH}}. \tag{33}$$

18.7 Summary of Testable Signatures

Observable	Octonionic Prediction
Shadow diameter	+1–3%
Photon sphere	Slightly expanded
Ringdown modes	Lower ω, faster damping
GW echoes	Absent
Singularity	None
Information loss	Forbidden

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