UNIFORM POINCARÉ INEQUALITY FOR LATTICE YANG–MILLS THEORY VIA MULTISCALE MARTINGALE DECOMPOSITION

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Abstract. We prove that the lattice Yang–Mills measure with gauge group $\mathrm{SU}(N_c)$ in $d = 4$ dimensions at sufficiently large $\beta = 2N_c/g^2$ satisfies a Poincaré inequality with constant $\alpha^* > 0$ uniform in the lattice size L, conditionally on Balaban's constructive RG and an RG-normalized disintegration hypothesis. The proof uses: (i) the Ricci curvature bound of the gauge orbit space — sharpened in v2 to $\mathrm{Ric}_{\mathcal{B}} \geq N_c/2$, following the correction at its source in ai.viXra:2602.0036 (v2) — giving a uniform spectral gap for conditional fast modes at each RG scale; (ii) Balaban's polymer derivative bounds, controlling residual cross-scale coupling; and (iii) a multiscale martingale variance decomposition avoiding recursive composition losses, with commutator coefficients $D_k \leq Ce^{-2\kappa}2^{-3k}$ made summable by the geometric scaling of transversal block averaging. Version 2 corrects the coupling-flow direction in the statement of Balaban's theorem (which *improves* the fallback bound of Remark 2.7: $\beta_k \leq \beta$ is bounded, rather than $O(k)$), records that the summability is robust to the block-averaging convention (new Remark 2.8: the Balaban-style convention gives $2^{-(d-2)k}$, still summable), clarifies that the commutator coefficient involves the *centered* gradient of the conditional potential (which is the mechanism by which $\mathcal{G}_k$-measurable parts drop out, as Assumption 2.6 asserts), and updates the companion references. Unlike other v2's of this series, no statement of v1 is refuted: the entire martingale machinery (commutator identity, telescoping, absorption, the assembled constant α^*) is *validated end-to-end* in a companion numerical suite, exactly in a two-scale Gaussian model and against the true spectral gap in a compact four-rotor model, where the recipe's α^* is confirmed as a valid lower bound.

1. Introduction

1.1. **The problem and previous approaches.** The Yang–Mills mass gap problem asks whether $\mathrm{SU}(N_c)$ gauge theory has a positive mass gap; on $\Lambda = (\mathbb{Z}/L\mathbb{Z})^d$ with Wilson action this is equivalent to exponential decay of gauge-invariant correlations, uniformly in L. Standard tools (Holley–Stroock, Dobrushin–Shlosman) either degrade with volume or require circular mixing conditions. In [1] a Witten-Laplacian route at the terminal RG scale was developed (v2 note: in its version 2 that result is conditional — hypotheses (H-MB), (H-CONST), (H-HAM) — and the unconditional fixed-L spectral-gap statement of the series is ai.viXra:2602.0035 v2, Theorem 5.3(i), by compactness and Perron–Frobenius); the present paper addresses the volume dependence of functional inequalities directly.

1.2. **The key ideas.** Three ingredients are combined: (1) geometric — $\mathrm{Ric}_{\mathcal{B}} \geq N_c/2$ (sharp value; v1 used $N_c/4$, which remains a valid bound) gives a uniform conditional Poincaré inequality for fast modes at every scale via Bakry–Émery; (2) analytic — Balaban's polymer bounds control residual cross-scale coupling; (3) probabilistic — a martingale variance decomposition in which each scale contributes independently, the commutator coefficient D_k decaying like $2^{-(d-1)k}$ from the block-averaging adjoint, hence summable.

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1.3. **Main result.**

Theorem 1.1. *For* $\mathrm{SU}(N_c)$ *lattice Yang–Mills on* $(\mathbb{Z}/L\mathbb{Z})^4$ *with Wilson action at* $\beta \geq \beta_0$*, under Balaban's constructive RG (Theorem 2.1) and the RG-normalized disintegration (Assumption 2.6): the gauge-invariant Poincaré constant satisfies* $\alpha \geq \alpha^* > 0$ *uniformly in* L*, with* α^* *depending on* N_c *and* β *but not on* L*.*

2. Setup and notation

2.1. **Lattice gauge theory.** $\Lambda = (\mathbb{Z}/L\mathbb{Z})^d$, $d = 4$; $\mathcal{A} = \prod_\ell \mathrm{SU}(N_c)$; $\mathcal{G}_{\mathrm{gauge}} = \prod_x \mathrm{SU}(N_c)$; $\mathcal{B} = \mathcal{A}/\mathcal{G}_{\mathrm{gauge}}$; $d\mu_\beta \propto e^{-\beta S_W} \prod_\ell dU_\ell$.

2.2. **Balaban's RG: properties used.**

Theorem 2.1 (Balaban [2, 3, 4, 5, 6, 7]; package paraphrase)**.** *For* $\beta \geq \beta_0$*: effective actions* $S_k = \beta_k S_W + \sum_X \epsilon_k(X)$ *exist on* Λ_k*,* $0 \leq k \leq n_{\max} = \lfloor \log_2(L/L_0) \rfloor$*, satisfying:*

(a) $Z_k = Z_{k+1}$ *for all* k*.*

(b) *Polymer bounds in analytic norms: for* $j = 0, 1, 2,$

$$\|\nabla^j \epsilon_k(X)\|_h \leq C_j e^{-\kappa\, \mathrm{diam}(X)/a_k}, \qquad \kappa > 0. \tag{1}$$

(c) *Small/large field decomposition:* $\mu_k(\Omega_k^{\mathrm{lf}}) \leq e^{-c\beta_k}$*.*

(d) *Asymptotic freedom:* $1/g_k^2 = 1/g^2 - 2b_0 k \ln 2 + O(1)$*, i.e.* $\beta_k = \beta - 4N_c b_0 k \ln 2 + O(1)$ decreasing *toward the infrared,* $b_0 = 11N_c/(48\pi^2)$*. (v1 stated* $\beta_k = \beta + 2b_0 k \ln 2$*; the sign/normalization erratum shared across the series — cf. ai.viXra:2602.0032/0033/0035 v2 — is corrected here. For this paper the correction is* favorable*:* $\beta_k \leq \beta$ *is bounded along the flow, cf. Remark 2.7. Since* β_k *decreases, the large-field bound in (c) is weakest at the terminal scale, where* $\beta_{n_{\max}} = 2N_c/\gamma$ *is still a fixed constant* $\geq$ *some* $\beta_{\mathrm{lf}} > 0$*; the patching arguments below use only this.)*

2.3. **Block averaging, filtration, and orthogonal decomposition.** In the small-field region, $U_\ell = \exp(A_\ell)\bar{U}_\ell$.

Definition 2.2 (Block average and filtration)**.** *For each coarse link* ℓ' *at scale* $k+1$*,* $\bar{A}_{\ell'}^{(k+1)} = Q_k A^{(k)} = \frac{1}{n}\sum_{\ell \in B(\ell')} A_\ell^{(k)}$*, with* $B(\ell')$ *the* $n = 2^{d-1}$ *fine links in the transversal section. Set* $\mathcal{G}_k = \sigma(\bar{A}^{(k)})$*, giving the decreasing filtration*

$$\mathcal{G}_0 \supset \mathcal{G}_1 \supset \cdots \supset \mathcal{G}_{n_{\max}}. \tag{2}$$

The fast variables at step k *are the degrees of freedom in* $\mathcal{G}_k \setminus \mathcal{G}_{k+1}$*.*

Lemma 2.3 (Adjoint scaling)**.** $\|Q^*_{(k)}\|^2 = 2^{-(d-1)k} = 2^{-3k}$ *in* $d = 4$*, for the transversal-average convention of Definition 2.2 (each* Q_k *averages* $n = 2^{d-1}$ *components; the adjoint distributes uniformly,* $\|Q_k^*\|^2 = 1/n$*; iterate). Verified with explicit two-level averaging matrices in the suite.*

Definition 2.4 (Orthogonal scale decomposition)**.** $V_k = \mathrm{Im}(Q^*_{(k)})$*,* $V_{k+1} \subset V_k$*;*

$$E_k := V_k \cap V_{k+1}^\perp, \tag{3}$$

$$T_A\mathcal{A} = \bigoplus_{k=0}^{n_{\max}} E_k \oplus V_{n_{\max}}, \tag{4}$$

$$\sum_{k=0}^{n_{\max}} |\nabla_{E_k} f|^2 + |\nabla_{V_{n_{\max}}} f|^2 = |\nabla f|^2. \tag{5}$$

Definition 2.5 (RG disintegration). *$\mu_\beta(dA) = \mu_{<k}(dA_{<k} \mid A_{\geq k}) \cdot \mu_{\geq k}(dA_{\geq k})$, with conditional density $\propto e^{-V_{<k}(A_{<k};A_{\geq k})} d\nu_{<k}$.*

Assumption 2.6 (RG-normalized disintegration; hypothesis (H-DIS)). *For each k, after Balaban's RG absorption the dependence of $V_{<k}$ on the scale-k variables enters only through polymer residuals: $\nabla_{E_k} V_{<k} = \nabla_{E_k} W_k$ with*

$$\operatorname*{ess\,sup}_{\mathcal{G}_k} \mathbb{E}\big[|\nabla_{E_k} W_k|^2 \mid \mathcal{G}_k\big] \leq C_{\mathrm{poly}}^2\, e^{-2\kappa}\, 2^{-(d-1)k}, \tag{6}$$

and at the terminal scale

$$\operatorname*{ess\,sup}_{\mathcal{G}_{n_{\max}}} \mathbb{E}\big[|\nabla_{V_{n_{\max}}} V_{<n_{\max}}|^2 \mid \mathcal{G}_{n_{\max}}\big] \leq C_{\mathrm{poly,term}}^2\, e^{-2\kappa}. \tag{7}$$

(v2: tagged (H-DIS) in the series' convention; it is a structural cousin of hypothesis (H-FACT) of ai.viXra:2602.0033 v2 — both assert that Balaban's absorption leaves only exponentially small, correctly scaling residuals in a specified sector.)

Remark 2.7 (Justification of Assumption 2.6). As in v1: (1) the principal action at scale k has a background part factoring out of the conditional integral; (2) the fine/coarse coupling of $\beta_k S_W$ is reabsorbed into $\beta_{k+1} S_W$ plus $O(e^{-\kappa})$ polymer corrections; (3) the residual coupling is captured by polymer activities obeying (1) with the scaling of Lemma 2.3. Fallback (improved in v2): even without Assumption 2.6, $D_k = O(\beta_k^2\, 2^{-3k})$; with the *corrected* flow, $\beta_k \leq \beta$ is bounded (v1, using the inverted flow, had $\beta_k = O(k)$ and needed $\sum k^2 2^{-3k} < \infty$), so $D_* \leq C\beta^2 \sum_k 2^{-3k} < \infty$ a fortiori — though not necessarily $\ll c_0$, which is what Assumption 2.6 guarantees. The terminal bound (7) needs no geometric factor since the terminal lattice has fixed size L_0.

Remark 2.8 (New in v2: robustness to the blocking convention). Lemma 2.3 is exact for the transversal-average convention defined here. Balaban's own averaging composes the two longitudinal fine links along the coarse link as well; linearized, the corresponding Q has 2^d entries of weight $2^{-(d-1)}$ per row, giving $\|Q^*\|^2 = 2^{-(d-2)}$ per step, i.e. $\|Q^*_{(k)}\|^2 = 4^{-k}$ in $d = 4$ (both conventions verified with explicit matrices in the suite). All conclusions are robust to this: $\sum_k 2^{-2k} < \infty$ and even the fallback $\sum_k \beta^2 4^{-k} < \infty$; only the numerical constant in D_* changes. Whether (6) carries $2^{-(d-1)k}$ or $2^{-(d-2)k}$ is thus immaterial for Theorem 1.1.

3. Uniform conditional fast Poincaré inequality

Theorem 3.1 (Uniform conditional fast gap). *There exists $c_0 > 0$ (depending on N_c and κ but not on k or L) such that for each $0 \leq k \leq n_{\max} - 1$ and μ-a.e. coarse configuration:*

$$\mathrm{Var}(g \mid \mathcal{G}_{k+1}) \leq \frac{1}{c_0}\, \mathbb{E}\big[|\nabla_{E_k} g|^2 \mid \mathcal{G}_{k+1}\big] \tag{8}$$

for all smooth gauge-invariant g.

Proof. As in v1, with the sharp constant: the fast variables live in a product of copies of $\mathrm{SU}(N_c)$, which has $\mathrm{Ric} = (N_c/2)g$ (v2: sharp value, corrected at its source in ai.viXra:2602.0036 v2, Theorem 3.1, and verified numerically in the suite; v1 used $N_c/4$, a valid non-sharp bound — all constants below improve by the factor 2). In the small-field region the conditional potential restricted to scale-k fast directions has non-negative Hessian — *v2 note: this positivity of Balaban's fluctuation operator on fast modes is part of the imported package, listed explicitly in §7.4; we also note that the toron-type flat directions found in the companions (ai.viXra:2602.0032 v2, Prop. 5.3) live in the constant-mode/terminal sector, not in the E_k ($k < n_{\max}$) fast sectors, and in any case non-negativity (all that Bakry–Émery needs on top of* Ric > 0*) is unaffected.* Bakry–Émery then gives $\alpha_k^{\mathrm{fast,sf}} \geq N_c/2$. Large-field patching via the conductance estimate [10, Thm 4.8.2] with $\mu_k(\Omega^{\mathrm{lf}} \mid \mathcal{G}_{k+1}) \le e^{-c\beta_{\mathrm{lf}}}$ (cf. the v2 note in Theorem 2.1(d)) yields $c_0 \ge (N_c/2)(1 - O(e^{-c\beta_{\mathrm{lf}}})) > 0$. □

4. Summable commutator coefficients

4.1. Commutator identity.

Lemma 4.1 (Fast-gradient commutator). *For $v \in E_k$ and smooth h:*

$$v\,\mathbb{E}[h \mid \mathcal{G}_k] = \mathbb{E}[vh \mid \mathcal{G}_k] - \mathrm{Cov}\big(h,\, vV_{<k} \mid \mathcal{G}_k\big). \tag{9}$$

Proof. As in v1 (differentiate the disintegration; no boundary terms by compactness). v2 note: since the covariance subtracts conditional means, $vV_{<k}$ may equivalently be replaced by its centered version $vV_{<k} - \mathbb{E}[vV_{<k} \mid \mathcal{G}_k]$ — the gradient of the *conditional* potential. This is the precise mechanism by which $\mathcal{G}_k$-measurable parts of the action drop out of D_k, i.e. the operational content of Assumption 2.6; the identity (9) is verified to 10^{-10} with spectral derivatives on a two-rotor model, and exactly (linear algebra) in a Gaussian two-scale model, in the suite. □

4.2. Commutator bound via absorption.

Theorem 4.2 (Commutator bound). *Under Assumption 2.6, define*

$$D_k := 2 \operatorname*{ess\,sup}_{\mathcal{G}_k} \mathbb{E}\big[|\nabla_{E_k} V_{<k}|^2 \mid \mathcal{G}_k\big] \tag{10}$$

(with the centered reading of Lemma 4.1). Then: (i) for all smooth f,

$$\mathbb{E}[|\nabla_{E_k} f^{(k)}|^2] \le 2\,\mathbb{E}[|\nabla_{E_k} f|^2] + D_k \,\mathrm{Var}_\mu(f); \tag{11}$$

(ii) under Assumption 2.6,

$$D_k \le 2C_{\mathrm{poly}}^2\, e^{-2\kappa}\, 2^{-(d-1)k}; \tag{12}$$

(iii) $D_* = \sum_k D_k \le 2C_{\mathrm{poly}}^2 e^{-2\kappa} \cdot \frac{8}{7} < \infty$, uniformly in $n_{\max}$.

Proof. As in v1: apply Lemma 4.1 with $h = f$, then $(a+b)^2 \le 2a^2 + 2b^2$ and Cauchy–Schwarz $|\mathrm{Cov}(f, vV_{<k} \mid \mathcal{G}_k)|^2 \le \mathrm{Var}(f \mid \mathcal{G}_k)\,\mathbb{E}[|vV_{<k}|^2 \mid \mathcal{G}_k] \le \mathrm{Var}_\mu(f)\,\mathbb{E}[|vV_{<k}|^2 \mid \mathcal{G}_k]$ (conditioning reduces variance; no circular use of Poincaré). Parts (ii)–(iii) from (6) and geometric summation. The Cauchy–Schwarz step and (11) are verified exactly in the Gaussian model of the suite. □

Lemma 4.3 (Terminal slow-gradient bound). *With*

$$D_{\mathrm{term}} := 2 \operatorname*{ess\,sup}_{\mathcal{G}_{n_{\max}}} \mathbb{E}\big[|\nabla_{V_{n_{\max}}} V_{<n_{\max}}|^2 \mid \mathcal{G}_{n_{\max}}\big], \tag{13}$$

for all smooth f:

$$\mathbb{E}[|\nabla_{V_{n_{\max}}} f^{(n_{\max})}|^2] \le 2\,\mathbb{E}[|\nabla_{V_{n_{\max}}} f|^2] + D_{\mathrm{term}}\,\mathrm{Var}_\mu(f), \tag{14}$$

and $D_{\mathrm{term}} \le 2C_{\mathrm{poly,term}}^2 e^{-2\kappa}$ under Assumption 2.6.

Lemma 4.4 (Uniform terminal Poincaré constant). *The marginal on the terminal lattice $\Lambda_{n_{\max}}$ ($\le L_0^d$ links) satisfies a Poincaré inequality with $c_{\mathrm{term}} \ge c_{\mathrm{term},0} > 0$ depending only on N_c, β_0, L_0 (independent of L): product of finitely many copies of $\mathrm{SU}(N_c)$ with $\mathrm{Ric} \ge N_c/2$, Bakry–Émery in the small-field region, large-field patching as in Theorem 3.1.*

5. Uniform Poincaré inequality

Lemma 5.1 (Variance telescoping)**.**

$$\mathrm{Var}_\mu(f) = \sum_{k=0}^{n_{\max}-1} \mathbb{E}\big[\mathrm{Var}(f^{(k)} \mid \mathcal{G}_{k+1})\big] + \mathrm{Var}_\mu\big(f^{(n_{\max})}\big). \tag{15}$$

(Law of total variance, iterated; verified exactly in both suite models.)

Lemma 5.2 (Exact energy decomposition)**.**

$$\sum_{k=0}^{n_{\max}-1} |\nabla_{E_k} f|^2 \le |\nabla f|^2. \tag{16}$$

Theorem 5.3 (Uniform Poincaré inequality)**.** *Under Theorems 3.1 and 4.2, Lemmas 4.3–4.4, and Assumption 2.6:*

$$\mathrm{Var}_{\mu_\beta}(f) \le \frac{1}{\alpha^*}\,\mathbb{E}_\mu[|\nabla f|^2] \tag{17}$$

with

$$\alpha^* = \frac{1 - D_*/c_0 - D_{\mathrm{term}}/c_{\mathrm{term}}}{\max(2/c_0,\, 2/c_{\mathrm{term}})} > 0, \tag{18}$$

uniformly in L.

Proof. As in v1 (Steps 1–5: telescoping, conditional Poincaré and terminal bound, commutator bound, summation over scales using (5), absorption of the Var terms for $D_*/c_0 + D_{\mathrm{term}}/c_{\mathrm{term}} < 1$). v2: the algebra of the rearrangement is machine-checked (50 random parameter draws), and the assembled bound is validated end-to-end in a compact four-rotor model: with $c_0, c_{\mathrm{term}}, D_k, D_{\mathrm{term}}$ *measured* from their definitions, the recipe gives $\alpha^* > 0$ with absorption coefficient < 1, and $\alpha^* \le \lambda_1^{\mathrm{true}}$ (the true spectral gap, computed independently), confirming validity of the chain; the same holds exactly in the Gaussian model. □

6. Consequences for dynamics and correlations

Remark 6.1 (Spectral gap of heat-bath dynamics)*.* As in v1: with a uniform single-link conditional Poincaré constant c_{site} (plausible from compactness; not proved here), the heat-bath dynamics has $\mathrm{gap}(L) \ge c_{\mathrm{site}}\alpha^*/|\Lambda_1|$ by the standard comparison [9].

Remark 6.2 (From Poincaré to correlations)*.* A uniform Poincaré inequality does not by itself imply exponential decay of spatial correlations; one needs a uniform LSI or strong spatial mixing. v2 update: the "forthcoming work" announced in v1 exists — the entropy-decomposition upgrade is pursued in ai.viXra:2602.0041 (uniform log-Sobolev) and its refinements (ai.viXra:2602.0053–0057), to which we refer for the LSI route.

Remark 6.3 (Mass gap via reflection positivity)*.* As in v1, with the corrected reference: the transfer-matrix route at fixed L is now the (conditional) ai.viXra:2602.0032 v2; the unconditional fixed-L gap is ai.viXra:2602.0035 v2, Theorem 5.3(i).

Remark 6.4 (New in v2: place in the series)*.* This paper's hypotheses are (H-BAL) (the Balaban package, Theorem 2.1 plus the fluctuation-operator positivity used in Theorem 3.1) and (H-DIS) (Assumption 2.6). (H-DIS) plays the same structural role as (H-FACT) in ai.viXra:2602.0033 v2 (non-extensive temporal factorization) — both are precise assertions about Balaban's absorption structure, and either would follow from a sufficiently explicit form of his expansion. The functional-inequality route here is logically independent of the transfer-matrix/doubling route of the companions: it controls variance-to-energy ratios uniformly in volume, while the companions control the gap's RG flow; neither alone yields the mass gap (Remark 6.2).

7. Discussion

7.1. **What is new.** (1) Ricci curvature as conditional gap at every scale; (2) martingale decomposition with absorption (no recursive losses); (3) geometric summability of D_k — robust to the blocking convention (Remark 2.8).

7.2. **Why previous approaches failed and why the martingale method succeeds.** As in v1: Holley–Stroock pays $\mathrm{osc}(\beta S_W) = O(\beta L^d)$; recursive Poincaré composition loses $2^{n_{\max}}$; Dobrushin-type conditions fail at coupling $O(\beta)$. The martingale method avoids all three. Fallback without Assumption 2.6 (corrected in v2): $D_k \le C(\beta_k^2 + e^{-2\kappa})2^{-3k}$ with $\beta_k \le \beta$ *bounded* (corrected flow), so $D_* \le C'\beta^2 < \infty$; Assumption 2.6 strengthens this to $D_* = O(e^{-2\kappa}) \ll c_0$, guaranteeing absorption.

7.3. **Relation to the Millennium Problem.** Theorem 1.1 gives functional-analytic control at fixed lattice spacing, uniform in volume. It does not address the continuum limit or OS axioms, and by Remark 6.2 does not by itself give exponential clustering.

7.4. **Conditional nature of the result.** The proof is conditional on: (1) Balaban's effective actions with polymer bounds (1); (2) the derivative bounds for $j \le 2$; (3) the small/large-field decomposition; (3') the positivity of Balaban's fluctuation operator on fast modes (used in Theorem 3.1; made explicit in v2); (4) the RG-normalized disintegration (H-DIS). Conditions (1)–(3') constitute the series' package hypothesis (H-BAL); condition (4) is the delicate one, asserting that the principal action at each scale is properly absorbed leaving only exponentially decaying polymer corrections.

Note added in v2

Prepared with the companion suite `verify_2602_0040.py` (repository `THE-ERIKSSON-PROGRAMME`, `verification/2602-0040/`; 12 checks, all passing, ∼30 s, NumPy/SciPy). Unlike the other v2's of this series, no v1 statement is refuted here; the machinery is *validated*: the commutator identity (9) to 10^{-10} (spectral derivatives) and exactly (Gaussian); the telescoping (15) exactly; the Cauchy–Schwarz step and (11); and the assembled α^* of (18) confirmed as a valid lower bound for the true spectral gap in a compact four-rotor model with all constants measured from their definitions. Changes: (1) Theorem 2.1(d): coupling-flow sign/normalization corrected (series erratum); the Remark 2.7 fallback *improves* ($\beta_k \le \beta$ bounded); the large-field bound's k-dependence is re-read accordingly; (2) $\mathrm{Ric} = N_c/2$ sharp (source: ai.viXra:2602.0036 v2), improving c_0 and c_{term} by a factor 2; (3) new Remark 2.8: robustness of the summability to the block-averaging convention (8^{-k} vs 4^{-k}; both verified with explicit matrices); (4) Lemma 4.1's v2 note: the commutator coefficient involves the centered (conditional-potential) gradient — the operational mechanism of Assumption 2.6, and the definition used in the suite's measurements; (5) Assumption 2.6 tagged (H-DIS) and related to (H-FACT) of ai.viXra:2602.0033 v2 (new Remark 6.4); (6) reference [1] identified as ai.viXra:2602.0032, updated to its v2 (conditional reduction), with the introduction's fixed-L claim re-pointed to ai.viXra:2602.0035 v2, Theorem 5.3(i); the LSI "forthcoming work" of Remark 6.2 now cited as ai.viXra:2602.0041 and refinements; (7) the toron caveat of the companions recorded in Theorem 3.1's proof (it does not affect the E_k sectors used here). All v1 numbered statements (1.1–6.3) and equations (1)–(18) keep their numbers and positions; new items are Remark 2.8 and Remark 6.4.

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