Erratum (v3): Theorem 3.1 not established; withdrawal of Corollaries 3.2–3.3

Geodesic Convexity and Structural Limits of Curvature Methods for the Yang–Mills Mass Gap on the Lattice

Lluis Eriksson • ai.viXra:2602.0036 • July 2026

Corollaries 3.2 and 3.3 are withdrawn, and Theorem 3.1 is not established by its printed proof. The corollaries pass from a Ricci bound at regular points to a spectral gap for a global operator, by a one-sentence appeal to codimension; that step is not established, and the reference given for it does not supply it (§1). But the regular-point bound itself does not survive either: the O'Neill trace in equation (19) omits the mixed horizontal–vertical sectional terms, and the omitted terms have the unfavourable sign (§2). A correct global statement exists — but it concerns a measure and an operator that *have not been identified with those of the withdrawn corollaries*, and is therefore added here as a separate result rather than presented as a repair (§3).

An earlier draft of this note stated that "Theorem 3.1 stands exactly as printed". That was wrong, and is withdrawn here. It is recorded rather than quietly amended: it is the fourth time in this correction campaign that a list of surviving statements was closed by checking dependencies instead of reading a proof.

1 What is withdrawn

Corollary 3.2 concludes $\lambda_1 \geq N_c/2$ for the Laplacian on the compact quotient B, arguing that "*the singular set (reducible connections) has codimension* ≥ 2 *and does not affect the spectrum*", with a single citation. Corollary 3.3 converts that into a kinetic gap $E_1^{\text{kin}} - E_0^{\text{kin}} \geq g^2 N_c/4$ uniform in L.

Three things are wrong with the step, and each alone suffices.

(a) The cited reference is about a different situation. The citation is to Bérard-Bergery–Bourguignon, *Laplacians and Riemannian submersions with totally geodesic fibres*. That is a theorem about *smooth* submersions whose fibres are totally geodesic. The gauge orbits are not totally geodesic in general, and the submersion structure degenerates exactly at the reducible connections — which is precisely where the statement is needed. It is not a spectral-removability theorem for a singular stratum.

(b) Codimension ≥ 2 is not by itself a removability criterion. Two distinct levels are involved, and they must not be run together. To identify the Friedrichs/Dirichlet-form realisation across the singular set one would need an appropriate removability or capacity theorem for the relevant *form* — that the closure of the form on $C_c^\infty(B_{\text{reg}})$ coincides with the intended global form. *Essential self-adjointness* of the minimal differential operator, and hence uniqueness among *all* self-adjoint extensions, is a separate and strictly stronger question, and is not established here either. Codimension alone gives neither. Moreover the stabilisers of the reducible configurations in the singular locus have positive dimension, so the quotient is a stratified space, not merely a manifold with finitely many quotient singularities.

(c) The paper specifies and justifies no global self-adjoint realisation of $\mathcal{K}_B^{\text{reg}}$. B_{reg} is the regular stratum with the singular locus removed and is in general *incomplete*. The unweighted Laplace–Beltrami expression on $C_c^\infty(B_{\text{reg}})$ is symmetric and non-negative, but it has not been shown to be essentially self-adjoint; it may admit several self-adjoint extensions. Realisations *can* be constructed — the Friedrichs extension of a chosen form, for one — so the objection is not that none exists; it is that the paper picks none and justifies none, while Corollary 3.2 quantifies the spectrum of one.

2 Theorem 3.1: the printed O'Neill trace is not the right one

The corollaries would fall on §1 alone. But the regular-point bound they globalise does not survive its own proof either.

Equation (19) reads

$$\mathrm{Ric}_B(X,X) = \mathrm{Ric}_{\mathcal{A}}(X^h,X^h) + \tfrac{3}{4}\sum_i \big\|[X^h,E_i^h]^v\big\|^2 \;\geq\; \mathrm{Ric}_{\mathcal{A}}(X^h,X^h).$$

That is not the trace of O'Neill's horizontal equation. With $\{X, E_2, \ldots, E_b\}$ an orthonormal basis of $T_{[U]}B$ and $\{V_1, \ldots, V_r\}$ an orthonormal vertical basis, O'Neill gives $K_B(X,E_i) = K_{\mathcal{A}}(X^h,E_i^h) + \frac{3}{4}\|[X^h,E_i^h]^v\|^2$, so tracing over *horizontal* directions only,

$$\mathrm{Ric}_B(X,X) = \sum_{i=2}^{b} K_{\mathcal{A}}(X^h,E_i^h) + \tfrac{3}{4}\sum_{i=2}^{b}\big\|[X^h,E_i^h]^v\big\|^2.$$

The total-space Ricci tensor, however, traces over horizontal *and* vertical directions:

$$\mathrm{Ric}_{\mathcal{A}}(X^h,X^h) = \sum_i K_{\mathcal{A}}(X^h,E_i^h) + \sum_a K_{\mathcal{A}}(X^h,V_a).$$

Hence the correct identity is

$$\boxed{\begin{aligned}\mathrm{Ric}_B(X,X) = {}& \mathrm{Ric}_{\mathcal{A}}(X^h,X^h) - \sum_{a=1}^{r} K_{\mathcal{A}}(X^h,V_a)\\ &+ \tfrac{3}{4}\sum_{i=2}^{b}\big\|[X^h,E_i^h]^v\big\|^2\end{aligned}}$$

and (19) has dropped $-\sum_a K_{\mathcal{A}}(X^h,V_a)$.

The omitted term has the unfavourable sign. For a bi-invariant metric on a compact group all sectional curvatures satisfy $K(X,Y) = \frac{1}{4}\|[X,Y]\|^2 \geq 0$, so each $K_{\mathcal{A}}(X^h,V_a) \geq 0$ and the omitted term *subtracts*. Whether the O'Neill term still dominates it is exactly the computation the proof would have to do, and does not. It cannot be waved through on general grounds: Riemannian submersions do not in general preserve positive Ricci curvature.

The omitted mixed sectional-curvature trace must be retained *independently* of whether the fibres are totally geodesic: it arises simply because $\mathrm{Ric}_{\mathcal{A}}$ traces over vertical directions as well as horizontal ones, while Ric_B traces only over horizontal ones. Total geodesicity controls the O'Neill T-tensor, which enters the vertical and mixed equations, not this horizontal trace. The non-total-geodesicity of the gauge orbits is therefore a *separate* mismatch — with the theorem cited in §1(a) — and not the reason these mixed terms occur.

Status. The statement $\mathrm{Ric}_B(X,X) \geq \frac{N_c}{2}|X|^2$ on B_{reg} **is not refuted**; what is refuted is the printed derivation. **Theorem 3.1 is not established by its proof**, and a new argument specific to the gauge quotient would be required. Until then, **no pointwise Ricci bound on B_{reg} may be cited from this paper as available** — and it is so cited, in `ai.viXra:2602.0033` and `2602.0035`.

3 The one sound geometric input, on the correct object

A global result is available, and it does not go through removability at all. With $\mathcal{A} = SU(N_c)^E$ carrying the product Haar measure, and $\nu = \pi_\#\mathrm{Vol}_\mathcal{A}$ the pushforward to B, the metric-measure space (B, d_B, ν) satisfies $\mathrm{RCD}^*(N_c/4, \dim \mathcal{A})$, by stability of the curvature-dimension condition under quotients by compact groups of measure-preserving isometries. This is ai.viXra:2602.0046 v3, and it handles the singular stratum automatically — its own text records that B is generally not a manifold, and that the codimension analysis is *not needed* for its argument.

That paper also states that its route *bypasses the O'Neill computation*. In the light of §2 that remark is worth more than it looked: the synthetic route is now the *only* sound geometric input in this chain, because the O'Neill route printed here does not establish its own conclusion.

That result is **not** a proof of Corollary 3.2. It is a statement about K_ν, the self-adjoint operator of the closed Dirichlet form on $L^2(B, \nu)$; Corollary 3.2 was about a realisation of $\mathcal{K}_B^{\mathrm{reg}}$ on $L^2(B, d\mathrm{Vol}_B)$. On the regular stratum, writing $d\nu = w\, d\mathrm{Vol}_B$ with w the orbit-volume density, the two differential expressions differ by $\langle \nabla \log w, \nabla \cdot \rangle$, and they agree only if w is constant on each relevant component — which constant stabiliser *type* does not imply, since the induced metric on the orbits may vary even where all stabilisers are conjugate.

What makes the ν-side statement the physically natural one is that it is exactly the gauge-invariant sector: pullback along π is an isometry

$$L^2(B, \nu) \;\cong\; L^2(\mathcal{A}, \mathrm{Vol}_\mathcal{A})^\mathcal{G},$$

because $\int_B |f|^2 d\nu = \int_\mathcal{A} |f \circ \pi|^2 d\mathrm{Vol}_\mathcal{A}$ is the definition of the pushforward. So the replacement for Corollaries 3.2–3.3 should be read as a statement about gauge-invariant functions on the smooth compact $\mathcal{A}$, where no singular geometry is involved at all.

4 Metric normalisation

$\mathrm{Ric} \geq N_c/2$ for $\langle X, Y \rangle = -\operatorname{tr}(XY)$ and $\mathrm{Ric} \geq N_c/4$ for $-2\operatorname{tr}(XY)$ are *the same bound in two normalisations*, not a sharpening of one by the other. This paper's Theorem 3.1 uses the first; ai.viXra:2602.0046 states its condition in the second. Any downstream use must name its inner product.

The conversion is correct as a formal statement; what is not correct is to read either side as available. **Since Theorem 3.1 is not established (§2), neither the $N_c/2$ nor the $N_c/4$ pointwise bound may presently be cited from this paper**, and the v2 parenthetical "*v1 stated the bound with $N_c/4$; that is true but not sharp*" is **withdrawn**: it belongs to the same defective derivation.

5 Three citations to a version that does not exist

This paper cites `ai.viXra:2602.0035 v2` in three places — in Remark 3.4, in Proposition 4.6, and in the discussion of Theorem 5.2 — for the corrected ground-state exponent $f = -\log \psi_0^2$ and related material. **No v2 of `2602.0035` exists in the archive.** The file prepared as that version was mis-filed onto a different record during the replacement campaign of 2026-07-06; the record `2602.0035` still shows only its v1. Readers following those citations will not find what is cited. The three references become live when that record is corrected; until then they should be read as pointing to material that is prepared but unpublished.

6 Scope of this note

This note audits Section 3 and the object-provenance failure it caused. It is not a re-audit of the whole paper, and says so rather than leaving the impression of one.

Read for this note and unaffected by §1–§2: **Corollary 2.3** in its corrected v2 scope, together with the v2 refutation of the v1 claim it replaces; and **Theorem 2.5**, whose v1 convexity route is withdrawn in v2 and whose statement rests instead on the classical Osterwalder–Seiler strong-coupling cluster expansion — an external result, correctly cited and not re-derived here. **Theorem 2.1** is not affected by the present findings and was not re-audited.

Not audited here, and flagged because the paper's headline rests on them: Propositions 4.3–4.6 are labelled "from v1", were not revised in the v2 pass, and are stated in this version *without proof*; Remark 4.7 makes the $O(g^2)$ structural ceiling rest on them together with Theorem 4.2. Proposition 4.6 in particular invokes Ric_B on the full quotient and therefore inherits the object-provenance question of §1–§2, although it does so in the favourable direction, as an obstruction rather than a positive bound. Their status is left open by this note.

Provenance. The defect of §1 was found by tracing which paper supplies the geometric input consumed by the companions, and checking the cited reference rather than the citing sentence. The distinction that makes it a defect rather than a citation slip — a correct theorem about the wrong object for the consuming dependency — is not covered by the usual separation of false statement from invalid proof, and is recorded as its own category on the page that follows. **The defect of §2 came from an external audit of the first draft of this very note**, which had certified Theorem 3.1 as standing; it was found by re-deriving the O'Neill trace rather than by reading the surrounding text, and it is the reason §2 exists. The page that follows is reproduced verbatim, by instruction: paraphrase is how the columns drift.

Object Provenance for the Orbit-Space Gap Chain

A common page, reproduced verbatim in ai.viXra:2602.0033, 2602.0035 and 2602.0036

Lluis Eriksson • July 2026

Why this page exists. Four papers of this series state, prove, or consume spectral claims about objects that are routinely called "the gap on the orbit space". They are not the same object. A pointwise curvature *claim* stated in one of them was consumed, in three places, as if it were established and as if it applied to the same object. It is neither: its printed proof is not presently established, and the sound synthetic result available elsewhere concerns a measure and an operator that *have not been identified* with the ones the consuming steps need. The error was invisible because every paper used the same word. This page fixes the dictionary. **A citation discharges a hypothesis only when all four columns agree.**

Notation barrier

This page writes ν for $\nu_{\text{push}} := \pi_{\#}\text{Vol}_{\mathcal{A}}$, and never for anything else. The host papers of this chain use the same letter for a different object: `2602.0033` (Theorem 3.1, Step 3) and `2602.0035` (Theorem 5.3, Step 2) both write "the uniform measure ν" for the normalised Riemannian volume

$$\nu_{\text{Vol}} := d\text{Vol}_B/\text{Vol}_B(B).$$

ν_{push} **and** ν_{Vol} **are not identified**, and identifying them is exactly what (I2b) below forbids without a statement about the orbit-volume density. A reader who carries the host's ν into this page, or this page's ν into the host's Step 3, will read a printed derivation as though it already used the corrected measure. *Where the distinction bites, the subscripts are written out.*

The dictionary

Source	Space, measure	Operator	What it can conclude
`2602.0036`	B_{reg}, $d\text{Vol}_B$	$\mathcal{K}_B^{\text{reg}}$: unweighted Laplace–Beltrami *expression*; global realisation unspecified	*targets* a pointwise Ricci bound on the regular stratum; printed proof not presently established
`2602.0046`	B, $\nu = \pi_{\#}\text{Vol}_{\mathcal{A}}$	$K_\nu \geq 0$, operator of the form on $L^2(B,\nu)$	global Poincaré/LSI, singular locus included
`2602.0033`, as printed	spatial orbit space; printed *regular-stratum formula* $d\mu_{\text{print}} = Z_{\text{print}}^{-1} e^{-S_{\text{eff}}} d\text{Vol}_B$ on B_{reg}, **against** Vol_B, **not** ν; *no global extension specified*	weighted diffusion of μ_{print}, *if* a global closed form is constructed	neither a Poincaré/LSI statement nor a Hamiltonian gap is presently established
route (A): keep μ_{print}	(B,ν); a candidate *global extension* of μ_{print}, exponent $S_{\text{eff}} + \log w$	weighted form over K_ν	needs w globally, and $\text{osc}(S_{\text{eff}} + \log w)$
route (B): new measure	(B,ν); a *separately defined candidate* measure $d\tilde{\mu} = \tilde{Z}^{-1} e^{-S_{\text{eff}}} d\nu$	weighted form over K_ν	needs a theorem that this is the right object
`2602.0035`, as printed	$L^2(B, d\text{Vol}_B)$, global realisation not fixed; $\mu_{0,\text{print}} = \psi_{0,\text{print}}^2\, d\text{Vol}_B$	Hamiltonian on $L^2(B, d\text{Vol}_B)$; diffusion of $\mu_{0,\text{print}}$	parts (i) and (ii) are not established by the printed proofs
candidate physical restatement	$L^2(\mathcal{A})^{\mathcal{G}} \cong L^2(B,\nu)$; $\mu_{0,\nu} = \psi_{0,\nu}^2\, \nu$	$H_\nu = \frac{g^2}{2} K_\nu + V_{\text{pot}}$; then diffusion of $\mu_{0,\nu}$	qualitative fixed-volume positivity; a rate only under $(\text{A-osc})_\nu$

The four identifications that must never be silent

(I1) $L^2(B,\nu) \cong L^2(\mathcal{A}, \mathrm{Vol}_{\mathcal{A}})^{\mathcal{G}}$ **— exact, and free.** Pullback $f \mapsto f \circ \pi$ gives $\int_B |f|^2\, d\nu = \int_{\mathcal{A}} |f \circ \pi|^2\, d\mathrm{Vol}_{\mathcal{A}}$, which is the definition of $\nu = \pi_{\#}\mathrm{Vol}_{\mathcal{A}}$. This is an isometry, not an approximation, and it holds across the singular locus. It is the reason the metric-measure quotient is the natural physical object: gauge-invariant functions on the smooth compact $\mathcal{A}$ *are* $L^2(B,\nu)$.

(I2) K_ν **versus** $\mathcal{K}_B^{\mathrm{reg}}$ **— not free, and not even the same kind of object.** Fix the convention here, so that nothing depends on the host paper's. On the regular stratum write $d\nu = w\, d\mathrm{Vol}_B$, with w the orbit-volume density, and define the *formal differential expressions*

$$\mathcal{L}_B^{\mathrm{reg}} := \mathrm{div}_B \nabla, \qquad \mathcal{L}_\nu^{\mathrm{reg}} := w^{-1} \mathrm{div}_B(w\nabla), \qquad \mathcal{K}_\bullet^{\mathrm{reg}} := -\mathcal{L}_\bullet^{\mathrm{reg}},$$

so that

$$\mathcal{L}_\nu^{\mathrm{reg}} f = \mathcal{L}_B^{\mathrm{reg}} f + \langle \nabla \log w, \nabla f \rangle.$$

Let $K_\nu \geq 0$ denote the *global* self-adjoint operator associated with the closed Dirichlet form on $L^2(B,\nu)$.

No global self-adjoint realisation of $\mathcal{K}_B^{\mathrm{reg}}$ is fixed by this page. B_{reg} is the regular stratum with the singular locus removed and is in general an incomplete manifold; $\mathcal{K}_B^{\mathrm{reg}}$ on $C_c^\infty(B_{\mathrm{reg}})$ is symmetric and non-negative, but it has not been shown to be essentially self-adjoint, and it may admit several self-adjoint extensions. Choosing the Friedrichs extension is already a decision about behaviour across $B \setminus B_{\mathrm{reg}}$, and must be specified and justified separately — this is one of the gaps that forces the withdrawal of the global corollary in `2602.0036`.

The two *regular-stratum differential expressions* coincide only if w is constant on each relevant component. *Constant stabiliser type does not by itself imply constant orbit volume* — on the principal stratum all stabilisers are already conjugate, and the induced metric on the orbits, hence their volume, may still vary with the configuration; reducible connections make a global identification less tenable still. **A spectral bound for the global K_ν does not by itself transfer to any chosen self-adjoint realisation of $\mathcal{K}_B^{\mathrm{reg}}$, or conversely.** Comparison theorems under further hypotheses are not excluded; what does not exist is a free identification — and on the $\mathcal{K}_B^{\mathrm{reg}}$ side, there is not yet a single global operator for such a theorem to be about.

(I2b) $d\mathrm{Vol}_B$ **versus** ν ***at the level of the perturbed measure*** **— also not free.** This is a separate step from (I2), and it is the one most easily skipped, because both measures are written with the same exponent. On B_{reg}, $d\nu = w\, d\mathrm{Vol}_B$ with w the orbit-volume density, so the measure a paper prints against Vol_B is, written against ν,

$$d\mu_{\mathrm{print}} = Z_{\mathrm{print}}^{-1} e^{-S_{\mathrm{eff}}}\, d\mathrm{Vol}_B = Z_{\mathrm{print}}^{-1} e^{-(S_{\mathrm{eff}} + \log w)}\, d\nu \qquad \text{on } B_{\mathrm{reg}},$$

an *exact* rewriting of one and the same measure, with the same normaliser. The candidate ν-based measure $d\widetilde{\mu} = \widetilde{Z}^{-1} e^{-S_{\mathrm{eff}}} d\nu$ carries the exponent S_{eff} and, in general, a *different* normaliser. **On B_{reg} the printed measure has exponent $S_{\mathrm{eff}} + \log w$ relative to ν, whereas the candidate ν-based measure has exponent S_{eff}. They coincide only under an additional constancy or identification statement for w, none of which is established here. Replacing $d\mathrm{Vol}_B$ by $d\nu$ under a fixed exponent is therefore not a free rewriting.**

And the converse is not asserted either. This page does not claim the two measures are *proved* distinct: that would need w non-constant, up to normalisation, on the relevant domain. Reducible connections make a gratuitous global identification implausible, but implausible is not proved, and the discipline this page enforces cuts both ways — "no identification has been proved" must not become "an inequality has been proved".

Two routes are open, and they are not the same route. **(A) Keep the printed measure.** Put $\Phi_{\mathrm{print}} = S_{\mathrm{eff}} + \log w$, so that $d\mu_{\mathrm{print}} = Z_{\mathrm{print}}^{-1} e^{-\Phi_{\mathrm{print}}} d\nu$. This needs a global construction of the measure from the regular-stratum formula, control of w approaching the singular locus, and a bound on $\operatorname{osc}(S_{\text{eff}}+\log w)$ — for which a bound on $\operatorname{osc}(S_{\text{eff}})$ alone does not suffice: if w degenerates at strata of enlarged stabiliser, $\log w$ is exactly the term that destroys a bounded-perturbation argument. **No such control is supplied anywhere in this chain. (B) Change to the ν-natural measure $\widetilde{\mu}$,** which is *a separately defined candidate measure* and not the one the paper printed. Here `2602.0046` can supply the base functional inequality for K_ν, but a new theorem is required to show that $\widetilde{\mu}$, and not μ_{print}, is the correct Euclidean or transfer-theoretic object. Note that route (A) is not free either: "keep the printed measure" still means *constructing a global extension of a regular-stratum formula*, which is itself one of the open obligations. **A ν-based base inequality is therefore an input to one of these two routes, not a repair of a printed Vol_B-based derivation by change of reference measure.** Neither route by itself supplies the four-dimensional-action to three-dimensional-space step, the identification with μ_0, or a transfer-operator construction.

(I3) μ_{print} has not been identified with $\mu_0 = \psi_0^2\nu$ — not free, and the load-bearing gap. With $K_\nu \geq 0$ as in (I2) — equivalently, by (I1), the operator of the kinetic form $\mathcal{E}_\nu(f,f) = \int_B |\nabla f|^2\, d\nu$ on $L^2(\mathcal{A}, \mathrm{Vol}_{\mathcal{A}})^{\mathcal{G}}$ — **let V_{mult} be a specified real multiplication potential** and set $H_{\text{mult}} = \frac{g^2}{2}K_\nu + V_{\text{mult}}$, with a normalised, strictly positive ground state ψ_0, and put $d\mu_0 = \psi_0^2\, d\nu$. *The subscripted name is deliberate. This page is reproduced verbatim inside papers that have already bound V to a gauge-fixed potential and W to the Weyl group, so no bare letter is safe here: a page written to prevent symbol collisions must not cause one.* The ground-state transformation *then* gives

$$E_1 - E_0 = \tfrac{g^2}{2}\,\lambda_1(\mu_0), \qquad f_0 = -\log\psi_0^2,$$

where E_1 denotes the first spectral value strictly above the simple ground-state energy E_0, and, *defined explicitly to avoid the reciprocal ambiguity in the term "Poincaré constant"*,

$$\lambda_1(\mu_0) := \inf\Big\{\mathcal{E}_{\mu_0}(f,f)\ :\ f \in \operatorname{Dom}\mathcal{E}_{\mu_0},\ \textstyle\int f\, d\mu_0 = 0,\ \|f\|_{L^2(\mu_0)} = 1\Big\}.$$

That is, $\lambda_1(\mu_0)$ is the Poincaré *spectral gap* — the largest $\rho \geq 0$ with $\mathrm{Var}_{\mu_0}(f) \leq \rho^{-1}\mathcal{E}_{\mu_0}(f,f)$ for all admissible f — and *not* the constant $C_P = \rho^{-1}$ of the variance inequality, which some authors also call the Poincaré constant.

The Gibbs measure of a Euclidean effective action is *a priori* a distinct mathematical object, and its weighted Dirichlet-form gap is *a priori* a distinct spectral quantity. In the symbols fixed above,

No globally defined printed or corrected Euclidean measure has been identified with $\mu_0 = \psi_0^2\,\nu$.

That covers μ_{print}, route (A)'s global extension of it, and route (B)'s $\widetilde{\mu}$ alike — stated this way so that (I3) neither singles out one candidate nor silently presupposes (I2b). No such identification has been proved here; in special models the objects may coincide, but passing from one to the other in general requires a transfer-operator or Osterwalder–Seiler identification theorem, not a change of notation. **An oscillation bound for S_{eff} does not by itself bound $\operatorname{osc}(-\log\psi_0^2)$.**

Constants travel with their metric normalisation

$\mathrm{Ric} \geq N_c/2$ for $\langle X, Y\rangle = -\operatorname{tr}(XY)$ and $\mathrm{Ric} \geq N_c/4$ for $-2\operatorname{tr}(XY)$ are the *same* bound in two normalisations, not a sharpening of one by the other. Any statement quoting $N_c/2$ or $N_c/4$ must name its inner product.

How to use this page

Before writing that a companion result discharges a hypothesis, fill the four columns for the result and for the hypothesis. If any column differs, the citation does not discharge; at most it supplies an input to a further identification, which must then be stated as its own step. In particular:

- `2602.0046` supplies global geometry on the metric-measure object identified by (I1): a bound for K_ν on $L^2(B,\nu)$. It does not supply (I1) — that is free — and it does *not* supply a bound for any realisation of the unweighted $\mathcal{K}_B^{\mathrm{reg}}$; nor does it attempt pointwise bounds on B_{reg}, as its own text says. Note that it reaches its conclusion by synthetic quotient stability and *explicitly bypasses the O'Neill computation.* That is now the only sound geometric input in this chain: the O'Neill trace printed in `2602.0036` omits the mixed horizontal–vertical sectional terms and does not establish its own conclusion (see that record's erratum). **No pointwise Ricci bound on B_{reg} may be cited as available.**

- `2602.0033`, as printed, targets a weighted diffusion for $d\mu_{\mathrm{print}} = Z_{\mathrm{print}}^{-1} e^{-S_{\mathrm{eff}}} d\mathrm{Vol}_B$ on B_{reg} — against Vol_B, *not* against ν, and with no global extension or associated closed Dirichlet form specified. It does not presently establish a Euclidean Poincaré/LSI statement, and it does not establish a Hamiltonian or transfer-operator gap without (I3). **Citing `2602.0046` does not repair it by writing the same exponent against ν**: by (I2b) that is not a free rewriting. Either route (A) is taken and $\mathrm{osc}(S_{\mathrm{eff}} + \log w)$ is controlled, or route (B) is taken and the new measure is shown to be the right object.

- `2602.0035` v2 **already uses a ground-state measure**, so **(I3) is not the missing bridge in its printed quantitative proof** — that was the correction its v2 made to its own v1. Its printed-to-physical transition **crosses (I2) and (I2b)**: the unweighted realisation against K_ν, and $d\mathrm{Vol}_B$ against ν. The candidate physical restatement then requires $(\text{A-osc})_\nu$, a hypothesis about *its own* ground state. **Identification (I3) re-enters only if that Hamiltonian construction is used to identify a Euclidean or transfer-operator object**, as in the route towards (H1) of `2602.0033`. Among the four identifications on this page, its qualitative fixed-volume positivity needs only (I1); it additionally uses the standard compact-elliptic and positivity-improving inputs — self-adjoint realisation, compact resolvent, connectedness — and it does not require any treatment of the quotient singularities. *An earlier version of this page said flatly that the paper needs (I3) for its quantitative bound. That was wrong, and it is the kind of error this page exists to catch: it named the wrong hypothesis for a citation to discharge.*

Provenance. This page was written after an audit found the same curvature *claim* consumed in three places under three different pairings of measure and operator, and after a proposed repair — citing the metric-measure result in place of the regular-stratum one — was found to change the object rather than fix the proof. A later pass found that the regular-stratum claim is not established by its own printed proof either, so the defect is now twofold: an invalid derivation in one paper, and a correct theorem in another about a different object. A third pass found the drift *on this page*: the row for `2602.0033` had recorded the printed measure against ν when the paper writes it against Vol_B, which is precisely the substitution (I2b) now forbids. It was introduced by writing the row from the intended repair rather than from the printed text. Hence the rule below, and hence (I2b) as a numbered step rather than a remark. The same pass then caught the correction overshooting: (I2b) had asserted the two measures *are* different, which needs w non-constant and is not proved here. Both directions are now stated at the strength they have. A fourth pass found that (I3) wrote its Hamiltonian as $\frac{g^2}{2}K_\nu + V$ — and V is already bound, in `2602.0035`, to the gauge-fixed $S_{\mathrm{YM}} - \log\det M$. A page reproduced verbatim inside that paper cannot borrow a symbol the host has spent. The first repair reached for a bare W, which that same paper binds to the Weyl group S_{N_c}: **no bare letter is safe in a page meant to be embedded, only a subscripted name**. The same pass found the collision that matters most, because it is in the *measure* column: both host papers write "the uniform measure ν" for the normalised $d\mathrm{Vol}_B$, which is what this page's own (I2b) forbids identifying with $\pi_{\#}\mathrm{Vol}_{\mathcal{A}}$. Hence the notation barrier above. A fifth pass found the row for `2602.0035` built from its intended repair rather than its printed text — the identical error this page had already corrected once, for `2602.0033` — and found the claim that that paper needs (I3) to be simply wrong: its v2 already uses a ground-state measure. It records a failure mode that is not covered by the usual distinction between a false statement and an invalid proof: **a correct theorem about the wrong object for the consuming dependency**. Reproduce it verbatim; do not paraphrase it, because paraphrase is how the columns drift.

GEODESIC CONVEXITY AND STRUCTURAL LIMITS OF CURVATURE METHODS FOR THE YANG–MILLS MASS GAP ON THE LATTICE

LLUIS ERIKSSON

ABSTRACT. Building on Mondal's observation that the orbit space of Yang–Mills theory has positive sectional curvature, we study curvature methods for $\mathrm{SU}(N_c)$ lattice gauge theory with Wilson action. First, we prove that $U \mapsto -\mathrm{Re}\,\mathrm{Tr}\,U$ is strictly geodesically convex on the geodesic ball $B_{\pi/2}(I) \subset \mathrm{SU}(N_c)$, with an explicit Riemannian Hessian formula (Theorem 2.1, verified numerically to machine precision; the Daleckii–Kreĭn-type symmetrization stated in v1's Remark 2.2 is corrected). Version 2 records, however, that the v1 extension of this convexity to the full Wilson action on the domain Ω_L (Corollary 2.3) is *false*: product geodesics are not mapped to geodesics by the plaquette map, and a numerical counterexample with all links within $\pi/8$ of the identity gives $d^2S_W/dt^2 = -0.11$; the strong-coupling gap of Theorem 2.5 therefore rests on the cluster expansion, not on the Prékopa route sketched in v1. Second, the orbit space $\mathcal{B} = \mathcal{A}/\mathcal{G}$ has $\mathrm{Ric}_{\mathcal{B}} \geq N_c/2$ — version 2 corrects the constant at its source (v1 stated $N_c/4$; the sharp bi-invariant value is $N_c/2 = -B/4$), improving the spectral gap of the gauge-invariant Laplacian to $\lambda_1 \geq N_c/2$, uniformly in the lattice size. Third — the main correction of v2 — the zero-mode Hessian computation behind the $O(g^2)$ ceiling (Theorem 4.2) is re-derived exactly and adjudicated numerically on the lattice: the correct Gram and curvature terms are $8N_c\tilde{I}_1(L)|s|^2$ and $2N_cX(L)|s|^2$ with two lattice integrals $\tilde{I}_1 \neq I_1/d$ and $X = 2I_1 - (N-1)/(Nd)$ (exact), there is no relative factor g^2 between them, and the net Faddeev–Popov zero-mode Hessian at the vacuum is $2N_c[4\tilde{I}_1(L) - X(L)]|s|^2 = O(1/L) \;\to\; 0^+$: a new lemma proves $X_\infty = 4\tilde{I}_{1,\infty}$ by partial integration. Consequently v1's formula (28) and the critical coupling (29) are withdrawn; the $O(g^2)$ ceiling itself survives, resting on the four independent arguments of Propositions 4.3–4.6 and on the corrected computation (which shows no $O(1)$ curvature, of either sign, is available at the vacuum zero modes). The Witten–Helffer–Sjöstrand program of Section 5 is updated with the toron degeneracy of the vacuum found in the companion papers. All claims are verified in a companion numerical suite.

1. Introduction

1.1. The mass gap problem. The existence of a positive mass gap in four-dimensional $\mathrm{SU}(N_c)$ Yang–Mills theory is one of the seven Clay Millennium Prize Problems [1]. On a periodic Euclidean lattice $\Lambda_{\mathrm{lat}} = (\mathbb{Z}/L\mathbb{Z})^d$ the Wilson action is

$$S_{\mathrm{YM}}(U) = \frac{\beta}{2N_c} \sum_{\square} \big(N_c - \mathrm{Re}\,\mathrm{Tr}\,U_\square\big), \qquad \beta = \frac{2N_c}{g^2}, \tag{1}$$

and the mass gap $m > 0$ (in lattice units) is the exponential decay rate of connected two-point functions of gauge-invariant observables. In the Hamiltonian formulation the mass gap equals the spectral gap of the Kogut–Susskind Hamiltonian

$$H = \frac{g^2}{2} \sum_{\ell} E_\ell^2 + \frac{2}{g^2} \sum_{\square} \big(1 - \tfrac{1}{N_c}\mathrm{Re}\,\mathrm{Tr}\,U_\square\big), \tag{2}$$

acting on $\mathcal{H}_{\mathrm{phys}} = L^2(\mathcal{A})^{\mathcal{G}}$. This paper concerns the spectral gap of the lattice theory, uniform in L.

Date: February 2026. Version 2: July 2026.

1.2. **Curvature of the orbit space.** A central object is the orbit space

$$\mathcal{B} = \mathcal{A}/\mathcal{G}, \tag{3}$$

with the Riemannian metric induced from the bi-invariant product metric on $\mathcal{A}$; $\pi : \mathcal{A} \to \mathcal{B}$ is a Riemannian submersion at regular points. Mondal [2] observed that the continuum orbit space has positive sectional curvature and argued heuristically for a mass gap $\propto g^2$ via Bakry–Émery. Our first two results make this rigorous on the lattice; the third identifies its structural limits.

1.3. **Main results.** (i) **Geodesic convexity (Theorem 2.1).** $f(U) = -\operatorname{Re}\operatorname{Tr} U$ is strictly geodesically convex on $B_{\pi/2}(I)$ with $(\operatorname{Hess} f)_U(X,X) = \sum_{j,k} |A_{jk}|^2 \cos\lambda_j$. (v2: the extension to S_{YM} on Ω_L claimed in v1 is false; Corollary 2.3.) (ii) **Ricci curvature (Theorem 3.1).** $\operatorname{Ric}_{\mathcal{B}} \geq (N_c/2) g_{\mathcal{B}}$ (v1: $N_c/4$; corrected at the source), giving $\lambda_1(\Delta_{\mathcal{B}}) \geq N_c/2$ uniformly in L. (iii) **The $O(g^2)$ ceiling (Theorem 4.2, corrected).** The exact zero-mode Faddeev–Popov Hessian at the trivial vacuum is $2N_c[4\tilde{I}_1(L) - X(L)]\,|s|^2 = O(1/L) \to 0^+$: no $O(1)$ curvature of either sign is available at the vacuum, and v1's critical coupling g^2_{crit} is withdrawn. The ceiling — convexity methods produce at most $O(g^2)$ — survives via Propositions 4.3–4.6.

1.4. **Relation to prior work.** The Gribov–Zwanziger approach [4, 5, 6] restricts the functional integral to the first Gribov region. In [3] (Paper I) we showed that the Gribov–Zwanziger lattice measure has a mass gap via Prékopa–Brascamp–Lieb convexity. Mondal [2] initiated the orbit-space curvature program in the continuum. The lattice orbit-space metric was studied by Laufer and Orland [19]; log-Sobolev inequalities for gauge theories by Driver and Lohrenz [20]; constructive renormalization by Balaban [21, 22]. (v2) The present paper is Paper II of the series continued in [27] (Paper III = ai.viXra:2602.0035) and the companions ai.viXra:2602.0032/0033 [28, 29].

1.5. **Organization.** Section 2: geodesic convexity and its limits. Section 3: the Ricci bound. Section 4: the corrected ceiling computation. Section 5: the Witten–HS program. Section 6: discussion.

2. Geodesic convexity of the Wilson action

2.1. **The bi-invariant metric on $\mathrm{SU}(N_c)$.** We equip $G = \mathrm{SU}(N_c)$ with

$$\langle X, Y\rangle_U = -\operatorname{Tr}(XY), \qquad X, Y \in T_U G \cong \mathfrak{su}(N_c). \tag{4}$$

Geodesics through U are

$$\gamma(t) = U\exp(tX'), \qquad X' = U^{-1}X \in \mathfrak{su}(N_c), \tag{5}$$

and the geodesic distance from the identity is

$$\operatorname{dist}(U, I) = \|H\|_2 = \Big(\sum_j \lambda_j^2\Big)^{1/2}, \tag{6}$$

where $U = e^{iH}$, H hermitian traceless with eigenvalues $\lambda_j \in (-\pi, \pi]$.

2.2. **The Hessian formula.**

Theorem 2.1 (Geodesic convexity of $-\operatorname{Re}\operatorname{Tr}$)**.** *Let $f(U) = -\operatorname{Re}\operatorname{Tr} U$. For $U = e^{iH} \in B_{\pi/2}(I)$ and $X = iA$ with $A = A^\dagger$, the Riemannian Hessian of f at U is*

$$(\operatorname{Hess} f)_U(X, X) = \sum_{j,k=1}^{N_c} |A_{jk}|^2 \cos\lambda_j, \tag{7}$$

where $A_{jk} = \langle e_j, Ae_k\rangle$ *in the eigenbasis* $\{e_j\}$ *of* H*. In particular*

$$(\mathrm{Hess}\, f)_U(X,X) \;\ge\; \cos\big(\|H\|_{\mathrm{op}}\big)\,\|A\|_F^2 \;>\; 0 \qquad \text{on } B_{\pi/2}(I). \tag{8}$$

Proof. Along the geodesic $\gamma(t) = e^{iH}e^{itA}$:

$$f(\gamma(t)) = -\mathrm{Re}\,\mathrm{Tr}\big(e^{iH}e^{itA}\big), \tag{9}$$

$$\frac{d}{dt}f(\gamma(t)) = -\mathrm{Re}\,\mathrm{Tr}\big(e^{iH}\,iA\,e^{itA}\big) = \mathrm{Im}\,\mathrm{Tr}\big(e^{iH}Ae^{itA}\big), \tag{10}$$

$$\frac{d^2}{dt^2}f(\gamma(t)) = -\mathrm{Re}\,\mathrm{Tr}\big(e^{iH}(iA)^2e^{itA}\big) = \mathrm{Re}\,\mathrm{Tr}\big(e^{iH}A^2e^{itA}\big). \tag{11}$$

At $t = 0$:

$$(\mathrm{Hess}\, f)_U(X,X) = \mathrm{Re}\,\mathrm{Tr}\big(e^{iH}A^2\big). \tag{12}$$

Evaluating in the eigenbasis of H:

$$\mathrm{Tr}\big(e^{iH}A^2\big) = \sum_j e^{i\lambda_j}\langle e_j, A^2e_j\rangle = \sum_j e^{i\lambda_j}\sum_k |A_{jk}|^2, \tag{13}$$

whence

$$(\mathrm{Hess}\, f)_U(X,X) = \sum_{j,k}|A_{jk}|^2\cos\lambda_j. \tag{14}$$

Since $|\lambda_j| < \pi/2$ on $B_{\pi/2}(I)$, $\cos\lambda_j > 0$ and (8) follows. (v2: the sign discussion interrupting the v1 proof — an editorial artifact — is removed; the formula (7) is verified against finite differences of $f(e^{iH}e^{itA})$ to 10^{-5} over 40 random cases in $\mathfrak{su}(2)$ and $\mathfrak{su}(3)$ in the companion suite.) □

Remark 2.2 (Corrected in v2). Since $|A_{jk}|^2$ is symmetric in (j,k), (7) can be symmetrized using $\frac{1}{2}(\cos\lambda_j + \cos\lambda_k) = \cos\frac{\lambda_j+\lambda_k}{2}\cos\frac{\lambda_j-\lambda_k}{2}$:

$$(\mathrm{Hess}\, f)_U(X,X) = \sum_{j,k}\cos\Big(\frac{\lambda_j+\lambda_k}{2}\Big)\cos\Big(\frac{\lambda_j-\lambda_k}{2}\Big)|A_{jk}|^2,$$

which is again manifestly positive on $B_{\pi/2}(I)$. v1 stated instead a Daleckii–Kreĭn-type expression with $\mathrm{sinc}\big(\frac{\lambda_j-\lambda_k}{2}\big)$ in place of the second cosine and claimed it coincides with (7); this is false (suite: 1.84 vs 1.53 on the counterexample $H = 0.7\sigma_z$, $A = \sigma_x$). The sinc factor pertains to the differential of the matrix exponential, not to this Hessian.

2.3. **Convexity of the Wilson action: the v1 claim is false.**

Corollary 2.3 (Corrected scope; v1's statement refuted)**.** *Each plaquette term* $U_\square \mapsto -\mathrm{Re}\,\mathrm{Tr}\,U_\square$ *is strictly geodesically convex* as a function of the plaquette variable $U_\square \in B_{\pi/2}(I)$*, by Theorem 2.1. However, the Wilson action* S_{YM} *is* not *geodesically convex on*

$$\Omega_L = \Big\{U:\ \mathrm{dist}(U_\square, I) < \frac{\pi}{2}\ \ \forall\square\Big\}$$

as a function of the link variables, contrary to v1: along product geodesics $\gamma(t) = \{U_\ell e^{tX_\ell}\}$ *the plaquette path* $U_\square(t)$ *is not a geodesic of* $\mathrm{SU}(N_c)$*, and the acceleration terms omitted by the v1 proof can dominate. Numerical counterexample (suite): a single* $\mathrm{SU}(2)$ *plaquette with all four links within* $\pi/8$ *of* I *(hence* $\mathrm{dist}(U_\square, I) \le 0.55 < \pi/2$*, certified) and a unit product direction with*

$$\frac{d^2}{dt^2}S_W(\gamma(t))\Big|_{t=0} = -0.11 < 0,$$

with 73 *such violations in* 4000 *random trials.*

Remark 2.4. The set Ω_L contains the set where all links are within distance $\pi/8$ of the identity, since $\operatorname{dist}(U_\square, I) \le 4 \cdot \pi/8 = \pi/2$ by bi-invariance and the triangle inequality. (This inclusion, used to certify the counterexample above, is from v1 and is correct.)

Theorem 2.5 (Mass gap at strong coupling; status revised in v2)**.** *Let $d \ge 3$ and $N_c \ge 2$. There exists $\beta_{\mathrm{crit}} > 0$ such that for $\beta \le \beta_{\mathrm{crit}}$, lattice Yang–Mills with Wilson action on $(\mathbb{Z}/L\mathbb{Z})^d$ has a mass gap*

$$m \ \ge \ c(\beta, N_c, d) \ > \ 0, \tag{15}$$

uniformly in $L \ge 2$.

Status. This statement is true and classical: it follows from the strong-coupling cluster expansion of Osterwalder–Seiler [17] (cf. also [18]), which gives exponential clustering of all gauge-invariant correlations for β small. The v1 proof sketch via geodesic convexity and Riemannian Prékopa is *withdrawn* in v2, for two reasons: (a) its Step 1 rested on the convexity of S_{YM} on Ω_L, now refuted (Corollary 2.3); (b) its premise that "at strong coupling the measure concentrates on Ω_L" is inverted — as $\beta \to 0$ the Wilson measure approaches the *uniform* measure on $\mathcal{A}$, which does not concentrate near flat configurations (concentration near Ω_L occurs at *weak* coupling, where in turn the ceiling analysis of Section 4 applies). v1's Remark 2.6 already noted that the cluster expansion gives stronger results; in v2 it is the proof. □

Remark 2.6. Whether a corrected convexity route to (15) exists (e.g. convexity along plaquette-adapted variations, or on a smaller domain with quantitative control of the acceleration terms) is left open; the counterexample of Corollary 2.3 shows any such route must use more than termwise plaquette convexity.

3. Ricci curvature of the orbit space

3.1. **Setup.** $\mathcal{A} = \prod_\ell \mathrm{SU}(N_c)$ with the product metric

$$\langle V, W \rangle_{\mathcal{A}} = -\sum_\ell \operatorname{Tr}(V_\ell W_\ell), \tag{16}$$

and $\mathcal{G} = \prod_v \mathrm{SU}(N_c)$ acting by isometries

$$(g \cdot U)_\ell = g_{s(\ell)} U_\ell g_{t(\ell)}^{-1}. \tag{17}$$

At regular (irreducible) points, $\mathcal{B} = \mathcal{A}/\mathcal{G}$ inherits a metric making π a Riemannian submersion; $T_{[U]}\mathcal{B} \cong$ the horizontal subspace (lattice Coulomb gauge).

3.2. The Ricci curvature bound.

Theorem 3.1 (Ricci curvature of the orbit space; constant sharpened in v2)**.** *At every regular point $[U] \in \mathcal{B}$:*

$$\operatorname{Ric}_{\mathcal{B}}(X, X) \ \ge \ \frac{N_c}{2}\,|X|^2 \qquad \forall X \in T_{[U]}\mathcal{B}. \tag{18}$$

(v1 stated the bound with $N_c/4$; that is true but not sharp.)

Proof. O'Neill's formula [7, 8] for horizontal X^h:

$$\operatorname{Ric}_{\mathcal{B}}(X, X) = \operatorname{Ric}_{\mathcal{A}}(X^h, X^h) + \frac{3}{4}\sum_i \big|[X^h, E_i^h]^v\big|^2 \ \ge \ \operatorname{Ric}_{\mathcal{A}}(X^h, X^h). \tag{19}$$

Each factor $\mathrm{SU}(N_c)$ with the metric (4) is Einstein with $\mathrm{Ric} = -\frac{1}{4}B$, and the Killing form is $B(X,Y) = 2N_c \operatorname{Tr}(XY) = -2N_c\langle X, Y\rangle$, so $\mathrm{Ric}_{\mathrm{SU}(N_c)} = \frac{N_c}{2}\, g$; hence for the product

$$\mathrm{Ric}_{\mathcal{A}}(V,V) = \sum_{\ell} \mathrm{Ric}_{\mathrm{SU}(N_c)}(V_\ell, V_\ell) = \frac{N_c}{2}|V|^2_{\mathcal{A}}. \tag{20}$$

(v1 quoted $N_c/4$ from a different normalization of [8, Ch. 7]; the suite verifies $\mathrm{Ric}(X,X)/\|X\|^2 = 1.000,\ 1.500,\ 2.000$ for $N_c = 2, 3, 4$ via the structure-constant formula $\mathrm{Ric}(X,X) = \frac{1}{4}\sum_i \|[X, e_i]\|^2$.) For horizontal vectors $|X^h|_{\mathcal{A}} = |X|_{\mathcal{B}}$, giving (18). ☐

Corollary 3.2 (Spectral gap of the Laplacian). *The first nonzero eigenvalue of $\Delta_{\mathcal{B}}$ on the compact (orbifold) $\mathcal{B}$ satisfies*

$$\lambda_1(\Delta_{\mathcal{B}}) \ \geq\ \frac{N_c}{2}, \tag{21}$$

uniformly in L: by the Bakry–Émery criterion [9], $CD(N_c/2, \infty)$ holds and $\lambda_1 \geq N_c/2$ independently of the dimension; the singular set (reducible connections) has codimension ≥ 2 and does not affect the spectrum [10]. (v1: $N_c/4$.)

Corollary 3.3 (Kinetic mass gap). *The kinetic part $H_{\mathrm{kin}} = \frac{g^2}{2}(-\Delta_{\mathcal{B}})$ of (2) has*

$$E_1^{\mathrm{kin}} - E_0^{\mathrm{kin}} \ \geq\ \frac{g^2 N_c}{4}, \tag{22}$$

uniformly in L. (v1: $g^2N_c/8$.)

Remark 3.4. The full Hamiltonian includes the magnetic potential; adding a positive potential does not automatically preserve the gap. The Bakry–Émery criterion for the full measure requires $\mathrm{Ric}_{\mathcal{B}} + \mathrm{Hess}(f) \geq \kappa > 0$ for the appropriate ground-state exponent f; the potential Hessian includes terms of order $O(L^d/g^4)$ that can overwhelm the Ricci term at weak coupling. (v2: see ai.viXra:2602.0035 v2, Theorem 5.3, where the correct exponent $f = -\ln \psi_0^2$, with oscillation $O(L^d/g^2)$, replaces bookkeeping of this type.)

4. The $O(g^2)$ structural ceiling

This section contained the main new result of v1: that convexity-based methods produce a mass gap of at most $O(g^2)$. Version 2 re-derives the central computation exactly and adjudicates it numerically; the outcome (Theorem 4.2) differs from v1 quantitatively — v1's formula (28) and critical coupling (29) are withdrawn — while the ceiling itself survives on the independent grounds of Propositions 4.3–4.6.

4.1. The lattice tadpole integral.

Definition 4.1.

$$I_1 = \frac{1}{N}\sum_{q \in \Lambda^* \setminus \{0\}} \frac{1}{\hat{q}^2}, \qquad \hat{q}^2 = 4\sum_{\mu=1}^{d} \sin^2\frac{q_\mu}{2}, \qquad N = L^d, \tag{23}$$

with infinite-volume limits

$$I_1 \ \to\ \int_{[-\pi,\pi]^d} \frac{d^dk}{(2\pi)^d}\frac{1}{\hat{k}^2} = \begin{cases} 0.2527 & d = 3, \\ 0.1549 & d = 4, \end{cases} \tag{24}$$

verified numerically by extrapolation in the suite. (v2) Two further lattice integrals appear in the corrected computation:

$$\tilde{I}_1(L) = \frac{1}{N}\sum_{q\neq 0}\frac{\sin^2 q_\mu}{\hat{q}^4}, \qquad X(L) = \frac{1}{N}\sum_{q\neq 0}\frac{2\cos q_\mu}{\hat{q}^2}$$

(independent of μ by cubic symmetry).

4.2. **The Faddeev–Popov operator on the group.** In lattice Landau gauge the FP operator acts on $\psi \in L^2(\Lambda, \mathfrak{su}(N_c))$ as

$$[\mathcal{M}(U)\psi](x) = -\sum_{\mu}\Big[R(U_\mu(x))\psi(x+\hat{\mu}) + R\big(U_\mu^\dagger(x-\hat{\mu})\big)\psi(x-\hat{\mu}) - 2\psi(x)\Big], \tag{25}$$

with $R = \mathrm{Ad}$; at $U = I$, $\mathcal{M}_0 = -\hat{\partial}^2$. The Gribov–Zwanziger potential is

$$V(U) = S_{\mathrm{YM}}(U) - \ln\det\mathcal{M}(U). \tag{26}$$

4.3. **Geodesic Hessian at the trivial vacuum: the corrected computation.** Consider the geodesic through $U_\ell = I$,

$$U_\mu(x,t) = e^{tX_\mu(x)}, \qquad X_\mu(x) = s_\mu/\sqrt{N}, \quad s_\mu \in \mathfrak{su}(N_c) \text{ constant}, \tag{27}$$

and write $|s|^2 = \sum_\mu \|s_\mu\|_F^2$.

Theorem 4.2 (Zero-mode FP Hessian at the vacuum; corrected in v2)**.** *The geodesic Hessian of $V = S_{\mathrm{YM}} - \ln\det{}'\mathcal{M}$ at $U = I$ along (27) (with $\det{}'$ over the $q\neq 0$ modes; the $q = 0$ block is discussed in Remark 4.9) is, exactly,*

$$\frac{d^2V}{dt^2}\Big|_{t=0} = 0 + \underbrace{8N_c\,\tilde{I}_1(L)\,|s|^2}_{\textit{Gram (II)}} - \underbrace{2N_c\,X(L)\,|s|^2}_{\textit{curvature (III)}} = 2N_c\big[4\tilde{I}_1(L) - X(L)\big]|s|^2, \tag{28}$$

which is strictly positive at finite L and vanishes as $L\to\infty$:

$$0 \;<\; 2N_c\big[4\tilde{I}_1(L) - X(L)\big] \;=\; O(1/L) \qquad (d=3;\ \textit{numerically } L\,[4\tilde{I}_1 - X] \to 0.15). \tag{29}$$

In particular: the FP determinant contributes neither a positive Gribov-type mass $O(g^2)$ nor a negative $O(1)$ curvature at the vacuum zero modes; v1's formula for this Hessian and the derived critical coupling $g^2_{\mathrm{crit}} = (2dI_1 - 1)/I_1$ are withdrawn (Remark 4.10).

Proof. The decomposition is as in v1:

$$\frac{d^2V}{dt^2} = \frac{d^2S_{\mathrm{YM}}}{dt^2} + \mathrm{Tr}\big[(\mathcal{M}^{-1}\dot{\mathcal{M}})^2\big] - \mathrm{Tr}\big[\mathcal{M}^{-1}\ddot{\mathcal{M}}\big] =: \mathrm{(I)} + \mathrm{(II)} - \mathrm{(III)}, \tag{30}$$

with $\mathcal{M}^{-1} = \mathcal{M}_0^+$ the pseudo-inverse on the $q\neq 0$ sector (which is what v1's $q\neq 0$ momentum sums compute). (I) $= 0$: for constant X_μ the plaquette derivative vanishes at linear order and S_{YM} is quartic (the toron directions; cf. ai.viXra:2602.0032 v2, Prop. 5.3). For constant X, $\mathcal{M}(t)$ is translation-invariant and block-diagonal in momentum; differentiating (25):

$$\dot{\mathcal{M}}(q) = -\sum_\mu 2i\sin q_\mu\,\mathrm{ad}_\mu, \qquad \ddot{\mathcal{M}}(q) = -\sum_\mu 2\cos q_\mu\,\mathrm{ad}_\mu^2, \qquad \mathrm{ad}_\mu = \mathrm{ad}(s_\mu)/\sqrt{N}. \tag{31}$$

Using $\mathrm{Tr}_{\mathrm{adj}}\big(\mathrm{ad}(s)^2\big) = 2N_c\,\mathrm{Tr}(s^2) = -2N_c\|s\|_F^2$ and cubic symmetry (cross terms $\mu\neq\nu$ cancel under $q_\mu \to -q_\mu$):

$$\mathrm{(II)} = \sum_{q\neq 0}\frac{-4}{\hat{q}^4}\,\mathrm{Tr}\Big[\Big(\sum_\mu \sin q_\mu\,\mathrm{ad}_\mu\Big)^2\Big] = 8N_c\sum_\mu \|s_\mu\|_F^2\,\frac{1}{N}\sum_{q\neq 0}\frac{\sin^2 q_\mu}{\hat{q}^4} = 8N_c\tilde{I}_1|s|^2. \tag{32}$$

Similarly, using $2\cos q_\mu = 2 - \hat{q}_\mu^2$ and the exact identity

$$X(L) = \frac{1}{N}\sum_{q\neq 0}\frac{2\cos q_\mu}{\hat{q}^2} = 2I_1 - \frac{N-1}{Nd} \tag{33}$$

(v1's (33) with the exact finite-N coefficient; the cubic-symmetry sum $\frac{1}{N}\sum_{q\neq 0}\hat{q}_\mu^2/\hat{q}^2 = (N-1)/(Nd)$ is exact, not merely $1/d + O(1/N)$):

$$(\mathrm{III}) = 2N_c\, X(L)\, |s|^2. \tag{34}$$

Combining gives (28). All of (32)–(34) are verified to relative precision 10^{-3} (finite-difference $\dot{\mathcal{M}}, \ddot{\mathcal{M}}$) against the exact 192×192 FP operator on the $L=4$, $d=3$, SU(2) lattice, for both commuting and non-commuting $\{s_\mu\}$ (the result depends only on $|s|^2$, as v1 asserted). The vanishing (29) follows from Lemma 4.8. □

Proposition 4.3 (Stochastic localization; from v1)**.** *The stochastic localization method applied to the Yang–Mills measure requires $m_0^2 \geq \|\operatorname{Hess} W_{\mathrm{int}}\|_{\mathrm{op}}$ with $m_0^2 = g^2 N_c I_1/d$; since $\|\operatorname{Hess} W_{\mathrm{int}}\| = O(1)$, the condition fails for $g^2 = O(1)$ small.*

Proposition 4.4 (Dobrushin criterion; from v1)**.** *The Dobrushin uniqueness condition for the Yang–Mills conditional measures gives $\sum_y c_{xy} = 1 - O(g^2)$, producing a gap of $O(g^2)$.*

Proposition 4.5 (Entropy of the Gribov region; from v1)**.** *The effective zero-mode potential from the entropy of the Gribov section, $V_{\mathrm{eff}}(s) = -\ln \mathrm{Vol}(\Omega_s)$, equals the tree-level Gribov–Zwanziger potential $V^{\mathrm{tree}}(s) = \frac{g^2N_cI_1}{2d}|s|^2 + O(|s|^4)$, with curvature $m_0^2 = g^2N_cI_1/d = O(g^2)$. (v2: this* entropic *mechanism, imported from Paper I, is where the g^2 of the Gribov mass genuinely lives; it is not present in the bare Hessian of Theorem 4.2.)*

Proposition 4.6 (Orbit space Ricci vs. potential; from v1)**.** *The Bakry–Émery criterion for the full Yang–Mills measure on $\mathcal{B}$ requires $\mathrm{Ric}_{\mathcal{B}} + \mathrm{Hess}(f) \geq \kappa > 0$; the Ricci contribution is $O(1)$ while the potential Hessian has worst-case extensive eigenvalues, which overwhelm it for $L^d \gg g^4$. (Cf. the refined treatment in ai.viXra:2602.0035 v2.)*

Remark 4.7 (Universality of the ceiling; revised in v2)*.* v1 claimed five independent derivations of the ceiling, the main one being the computation behind v1's (28)–(29). In v2 the main computation is corrected (Theorem 4.2): it no longer produces a sign change at a finite g_{crit}, but something cleaner — the vacuum zero-mode FP Hessian is asymptotically *zero*. The ceiling now rests on: (a) Propositions 4.3–4.6, whose common mechanism (an $O(1)$ obstruction competing with an $O(g^2)$ mass scale) stands; and (b) Theorem 4.2 itself, which shows that no $O(1)$ *positive* curvature is available at the vacuum zero modes either — the only mass scale convexity methods can access there is the entropic $m_0^2 = O(g^2)$ of Proposition 4.5. Hence: convexity-based methods produce a mass gap of at most $O(g^2)$ in lattice units.

Lemma 4.8 (New in v2: partial-integration identity)**.** *In the infinite-volume limit,*

$$X_\infty = \int_{[-\pi,\pi]^d}\frac{d^dq}{(2\pi)^d}\frac{2\cos q_\mu}{\hat{q}^2} = 4\int_{[-\pi,\pi]^d}\frac{d^dq}{(2\pi)^d}\frac{\sin^2 q_\mu}{\hat{q}^4} = 4\tilde{I}_{1,\infty} \qquad (d\geq 3), \tag{35}$$

so the bracket in (28) vanishes as $L\to\infty$.

Proof. Since $\partial_{q_\mu}(1/\hat{q}^2) = -2\sin q_\mu/\hat{q}^4$, integration by parts in q_μ over the periodic Brillouin zone gives

$$\int \frac{\cos q_\mu}{\hat{q}^2}\,dq_\mu = \left[\frac{\sin q_\mu}{\hat{q}^2}\right]_{-\pi}^{\pi} + \int \sin q_\mu\cdot\frac{2\sin q_\mu}{\hat{q}^4}\,dq_\mu = 2\int\frac{\sin^2 q_\mu}{\hat{q}^4}\,dq_\mu;$$

the boundary term vanishes by periodicity and the integrands are integrable at $q = 0$ for $d \geq 3$. Multiplying by 2 and integrating over the remaining components yields the claim. Numerically, $L\,[4\tilde{I}_1(L) - X(L)] \to 0.150$ ($d = 3$), confirming the $O(1/L)$ rate. □

Remark 4.9 (New in v2: the $q = 0$ block and reducibility). Along (27) the three (for SU(2)) $q = 0$ FP modes have eigenvalues $\varepsilon(t) = O(t^2)$: they are the global-gauge modes, whose vanishing at $t = 0$ reflects the reducibility of the trivial vacuum. Including them in $\ln\det \mathcal{M}(t)$ produces a $+O(-\ln t^2)$ log-repulsion from the reducible stratum, singular at $t = 0$; v1's momentum sums (and Theorem 4.2) use the primed determinant, which excludes them. Since for constant X the momentum blocks decouple exactly, the primed prescription is self-consistent. The treatment of this block is part of the definition of the gauge-fixed measure and should be stated explicitly in any use of (26).

Remark 4.10 (New in v2: what is withdrawn). v1's equation (28), $\mathrm{Hess}\, V = N_c[(1 + g^2 I_1)/d - 2I_1]|s|^2$, and the derived critical coupling (29), $g^2_{\text{crit}} = (2dI_1 - 1)/I_1$ (Table 1 of v1: $g^2_{\text{crit}} = 2.04$ ($d = 3$), 1.54 ($d = 4$)), are withdrawn: (a) the relative factor g^2 between the Gram and curvature terms is inconsistent (both derive from the same $\mathcal{M}(e^{tX})$ with the same normalization of X); (b) the Gram term has the shape $8N_c\tilde{I}_1$, not $N_c I_1/d$ (measured ratio 2.81 against v1's formula at $L = 4$); (c) the curvature term is $2N_c X$, twice v1's $N_c(2I_1 - 1/d)$ (measured ratio 2.17); (d) the resulting bracket does not change sign in g at all. The suite adjudicates all four points against the exact lattice FP operator.

5. Beyond convexity: the Witten–Helffer–Sjöstrand program

5.1. **Why convexity cannot reach Λ_{QCD}.** The physical mass gap is

$$m_{\text{phys}} \sim \Lambda_{\text{QCD}} = \mu \exp\Big(-\frac{1}{2b_0 g^2(\mu)}\Big), \qquad b_0 = \frac{11N_c}{48\pi^2}, \tag{36}$$

i.e. $m_{\text{lat}} \sim e^{-c/g^2}$, exponentially below any $O(g^2)$ bound. The discrepancy is structural: convexity methods see the local curvature at the minimum; the physical gap is set by the global topology of $\mathcal{B}$ (tunneling between approximate vacua).

5.2. **The orbit space as a Morse landscape.** Critical points of V_{pot} on $\mathcal{B}$: the vacuum (index 0), the sphaleron (index 1; Klinkhamer–Manton [15] in the continuum, energy $E_{\text{sph}} = c_{\text{sph}} 4\pi/g^2$), and higher saddles.

Remark 5.1. On the lattice $\mathcal{B}$ is simply connected, so there are no exact topological sectors; the structure emerges energetically, with barriers $O(1/g^2)$ between approximate-charge valleys.

5.3. **The Witten Laplacian.** The Kogut–Susskind Hamiltonian, conjugated by the ground state, is the Witten Laplacian

$$\Delta^{(0)}_{V,h} = -h\Delta_{\mathcal{B}} + \frac{1}{h}|\nabla V_{\text{pot}}|^2 - \Delta V_{\text{pot}}, \qquad h = g^2/2. \tag{37}$$

5.4. **The Helffer–Sjöstrand framework.**

Theorem 5.2 (Helffer–Sjöstrand, informal [13, 14]). *For a compact M and a Morse function ϕ, as $h \to 0$: the number of eigenvalues of $\Delta^{(0)}_{\phi,h}$ in $[0, h^{3/2}]$ equals the number of local minima; with a unique minimum, $\lambda_1 \asymp e^{-2d_{\text{Ag}}/h}$ with d_{Ag} the Agmon distance to the nearest index-1 saddle; prefactors are computable from the Hessians at the critical points.*

5.5. **Application to Yang–Mills.**

Conjecture 5.3 (Mass gap from sphaleron tunneling)**.** *For* $\mathrm{SU}(N_c)$ *lattice Yang–Mills on* $(\mathbb{Z}/L\mathbb{Z})^d$*,* $d \geq 3$*,*

$$m_{\text{gap}} \asymp g^{-p} \exp\Big(-\frac{S_{\text{inst}}}{g^2}\Big) \tag{38}$$

in the semiclassical regime, giving $m_{\text{phys}} \sim \Lambda_{\text{QCD}}$ *in the continuum limit.*

Remark 5.4. The exponential factor is the instanton weight, generating Λ_{QCD} by dimensional transmutation.

Remark 5.5 (New in v2: the vacuum is not a Morse point)*.* The companion papers sharpen the landscape picture in one essential respect: the vacuum critical point is *not* Morse–Bott non-degenerate — at $\theta = 0$ (and the whole orbifold locus of the flat moduli) the $k = 0$ non-Cartan toron modes are normal directions with quartic potential (ai.viXra:2602.0032 v2, Prop. 5.3; re-verified in ai.viXra:2602.0035 v2). Consequently Theorem 5.2 does not apply verbatim to V_{pot} on $\mathcal{B}$ (its Morse hypothesis fails exactly at the relevant minimum), and any execution of the program of this section must supplement Helffer–Sjöstrand with the small-volume quartic sector (Lüscher, Koller–van Baal); this is hypothesis (H-MB) of ai.viXra:2602.0032 v2, where the Witten route of Section 5.5 is carried out conditionally. The open problems below should be read with this amendment.

5.6. **Open problems.** (1) Helffer–Sjöstrand uniformly in the dimension (as $L \to \infty$); (2) rigorous lattice sphaleron (index-1 saddle for all $L \geq L_0$); (3) prefactor control (fluctuation determinants; connects to Balaban's program [21, 22]). (v2: plus the quartic-sector amendment of Remark 5.5.)

6. Discussion

6.1. **Physical interpretation.** The gap has a local component — now understood, per Theorem 4.2 and Proposition 4.5, as *entropic* $O(g^2)$ rather than bare-Hessian — captured by convexity methods, and a global component $O(e^{-c/g^2})$ from sphaleron tunneling, requiring the Witten–HS framework. In the continuum limit the local component is a lattice artifact; the global one is physical.

6.2. **The local-global decomposition.** One-sentence summary (revised in v2): *the Yang–Mills mass gap on* $\mathcal{B}$ *has an entropic local component* $O(g^2)$ *(the vacuum FP Hessian itself vanishes as* $O(1/L)$*: Theorem 4.2) and a global component* $O(e^{-c/g^2})$ *from tunneling; convexity methods reach only the former.*

6.3. **Comparison with lattice data.** The (entropic) Gribov mass in $d = 3$ for SU(2) is $m_0 = \sqrt{g^2 N_c I_1/d} = g\sqrt{2 \times 0.2527/3} \approx 0.41\, g$ (in the $3d$ dimensional convention often written $0.41\, g^2$, since g^2 carries the mass dimension). The lowest SU(2) glueball mass in 2+1 dimensions is $m_{0^{++}} \approx 0.5\, g^2$ [25], within 20%: the local component captures the dominant $d = 3$ physics. In $d = 4$ the gap is $\sim e^{-c/g^2}$ and the convexity bound is uninformative. (v2: Table 1's m_0 column is corrected — v1's "$0.168 N_c$" double-counted N_c; the correct entry is $m_0^2 = g^2 N_c I_1/d$, i.e. $0.0842\, g^2 N_c$ ($d = 3$), $0.0387\, g^2 N_c$ ($d = 4$).)

d	I_1	m_0^2 (entropic)	v1's g^2_{crit}
3	0.2527	$0.0842\, g^2 N_c$	2.04 (withdrawn in v2)
4	0.1549	$0.0387\, g^2 N_c$	1.54 (withdrawn in v2)

Table 1. Lattice tadpole values and the entropic Gribov mass (corrected); the v1 critical couplings are withdrawn per Theorem 4.2.

6.4. **Outlook.** (1) Strong coupling: solved by cluster expansions [17]; the v1 convexity alternative is withdrawn (Corollary 2.3, Theorem 2.5). (2) Finite lattice, all couplings: Witten–HS on $\mathcal{B}_L$, now with the toron amendment (Remark 5.5); executed conditionally in ai.viXra:2602.0032 v2. (3) Infinite volume and continuum: constructive control of the instanton calculus, building on Balaban; cf. the conditional syntheses ai.viXra:2602.0033/0035 v2.

Note added in v2

Prepared with the companion suite `verify_2602_0036.py` (repository `THE-ERIKSSON-PROGRAMME`, `verification/2602-0036/`; 12 checks, all passing, $\sim$25 s, NumPy/SciPy). Summary of changes: (1) Corollary 2.3 *refuted* as stated in v1 (numerical counterexample: $d^2S_W/dt^2 = -0.11$ with all links within $\pi/8$ of I; the plaquette map does not send product geodesics to geodesics); Theorem 2.5 restated with the cluster expansion as its proof and the Prékopa sketch withdrawn (its concentration premise was also inverted: $\beta \to 0$ gives the uniform measure); (2) Theorem 3.1 sharpened at the source: $\mathrm{Ric}_{\mathcal{B}} \geq N_c/2$ (v1: $N_c/4$), with Corollaries 3.2–3.3 updated ($\lambda_1 \geq N_c/2$, kinetic gap $g^2N_c/4$); the downstream papers quoted $N_c/4$, which remains a valid non-sharp bound; (3) Theorem 4.2 corrected: the exact zero-mode FP Hessian at the vacuum is $2N_c[4\tilde{I}_1(L) - X(L)]|s|^2 = O(1/L) \to 0^+$, adjudicated against the exact lattice FP operator; v1's (28)–(29) (and Table 1's $g^2_{\mathrm{crit}}, \beta_{\mathrm{crit}}$ columns) are withdrawn — the g^2 was inserted inconsistently in the Gram term, the Gram shape is $8N_c\tilde{I}_1$ (not N_cI_1/d), and the curvature term carries an extra factor 2; new Lemma 4.8 ($X_\infty = 4\tilde{I}_{1,\infty}$ by partial integration) and Remarks 4.9–4.10 (the $q = 0$ log-repulsion block; the withdrawal record); the $O(g^2)$ ceiling itself survives via Propositions 4.3–4.6 and the corrected computation; (4) Remark 2.2's sinc formula corrected (the true symmetrization is $\cos \cdot \cos$); (5) new Remark 5.5: the vacuum is not a Morse point (toron degeneracy; companions), amending the Witten–HS program and its open problems; (6) repairs: the editorial "Wait—" artifact inside the proof of Theorem 2.1, the placeholder reference "[Author], [Journal and reference for Paper I]" (now ai.viXra:2602.0038; the submitted title differs from the working title), the date typo "Februari", Table 1's m_0 column ($0.168N_c$ double-counted N_c). One consequential flag for the series: v1's Remark 5.2 of Paper III (and its v2, which retained the sentence) attributes the FP obstruction to the curvature term "overwhelming" the Gram matrix; per Theorem 4.2 the Gram term in fact dominates at finite L and the difference vanishes as $L \to \infty$ — the obstruction to convexity at weak coupling is the extensive-potential mechanism of Proposition 4.6, not a negative vacuum FP Hessian. This is recorded for the next revisions of the companions. All v1 numbered statements keep their numbers; new items (4.8–4.10, 5.5) are numbered after the v1 items. Numbered equations are preserved in count and position, with proof displays re-purposed where the corrected derivation replaces the withdrawn one (documented here).

References

[1] A. Jaffe and E. Witten, "Quantum Yang–Mills theory," Clay Mathematics Institute (2000).

[2] P. Mondal, "Spectral gap of pure Yang–Mills theory using the orbit space geometry," JHEP **2023**, 191 (2023).

[3] L. Eriksson, "Mass gap for the Gribov–Zwanziger lattice measure: a non-perturbative proof," ai.viXra:2602.0038, 2026 (Paper I; cited in v1 by a placeholder).

[4] V. N. Gribov, "Quantization of non-Abelian gauge theories," Nucl. Phys. B **139**, 1–19 (1978).

[5] D. Zwanziger, "Local and renormalizable action from the Gribov horizon," Nucl. Phys. B **323**, 513–544 (1989).

[6] D. Zwanziger, "Non-perturbative Faddeev–Popov formula and the infrared limit of QCD," Phys. Rev. D **69**, 016002 (2004).

[7] B. O'Neill, "The fundamental equations of a submersion," Michigan Math. J. **13**, 459–469 (1966).
[8] A. L. Besse, *Einstein Manifolds*, Springer, 1987.
[9] D. Bakry and M. Émery, "Diffusions hypercontractives," Sém. Prob. XIX, LNM 1123, Springer, 1985, pp. 177–206.
[10] L. Bérard-Bergery and J.-P. Bourguignon, "Laplacians and Riemannian submersions with totally geodesic fibres," Illinois J. Math. **26**, 181–200 (1982).
[11] H. J. Brascamp and E. H. Lieb, "On extensions of the Brunn–Minkowski and Prékopa–Leindler theorems," J. Funct. Anal. **22**, 366–389 (1976).
[12] D. Cordero-Erausquin, R. J. McCann, and M. Schmuckenschläger, "A Riemannian interpolation inequality à la Borell, Brascamp and Lieb," Invent. Math. **146**, 219–257 (2001).
[13] B. Helffer and J. Sjöstrand, "Puits multiples en mécanique semi-classique. IV," Comm. PDE **10**, 245–340 (1985).
[14] D. Le Peutrec, F. Nier, and C. Viterbo, "Precise Arrhenius law for p-forms," Ann. Henri Poincaré **14**, 543–600 (2013).
[15] F. R. Klinkhamer and N. S. Manton, "A saddle-point solution in the Weinberg–Salam theory," Phys. Rev. D **30**, 2212 (1984).
[16] M. García Pérez and P. van Baal, "Sphalerons on a lattice," Nucl. Phys. B **429**, 451–473 (1994).
[17] K. Osterwalder and E. Seiler, "Gauge field theories on a lattice," Ann. Phys. **110**, 440–471 (1978).
[18] E. Seiler, *Gauge Theories as a Problem of Constructive QFT and Statistical Mechanics*, LNP 159, Springer, 1982.
[19] A. Laufer and P. Orland, "The metric on the space of lattice gauge orbits," (1996).
[20] B. K. Driver and T. Lohrenz, "Logarithmic Sobolev inequalities for pinned loop groups," J. Funct. Anal. **140**, 381–448 (1996).
[21] T. Balaban, "Renormalization group approach to lattice gauge field theories. I," Comm. Math. Phys. **109**, 249–301 (1987).
[22] T. Balaban, "Large field renormalization. II," Comm. Math. Phys. **122**, 355–392 (1989).
[23] R. Eldan, "Thin shell implies spectral gap up to polylog via a stochastic localization scheme," GAFA **23**, 532–569 (2013).
[24] Y. Chen and R. Eldan, "Localization schemes: a framework for proving mixing bounds for Markov chains," FOCS 2022.
[25] M. Teper, "$SU(N)$ gauge theories in $2+1$ dimensions," Phys. Rev. D **59**, 014512 (1998).
[26] A. Athenodorou and M. Teper, "$SU(N)$ gauge theories in $2+1$ dimensions: glueball spectra and k-string tensions," JHEP **02** (2017) 015.
[27] L. Eriksson, "Morse–Bott spectral reduction and the Yang–Mills mass gap on the lattice," ai.viXra:2602.0035 (v2), 2026.
[28] L. Eriksson, "The Yang–Mills mass gap on the lattice: a conditional reduction via Witten Laplacian and constructive renormalization," ai.viXra:2602.0032 (v2), 2026.
[29] L. Eriksson, "The Yang–Mills mass gap on the lattice: a conditional synthesis," ai.viXra:2602.0033 (v2), 2026.

Independent researcher
Email address: `lluiseriksson@gmail.com`