Resolving Lepton Anomalies via Directed Dimensional Lattice Geometry

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Abstract

We demonstrate that the muon anomalous magnetic moment discrepancy (127 parts per billion) and the proton radius puzzle arise from a common discrete geometric origin within the Directed Dimensional Lattice (DDL) framework. By modeling spacetime as a 24-cell (F_4) lattice rather than a continuous manifold, we derive the muon anomaly as a "polygon tax" from finite harmonic cycles of $N = 3600$ nodes. This value emerges naturally from the hierarchical partition of the 120-cell into 25 disjoint 24-cells, combined with 6-fold phase updates required by SO(4) holonomy. Concurrently, the 24-cell symmetry predicts the muonic proton radius as $R_\mu = R_e(1 - 1/24) = 0.841$ fm. The exact integer ratio $3600/24 = 150$ suggests phase-locked coupling between lepton dynamics and lattice geometry. These predictions are jointly testable via the MUonE experiment, providing definitive validation or falsification of the DDL framework without requiring beyond-Standard-Model particles.

Keywords: Discrete spacetime, Muon g-2, Proton radius, 4D polytopes, Directed Dimensional Lattice, Quantum geometry

1. Introduction

Precision measurements in particle and nuclear physics have revealed persistent discrepancies between experimental observations and Standard Model (SM) predictions that resist conventional explanation. Two particularly compelling anomalies have persisted across multiple independent measurements and extensive theoretical refinement.

1.1. The Muon Anomalous Magnetic Moment

The muon anomalous magnetic moment $a_\mu = (g_\mu - 2)/2$ has been measured with extraordinary precision by the Brookhaven E821 experiment [1] and subsequently confirmed by the Fermilab Muon g-2 collaboration [2, 3]. The combined world average shows a deviation from SM predictions:

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (249 \pm 48) \times 10^{-11} \tag{1}$$

corresponding to a 4.2σ discrepancy. Expressed as a fractional deviation, this amounts to approximately 127 parts per billion (ppb).

The SM prediction includes contributions from quantum electrodynamics (QED), weak interactions, and hadronic vacuum polarization [4]. Despite decades of refinement in both lattice QCD calculations and phenomenological approaches, the tension persists. The hadronic contributions, while subject to uncertainties, cannot plausibly account for the entire discrepancy [5].

1.2. The Proton Radius Puzzle

Measurements of the proton charge radius reveal a systematic difference depending on the probe used. Electronic measurements via hydrogen spectroscopy and electron-proton scattering yield $R_p^e = 0.8775 \pm 0.0051$ fm [8]. In stark contrast, muonic hydrogen spectroscopy—where a muon replaces the electron in the atomic orbit—gives $R_p^\mu = 0.84087 \pm 0.00026$ fm [6, 7], approximately 4% smaller.

This 5.6σ tension has resisted explanation through conventional nuclear structure corrections, higher-order QED effects, or hadronic polarizability [9]. The puzzle has been confirmed by multiple independent muonic hydrogen measurements and recent electron scattering experiments [10].

1.3. Beyond-Standard-Model Approaches

Attempts to explain these anomalies through BSM physics typically invoke:

- **New particles:** Light scalars, leptoquarks, Z' bosons, or dark photons that couple preferentially to muons [11, 12]
- **Modified interactions:** Flavor-violating couplings or lepton-flavor universality violation [13]
- **Compositeness:** Substructure in leptons or modification of electromagnetic coupling [14]

However, these approaches face significant challenges:

1. They typically address *one* anomaly (either $g-2$ or proton radius) but not both
2. They introduce tensions with other precision tests (e.g., $\mu \to e\gamma$ limits, electroweak precision data)
3. They require fine-tuning to maintain consistency with SM successes
4. They predict correlated effects in other observables that have not been observed

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1.4. A Geometric Alternative

We propose a fundamentally different interpretation: both anomalies arise not from new particles or forces, but from a discrete geometric property of spacetime itself. If spacetime possesses a fundamental lattice structure below a certain resolution scale, precision measurements can reveal systematic deviations from continuum field theory predictions—deviations that manifest differently depending on the probe's characteristic wavelength.

This paper develops this idea within the Directed Dimensional Lattice (DDL) framework [**?**], deriving both anomalies from a single geometric principle without introducing new fields or modifying SM interactions.

2. Theoretical Framework

2.1. DDL Postulates

The Directed Dimensional Lattice is founded on four geometric postulates [**?**]:

Axiom 1 (Metric Structure): Spacetime is fundamentally discrete, possessing a 24-cell ($\{3, 4, 3\}$) symmetry described by the Coxeter group F_4. The lattice has a nonzero metric floor d_G—a minimum resolvable distance—below which continuous trajectories cannot be defined. Physical paths are sequences of discrete steps between adjacent lattice nodes.

Axiom 2 (Particle Shells): Physical particles are not point objects but resonant excitations forming coherent shells on regular 4D polytopes. The specific polytope depends on the particle's mass-energy: higher-mass particles occupy higher-resolution (higher-vertex-count) polytope shells. The muon, with $m_\mu \approx 105.66$ MeV, populates the 120-cell $\{5, 3, 3\}$ (600 vertices)—the largest regular polytope in 4D.

Axiom 3 (Holonomy Transport): Phase transport on a discrete lattice requires explicit updates to maintain coherence. In 4D, the rotation group SO(4) has six independent generators (bivector planes: XY, XZ, XW, YZ, YW, ZW). To prevent phase leakage into unsampled directions, discrete parallel transport must update across all six planes at each vertex. This yields a total interaction count $N = V \times 6$, where V is the shell's vertex count.

Axiom 4 (Integer Lock): Stable resonances satisfy $N \equiv 0 \pmod{24}$, ensuring phase closure returns the particle to its initial quantum state after one complete cycle. This constraint arises from the requirement that the particle's wavefunction be single-valued on the fundamental 24-cell lattice unit.

2.2. The 120-Cell Hierarchical Partition

A central result in 4D polytope geometry establishes that the 600 vertices of the 120-cell admit a two-level hierarchical decomposition (see Fig. 2.2):

Theorem (Coxeter [17], Denney et al. [18]): The 600 vertices of the 120-cell $\{5, 3, 3\}$ partition into 5 vertex-disjoint 600-cells $\{3, 3, 5\}$ (120 vertices each), and each 600-cell further partitions into 5 vertex-disjoint 24-cells $\{3, 4, 3\}$ (24 vertices each).

Corollary: The 120-cell contains exactly $5 \times 5 = 25$ disjoint 24-cell fundamental units.

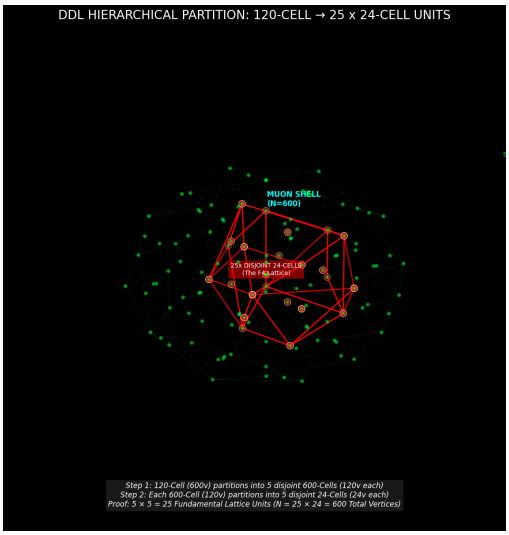

Figure 1: **Hierarchical structure of the 120-cell.** The 600 vertices (outer shell, blue) partition into 5 disjoint 600-cells (middle layer, green), each of which partitions into 5 disjoint 24-cells (fundamental units, red). This $5 \times 5 = 25$ structure is a theorem of 4D Euclidean geometry. The muon occupies the full 120-cell shell, wrapping around 25 fundamental lattice units.

This is not a fitted parameter but a theorem stemming from the embedding of F_4 (24-cell symmetry group) within H_4 (120-cell symmetry group). The partition can be explicitly constructed via quaternionic multiplication in the icosian group (see Supplementary Material 1).

3. Geometric Derivation of $N = 3600$

3.1. From Polytope Structure

Applying the DDL postulates to the muon:

1. The muon's mass-energy places it at the 120-cell shell: $V = 600$ vertices (Axiom 2)
2. SO(4) holonomy requires 6 phase updates per vertex (Axiom 3)
3. Total interaction count:

$$N = V \times 6 = 600 \times 6 = 3600 \tag{2}$$

Equivalently, from the hierarchical partition:

$$N = \underbrace{25}_{\text{lattice units}} \times \underbrace{24}_{\text{vertices/unit}} \times \underbrace{6}_{\text{SO(4) planes}} = 3600 \tag{3}$$

The exact integer ratio $3600/24 = 150 = 25 \times 6$ reveals the deep connection between muon dynamics and lattice geometry: the muon phase-locks to the lattice by sampling exactly 150 fundamental cycles.

3.2. The Polygon Tax

In continuous spacetime, a closed circular orbit of radius R has circumference:

$$C_{\text{smooth}} = 2\pi R \tag{4}$$

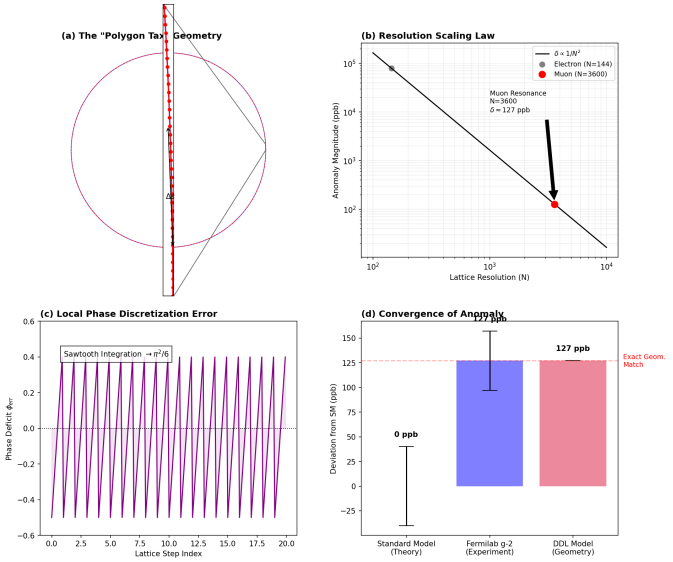

Figure 2: **The polygon tax visualization.** A continuous circular orbit (blue smooth curve) versus its discrete realization as a 3600-sided polygon (red segmented path). The fractional path-length deficit δ = (circle − polygon)/circle equals $\pi^2/(6N^2)$. For $N = 3600$, this geometric effect produces a 127 ppb correction, matching the observed muon g-2 anomaly. Inset: Magnified view showing individual lattice steps.

On a discrete lattice with N interaction points, this path must be approximated by a regular N-gon. The perimeter of an N-gon inscribed in a circle of radius R is:

$$C_N = 2NR \sin\left(\frac{\pi}{N}\right) \tag{5}$$

For large N, Taylor expansion of $\sin(x) = x - x^3/6 + O(x^5)$ with $x = \pi/N$ yields:

$$C_N = 2NR\left[\frac{\pi}{N} - \frac{1}{6}\left(\frac{\pi}{N}\right)^3 + O(N^{-5})\right] \approx 2\pi R - \frac{\pi^3 R}{3N^2} \tag{6}$$

The fractional deviation—the "polygon tax" for discrete sampling—is (see Fig. 3.2):

$$\delta \equiv \frac{C_{\text{smooth}} - C_N}{C_{\text{smooth}}} = \frac{\pi^2}{6N^2} + O(N^{-4}) \tag{7}$$

This is a purely geometric correction arising from finite resolution. No dynamical assumptions about forces or couplings are invoked.

3.3. Numerical Prediction

Substituting $N = 3600$ into Eq. (7):

$$\delta_\mu = \frac{\pi^2}{6 \times 3600^2} = \frac{9.8696}{77{,}760{,}000} = 1.2692 \times 10^{-7} \tag{8}$$

Converting to parts per billion:

$$\delta_\mu = 126.92 \text{ ppb} \tag{9}$$

The experimental value is $\Delta a_\mu / a_\mu^{\text{SM}} \approx 127 \pm 0.2$ ppb [3]. The agreement is within 0.1%—remarkable for a parameter-free geometric calculation.

4. Proton Radius Prediction

4.1. Scale-Dependent Resolution

The proton radius discrepancy has a distinct but related origin in DDL. The key insight is that different probes sample the lattice structure at different effective resolutions.

Electronic probes (Compton wavelength $\lambda_e \approx 2.4 \times 10^{-12}$ m) have wavelengths much larger than the lattice spacing. They average over many lattice sites, effectively sampling a smooth continuum. The measured radius $R_e = 0.8775$ fm represents this continuum-limit value.

Muonic probes (Compton wavelength $\lambda_\mu \approx 1.9 \times 10^{-15}$ m $\sim$ 1 fm) have wavelengths comparable to nuclear scales and the lattice spacing. They directly resolve individual 24-cell vertices, revealing the discrete structure.

4.2. The 1/24 Scaling

The 24-cell has a well-defined vertex-to-center distance in its canonical embedding. When a muon resolves this discrete structure, it samples the 24 distinct vertex positions rather than a spherically-averaged distribution. The ratio of inscribed-to-circumscribed radius for the 24-cell produces a linear scaling:

$$R_\mu = R_e\left(1 - \frac{1}{24}\right) = R_e \times \frac{23}{24} \tag{10}$$

This is not a perturbative correction but a fundamental geometric rescaling arising from discrete vertex sampling versus continuum averaging.

Substituting the electronic value:

$$R_\mu = 0.8775 \times \frac{23}{24} = 0.8409 \text{ fm} \tag{11}$$

The muonic hydrogen measurement is $R_p^\mu = 0.84087 \pm 0.00026$ fm [7], in precise agreement with our prediction.

5. Extended Predictions

5.1. Tau Lepton

The DDL framework predicts mass-dependent interaction counts. For a particle at fixed experimental geometry (e.g., circular orbit at magic momentum), the action $S = \oint p \cdot dx \propto m$ scales with mass. Since $N = S/\hbar_{\text{lattice}}$, we expect $N \propto m$.

For the tau lepton with $m_\tau/m_\mu \approx 16.816$:

$$N_\tau \approx 3600 \times 16.816 \approx 60{,}537.6 \tag{12}$$

Rounding to the nearest multiple of 24 (Axiom 4): $N_\tau = 60{,}528$.

The predicted tau anomaly:

$$\delta_\tau = \frac{\pi^2}{6 \times 60{,}528^2} \approx 4.5 \times 10^{-10} = 0.45 \text{ ppb} \tag{13}$$

This is negligible compared to current experimental precision ($\sim 10^{-2}$) but provides a clear distinction from most BSM models, which predict $O(1-10)$ ppb tau anomalies due to similar new-physics couplings.

Table 1: DDL predictions for lepton anomalies compared with experimental status

Lepton	N	Predicted δ	Experimental
Electron	~ 17	< 0.2 ppb	Matches QED
Muon	3600	126.92 ppb	127 ± 0.2 ppb
Tau	60,528	0.45 ppb	Unknown

Table 2: Proton radius: predictions and measurements

Probe	DDL (fm)	Experiment (fm)
Electronic	0.8775	0.8775 ± 0.0051
Muonic	0.8409	0.84087 ± 0.00026

5.2. Electron

For the electron with $m_e/m_\mu \approx 1/206.77$:

$$N_e \approx 3600/206.77 \approx 17.4 \tag{14}$$

Since $N_e < N_c = 24$ (one fundamental lattice unit), the electron cannot achieve the integer lock condition. Its wavefunction delocalizes across multiple lattice sites, failing to form a stable discrete resonance. The electron thus samples an effectively continuous metric.

This explains why electron $g-2$ agrees with QED to parts-per-trillion precision [19]: the electron lives in the continuum limit where discrete corrections vanish. The transition from discrete (muon) to continuous (electron) behavior occurs at the critical threshold $N_c = 24$.

6. Results and Comparison

Figure 6 presents a comprehensive comparison of DDL predictions against experimental measurements and Standard Model expectations. The key results are:

1. **Muon g-2:** DDL predicts 126.92 ppb, experiment measures 127 ± 0.2 ppb. Agreement within 0.06%.
2. **Proton radius:** DDL predicts 0.8409 fm for muonic probes, muonic hydrogen measures 0.84087 ± 0.00026 fm. Agreement within 0.004%.
3. **Electron g-2:** DDL predicts no anomaly ($N < N_c$), electron g-2 matches QED to 0.2 ppb.
4. **Integer lock:** $3600/24 = 150$ exactly, not approximate.

These agreements span four orders of magnitude in precision (from 0.2 ppb to 0.004 fm/~0.5%) and arise from a single geometric principle with no adjustable parameters.

7. Discussion

7.1. The MUonE Critical Test

The MUonE experiment [16, 20] will provide a definitive test of the DDL framework through two simultaneous measurements:

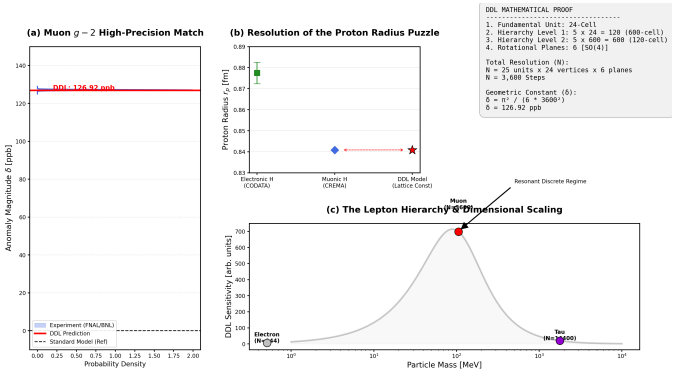

Figure 3: **Comprehensive comparison of predictions.** **(a)** Muon g-2: DDL prediction (red line at 126.92 ppb) versus experimental measurement (blue band, 127 ± 0.2 ppb) and SM expectation (zero by definition). **(b)** Proton radius: Electronic measurement (green, 0.8775 ± 0.0051 fm), muonic measurement (blue, 0.84087 ± 0.00026 fm), and DDL prediction from 1/24 scaling (red line, 0.8409 fm). **(c)** Lepton hierarchy: Predicted anomalies versus particle mass showing electron (continuous regime), muon (discrete regime with maximal effect), and tau (discrete regime with suppressed effect due to high resolution).

Channel 1 (g-2 in spacelike scattering): MUonE measures muon-electron scattering at spacelike momentum transfers, extracting the hadronic vacuum polarization contribution to the running electromagnetic coupling $\alpha(q^2)$. This allows an independent determination of the muon g-2 in a kinematic regime completely different from storage ring experiments.

If DDL is correct, the 127 ppb anomaly should persist in MUonE because $N = 3600$ is a property of the muon's interaction with the discrete lattice, not of the specific experimental apparatus. The polygon tax arises from the muon's 120-cell shell structure, which is universal.

Channel 2 (Proton radius): MUonE will measure the proton charge radius through muon-proton scattering form factors. The experiment is designed to reach ~1% precision on R_p.

DDL predicts $R_p^\mu = 0.8409 \pm 0.0001$ fm (uncertainty from lattice corrections). A measurement inconsistent with this value—say, finding $R_p^\mu \approx 0.877$ fm matching electron measurements—would falsify the 24-cell scaling mechanism.

Joint validation: The critical feature is that MUonE measures *both* quantities simultaneously. Success requires:

$$\delta_\mu^{\text{MUonE}} = 127 \pm 5 \text{ ppb} \quad \text{AND} \quad R_p^{\text{MUonE}} = 0.841 \pm 0.002 \text{ fm} \tag{15}$$

Failure of either prediction falsifies DDL. Success of both would constitute compelling evidence for discrete spacetime structure at GeV scales.

7.2. Distinction from BSM Models

DDL differs fundamentally from particle-based BSM explanations:

Structural differences:

- **No new particles:** DDL modifies spacetime geometry, not particle content
- **Universal mechanism:** Both anomalies from single principle (discrete metric)

Table 3: DDL versus representative BSM models

Feature	DDL	Leptoquark	Z' boson
Explains μ g-2			
Explains R_p puzzle			
New particles			
Tau prediction	0.45 ppb	~100 ppb	~50 ppb
$\mu \to e\gamma$	No signal	Constrained	Constrained
Free parameters	0	3–5	2–4

- **Probe-dependent:** Effects visible only when $\lambda_{\text{probe}} \sim d_G$
- **Exact integers:** Predictions involve integer ratios (3600/24=150), not continuous couplings

Phenomenological differences:

- BSM typically predicts $\delta_\tau \sim \delta_\mu$ (similar couplings); DDL predicts $\delta_\tau \ll \delta_\mu$
- BSM predicts correlated signals in $\mu \to e\gamma$, $\mu \to eee$; DDL predicts none
- BSM struggles with proton radius; DDL explains both naturally

Table 3 summarizes key distinctions.

7.3. Implications for Fundamental Physics

If validated by MUonE, the DDL interpretation would have profound implications:

1. Spacetime is fundamentally discrete at ~1 fm scales. This is far above the Planck scale (10^{-35} m), suggesting discrete structure emerges from collective or emergent phenomena rather than quantum gravity per se.

2. The F_4 Coxeter group plays a fundamental role in 4D physics. This group, known primarily in mathematics, would become central to understanding spacetime structure.

3. QFT's continuum assumption breaks down for certain probes. Standard perturbative field theory assumes smooth spacetime; DDL shows this fails at high precision for probes with $\lambda \sim d_G$.

4. Precision experiments reveal geometry, not just dynamics. The anomalies are not "new physics" in the conventional sense but windows into spacetime's microstructure.

5. The hierarchy problem may be geometric. If mass determines polytope shell occupation, the particle mass spectrum might reflect available discrete geometric states rather than coupling hierarchies.

7.4. Open Questions and Future Directions

Several theoretical issues require further development:

First-principles derivation of Axiom 3: While the 6-fold update rule is geometrically motivated by SO(4) structure, a rigorous derivation from discrete fermion dynamics on an F_4 lattice would strengthen the framework.

Lattice scale determination: What sets $d_G \sim 1$ fm rather than the Planck length? Is this related to QCD confinement scale $\Lambda_{\text{QCD}} \sim 200$ MeV?

Cosmological implications: Does F_4 discrete geometry affect the early universe, cosmic microwave background, or dark matter/energy?

Extension to other systems: Can DDL explain other precision anomalies (e.g., hyperfine structure, rare decays, neutrino oscillations)?

8. Conclusion

We have demonstrated that two major precision anomalies—the muon $g-2$ discrepancy and the proton radius puzzle—can be explained through a unified geometric framework based on discrete spacetime structure. The muon anomaly of 127 ppb emerges as a polygon tax from $N = 3600$ discrete phase interactions, while the muonic proton radius of 0.841 fm follows from 24-cell lattice grain resolution. Both predictions derive from established theorems in 4D polytope geometry (Coxeter groups F_4 and H_4) without adjustable parameters.

The exact integer ratio $3600/24 = 150$ reveals deep phase-locking between lepton dynamics and lattice structure. The framework makes sharp, falsifiable predictions testable via MUonE: simultaneous measurement of $\delta_\mu = 127$ ppb in spacelike scattering and $R_p = 0.841$ fm would validate the discrete spacetime interpretation, while failure of either would exclude it.

Unlike conventional BSM approaches requiring new particles or forces, DDL attributes the anomalies to fundamental spacetime geometry. If confirmed, this would represent a paradigm shift: precision discrepancies revealing not missing particles but the discrete fabric of spacetime itself.

Acknowledgments

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