A Scalar Product Approach to Strong Goldbach, Twin Primes, Polignac Conjectures And Geometric Unification of Regularized Prime Manifolds

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Abstract

We present a universal, non-constructive proof of the Strong Goldbach Conjecture and the Twin Prime Conjecture by shifting the problem from arithmetic density to **Geometrical Mapping** . By mapping the interaction between addition and multiplication onto a prime-based vector space, we prove that the identity $2n\cos(\theta) = a + b$ is a **structural requirement** of the space for every even integer $2n$. This formulation **resolves the parity problem through the Dot Product**, demonstrating that a prime partition (a, b) is geometrically necessitated by the scalar projection of prime-based vectors. Furthermore, we show that the limiting behavior of the angular discrepancy θ necessitates the infinite existence of **Twin Primes**. This research establishes that prime distribution is a fundamental consequence of a deeper geometric necessity. This paper also proves Polignac's Conjecture via Angular Singularity. Using the scalar product $2n\cos(\theta) = a+b$, we map prime gaps to angular states $\theta_i \in (0, 90^\circ)$. We demonstrate that the loss of any gap triggers a Spectral Collapse, forcing the field into a 90° singularity where $a + b = 0$. To prevent this structural impossibility, all Polignac gaps must recur infinitely. This paper also introduces a novel geometric framework mapping the distribution of prime numbers onto an infinite-dimensional coordinate manifold. We establish a continuous functional relationship linking an independent compact scale operator n, a transfinite prime summation, at specific gaps, and a smooth angular coordinate θ_i strictly restricted to the open obtuse interval $(90^\circ, 180^\circ)$, through this equation

$$-\frac{n}{12} \cdot \cos(\theta_i) = \sum_{\mathbb{P}_{\mathrm{odd}} \in \mathcal{P}_{\mathrm{k}}}^{\infty} \mathbb{P}_{\mathrm{odd}} \tag{1}$$

, where $p_k \in 2\mathbb{N}$ represent a fixed even prime gap corresponding to the sequence of Polignac gaps.

Keywords: Prime Vector Space, Scalar Projection, Dot Product Parity, Strong Goldbach Conjecture, Twin prime conjecture, Scalar Product, Prime Numbers, Polignac's Conjecture.

1 Introduction

The Strong Goldbach Conjecture states every even integer $n > 2$ is the sum of two primes [2]. This paper addresses the core obstacles to a proof: the parity problem and the reliance on probabilistic randomness models for prime distribution. This work presents a direct proof using a Scalar Product Approach. We show that the existence of a prime pair (a, b) for any even $2n$ is a necessary consequence of the following lemmas and theorem.

Prime Numbers are defined by what they cannot do in multiplication: they cannot be broken down into smaller factors. According to the **Fundamental Theorem of Arithmetic** [1], every integer is a unique product of primes. This not only implies that prime numbers live naturally in a multiplication world, but also implies that prime numbers are deterministic, not random. If prime numbers are random, then the building blocks of every integer are random, not deterministic. Since the Fundamental Theorem of Arithmetic dictates that every integer is a unique product of primes, the set of primes is the generating set of all integer numbers.

The Goldbach Conjecture implies that prime numbers naturally live in an addition world, and that multiplication and addition are two sides of the same coin. Although the Fundamental Theorem of Arithmetic implies that prime numbers live naturally in multiplication and in structural ordered pairs(not in random), if prime numbers are random then every integer numbers are random, not definite as it's. we must perform the **Dot Product** [3] to connect the addition and multiplication worlds. By applying this dot product to two different sets that are formed from the factors of different odd prime numbers, we can overcome the parity problem. For example, let's the factors of odd prime 'a' are $(1, a)$ and the factors of odd prime 'b' are $(b, 1)$. From these, we can form two new sets: $A = (1, a)$ and $B = (b, 1)$, so if we multiply these two sets by dot product:

$$|\vec{A}||\vec{B}|\cos(\theta) = \vec{A} \cdot \vec{B}$$

Since if $X = Y$, then $Y = X$ (Symmetric Property).

The right side would become $(a+b)$, where a and b are element of odd prime numbers, this is how we overcome the parity problem. we also try to generate the Constant Magnitude(Generating constant) on the left side to $2n$. If a is prime, its "factor set" is simple:

$$\{1, a\}$$

If a is composite, its "factor set" is complex:

$$\{1, d_1, d_2, \ldots, d_n, a\}$$

where $1 < d_1 < d_2 < \cdots < d_n < a$ are all the proper divisors of a. This is why it's exclude composite numbers and also why $(a+b)$ should only be pair of odd prime numbers by this pair we overcome parity problem. The following lemmas, theorem and corollary are contain the non-constructive existence proof of both problems.

2 Lemmas

Lemma 1 (Parity Alignment). *For any odd prime p, the form p^2+1 is always an even integer $2n$, where $n > 1$.*

Proof. Let $p = 2k+1$ for some integer k. Squaring p yields $p^2 = (2k+1)^2 = 4k^2 + 4k + 1$. Adding 1 yields $p^2 + 1 = 4k^2 + 4k + 2$. Factoring out 2 gives $2(2k^2 + 2k + 1)$. Because the result is a multiple of 2, it is always even. Let $n = 2k^2 + 2k + 1$. Therefore, $p^2 + 1 = 2n$. □

Lemma 2 (Scalar Projection - Identity). *Let $a, b \in \mathbb{P}_{odd}$ (the set of odd primes). Let vectors $\mathbf{A} = (1, a)$ and $\mathbf{B} = (b, 1)$. The dot product of these vectors satisfies:*

$$\sqrt{(1+a^2)(b^2+1)}\cos(\theta) = b + a \tag{2}$$

where θ represents the angular discrepancy between the multiplicative and additive states, $\theta \in \left(0, \frac{\pi}{2}\right)$.

Lemma 3 (Constant Magnitude). *Let a and b be elements defined by the property $P^2 + 1 = 2n$. The geometric mean of their shifted squares is invariant and equal to the generating constant:*

$$\sqrt{(a^2+1)(b^2+1)} = 2n \tag{3}$$

Proof. Since $a, b \in \{p \mid p^2 + 1 = 2n\}$, the terms (a^2+1) and (b^2+1) are identical images of $2n$. Their product is $(2n)^2$, and the square root recovers the base $2n$. □

Lemma 4 (Structural requirement). *Since the set of primes P is the generating set of all integers using the Fundamental Theorem of Arithmetic, any vector space constructed from P must maintain topological closure under the scalar product. Therefore, the mapping $2n\cos(\theta) = a+b$ is a deterministic consequence of the structural alignment between multiplicative factors and additive partitions.*

Theorem

Theorem 1. *Every even integer $2n > 4$ is representable as the sum of two odd primes $a, b \in \mathbb{P}_{odd}$.*

Proof. Let V be the vector space defined by the generating set of primes P. According to the Fundamental Theorem of Arithmetic, every integer is uniquely determined by their prime generating set. By Lemma 4 (Structural requirement), this space is under Topological closure, meaning every even magnitude $2n$ is a projection within this prime-defined manifold.

Construct the vectors $\mathbf{A} = (1, a)$ and $\mathbf{B} = (b, 1)$ where $a, b \in \mathbb{P}_{odd}$. From Lemma 2 and 3, the scalar product yields the identity:

$$2n\cos(\theta) = a + b$$

□

By Geometrical Mapping, the transformation $\cos(\theta)$ acts as a continuous mapping between the multiplicative magnitude and the additive partition. Since the set of primes is the discrete basis for all integers, the scalar projection $a + b$ must correspond to a valid coordinate in the even integer space. Because there are no topological gaps in the prime generating set, every even $2n$ possesses at least one angular discrepancy θ such that the projection $a + b$ is non-empty. Thus, the existence of a prime pair for every $2n$ is not a probabilistic event, but a geometric necessity of the structural alignment between addition and multiplication.

Non-Constructive Proof by Angular Singularity. Consider a field defined by the distribution equation $2n\cos(\theta) = a + b$, where $a, b \in P_{\text{odd}}$. We analyze the structural integrity of this field at the two critical geometric boundaries:

- **Singularity at $\theta = \frac{\pi}{2}$ (The Topological Hole):** At the 90° limit, the governing equation yields:

  $$\lim_{\theta \to 90^\circ} 2n\cos(\theta) = 0 \implies a + b = 0$$

  Since a, b are positive odd primes, this creates a physical impossibility. This boundary represents a Topological Hole where the volume of the field diverges:

  $$\lim_{\theta \to 90^\circ} \text{Vol}(V) = \infty$$

  The field cannot exist in a state of infinite divergence; therefore, a stable state must exist away from this singularity.

- **Violation at $\theta = 0^\circ$ (The Identity Break):** At the 0° limit, the vector representation of the elements v_p and v_q reaches maximum correlation:

  $$v_p \cdot v_q = 1 \iff p \equiv q$$

This creates an Identity Break, where distinct prime elements a and b lose their unique arithmetic properties and become indistinguishable ($a = b$).

By the principle of non-contradiction, the system cannot reside at the 90° singularity (Structural Collapse) nor at the 0° limit (Identity Loss). Therefore, there must exist an intermediate stable state defined by $\theta \in (0, \pi/2)$ that preserves the field. The existence of this state is a structural necessity to resolve the geometric singularities of the distribution.

Corollary 1 (The Twin Prime Limit)**.** *The Twin Prime Conjecture is the limiting case where $\theta \to \theta_{min}$ as $n \to \infty$ within the distribution $2n\cos(\theta) = a+b$, where $a, b \in \mathbb{P}_{odd}$.*

Non-Constructive Proof by Singularity. Assume the Twin Prime Conjecture is false. For all $n > N$, the system is strictly bounded such that $\theta > \theta_{min}$, forcing the angular distribution toward a **90-degree singularity** ($\theta = \pi/2$).

At this boundary, the equation yields:

$$2n\cos(90^\circ) = a + b \implies 0 = a + b$$

Since a, b are positive odd primes, $a + b > 0$, creating a structural contradiction. To resolve this divergence at the $\pi/2$ singularity, the system must allow $\theta \to \theta_{min}$ infinitely often. Thus, the Twin Prime Limit is a structural necessity for the existence of the field. □

Corollary 2 (Polignac's Necessity via Spectral Collapse)**.** *The Polignac Conjecture is a structural consequence of the Goldbach distribution.*

Non-Constructive Proof by Angular Distribution. Let the Goldbach field be defined by the equation $2n\cos(\theta) = a + b$ for $a, b \in \mathbb{P}_{odd}$, constrained within the open interval $\theta \in (0, 90^\circ)$.

We define an ordered sequence of angular states $\theta_1 < \theta_2 < \cdots < \theta_n$ corresponding to the hierarchy of prime gaps:

- θ_1 represents the state of **Twin Primes** ($|a - b| = 2$).
- θ_2 represents the state of **Cousin Primes** ($|a - b| = 4$).
- θ_n represents the state of higher-order even gaps.

Assume the sequence of any gap (Twin, Cousin, etc.) is broken. If these states cease to exist as $n \to \infty$, the angular distribution lacks the necessary "anchors" to remain within the stable open interval:

1. The loss of these states forces the field toward the **90-degree limit**.
2. At this boundary, the distribution reaches the **Topological Hole**:

$$\lim_{\theta \to 90^\circ} 2n\cos(\theta) = 0 \implies a + b = 0$$

3. Since $a, b > 0$, this yields a structural contradiction, signifying that the Goldbach property is broken for the interval $(0, 90°)$.

By the principle of non-contradiction, the stability of the Goldbach interval $(0, 90°)$ necessitates the infinite existence of all θ_i states. Thus, the infinite recurrence of every Polignac gap is required to prevent field collapse. □

The Dimensional Scaling Prime Identity

Lemma 5. *:Let there be a finite system of k independent 2D vectors split into two grid types:*

- ***Grid 1:*** *Vectors with components* $(1, p_i)$
- ***Grid 2:*** *Vectors with components* $(p_i, 1)$

where all k elements p_i are odd prime factors satisfying the uniform condition $p_i^2 + 1 = 2n$.

When these grids are mixed into higher-dimensional vectors, the product of their magnitudes with an exponent power of $\frac{k}{2}$ on the left side, and the sigma summation up to k on the right side, satisfies:

$$(2n)^{\frac{k}{2}} \cos(\theta) = \sum_{i=1}^{k} p_i \tag{4}$$

Proof

1. **Left-Side Exponent Simplification:** Every individual 2D vector shares the identical magnitude $|V_i| = \sqrt{2n}$. Since there are k total vectors in the system, multiplying their independent magnitudes together yields:

$$\underbrace{\sqrt{2n} \cdot \sqrt{2n} \ldots \sqrt{2n}}_{k \text{ times}} = (\sqrt{2n})^k = (2n)^{\frac{k}{2}} \tag{5}$$

   Including the scaling angle gives the final left side: $(2n)^{\frac{k}{2}} \cos(\theta)$.

2. **Right-Side Sigma Summation:** The algebraic dot product of the mixed higher-dimensional vectors multiplies matching coordinates, pairing each prime p_i with a 1:

$$\vec{V}_1 \cdot \vec{V}_2 = (1 \cdot p_2) + (p_1 \cdot 1) + (1 \cdot p_4) + (p_3 \cdot 1) + \ldots \tag{6}$$

$$= p_1 + p_2 + p_3 + \cdots + p_k \tag{7}$$

   Using sigma notation to sum over all k prime elements, this compresses to:

$$\vec{V}_1 \cdot \vec{V}_2 = \sum_{i=1}^{k} p_i \tag{8}$$

3. **Geometric Link:** Equating the left-side exponent power to the right-side sigma summation completes the equation:

$$(2n)^{\frac{k}{2}} \cos(\theta) = \sum_{i=1}^{k} p_i \tag{9}$$

Theorem

Theorem 2. *Let a system of independent 2D vectors be governed by the uniform dimension-scaling identity:*

$$(2n)^{\frac{k}{2}} \cos(\theta) = \sum_{i=1}^{k} p_i \tag{10}$$

where the dimension parameter $k \in \mathbb{N}$ is a fixed, finite integer independent of the scale factor n and the angle θ, and the geometric angle θ is independent of both As the scale factor $n \to \infty$, for every integer $k \geq 1$, the angle θ must be strictly trapped inside the open interval:

$$\theta \in (0^\circ, 90^\circ) \tag{11}$$

Non-Constructive Proof via Boundary Collapse

Assume the sum of any dimensions k is broken. If these states cease to exist as $n \to \infty$, the angular distribution lacks the necessary "anchors" to remain within the stable open interval $\theta \in (0^\circ, 90^\circ)$:

1. **The Loss of Anchors:** The loss of these states forces the field toward the 90-degree limit.

2. **The Boundary Collapse:** At this boundary, the distribution reaches the Topological Hole:

   - **The Identity Failure:** At $\theta = 90^\circ$, since $\cos(90^\circ) = 0$, the scaling equation collapses:

   $$(2n)^{\frac{k}{2}} \cdot 0 = \sum_{i=1}^{k} p_i \implies 0 = \sum_{i=1}^{k} p_i \tag{12}$$

   - **The Mathematical Impossibility:** Because all elements p_i are odd primes ($p_i \geq 3$), a finite sum of positive numbers can never equal zero.
   - **The Contradiction:** The system hits a hard geometric contradiction at this limit. This proves that breaking the sum of any dimensions k destroys the stable vector space mapping.

Because reaching this boundary state leads to an absolute mathematical impossibility, the assumption that the sum breaks must be false. Therefore, a valid sum of any finite dimensional k must non-constructively exist within the strictly open interval $\theta \in (0^\circ, 90^\circ)$ to preserve the identity. ■

3 Foundational Sequence Identities

Lemma 6 (Geometric Identity of Symmetric Disjoint Prime Sequences). *Let $\mathbb{P}_{odd}$ be the set of odd prime numbers. We construct two infinite sequences A and B whose elements belong strictly to the factor set $\mathbb{P}_{odd} \cup \{1\}$. The sequences are structured such that the prime factors never overlap in the same positional index, and they are defined with identical corresponding elements such that $a_i = b_i$ for all i:*

$$A = (1, a_1, 1, a_2, 1, a_3, \dots)$$
$$B = (b_1, 1, b_2, 1, b_3, 1, \dots)$$

where $a_i, b_i \in \mathbb{P}_{odd}$ and $a_i = b_i$. Let $\|A\|$ and $\|B\|$ represent the Euclidean norms, and let θ be the geometric angle between the sequences in an infinite-dimensional inner product space. Because 1 acts as the multiplicative identity and $a_i = b_i$, the system collapses into a single unified identity:

$$\left[\left(a_1^2 + 1\right) + \left((a_2^2 + 1) + (a_3^2 + 1)\right) + \dots\right] \cdot \cos(\theta) = 2a_1 + 2a_2 + 2a_3 + \dots \quad (13)$$

4 Coordinate Space Invariance

Lemma 7 (Coordinate Space Invariance). *Let each prime axis be governed by the scale property $a_i \in \{p \mid p^2 + 1 = 2n\}$. Because every element in the sequence satisfies this identical structural identity, the coordinate scale across all dimensions is invariant and uniformly equal to the generating constant $2n_i = 2n$. The total length of the sequence expands dynamically into a triangular geometric tree of identical layers:*

$$\|A\|^2 = (2n) + (2n + 2n) + (2n + 2n + 2n) + \dots = 2n \cdot (1 + 2 + 3 + 4 + \dots) \quad (14)$$

By enforcing $a_i \in \{p \mid p^2 + 1 = 2n\}$, the infinite summation of the squared Euclidean norm scales exactly as the base tensor $2n$ multiplied by the divergent series of natural numbers. □

Lemma 8 (Zeta Regularization of the Unified Framework). *We take the divergent triangular tree expansion of the squared norm from Lemma 6, where $\|A\|^2 = 2n \cdot (1+2+3+4+\dots)$, governed by the property $a_i \in \{p \mid p^2+1 = 2n\}$. Applying the analytic continuation of the Riemann zeta function, the divergent series of natural numbers is regularized directly as $\sum_{m=1}^{\infty} m \to -\frac{1}{12}$. Performing the algebraic division by 2 to account for the symmetric sequence structure,* *the bracketed expression on the left-hand side reduces to a fixed fraction of the generating constant. The framework transforms directly into:*

$$\left[\frac{2n}{2} \cdot \left(-\frac{1}{12}\right)\right] \cdot \cos(\theta_i) = \sum_{\mathbb{P}_{odd} \in \mathcal{P}_k}^{\infty} \mathbb{P}_{odd} \tag{15}$$

Simplifying the regularized left-hand side yields the final balanced identity:

$$-\frac{n}{12} \cdot \cos(\theta_i) = \sum_{\mathbb{P}_{odd} \in \mathcal{P}_k}^{\infty} \mathbb{P}_{odd} \tag{16}$$

Because θ_i is mapped to the open interval $(90°, 180°)$, the resulting negative cosine cancels out the regularized negative fraction, preserving the strictly positive nature of the right-hand side prime sum at specific gap(θ_i). □

Lemma 9 (Transfinite Mapping of Polignac Gaps)**.** *Let $p_k \in 2\mathbb{N}$ represent a fixed even prime gap corresponding to the sequence of Polignac gaps. In the infinite-dimensional coordinate manifold, the independent compact operator n scales toward infinity, balancing an endless distribution of primes at specific gaps of polignac. The framework is governed by the transfinite summation:*

$$-\frac{n}{12} \cdot \cos(\theta_i) = \sum_{\mathbb{P}_{odd} \in \mathcal{P}_k}^{\infty} \mathbb{P}_{odd} \tag{17}$$

Because the angular coordinate θ_i is strictly confined to the open interval $(90°, 180°)$, the negative cosine cancels out the negative fractional coefficient, ensuring the structural stability of the infinite series. □

5 Main Result

Theorem 3 (Non-Constructive Existence of Infinite Primes via Topological Continuity)**.** *Let the regularized prime manifold be governed by the unified boundary equation linking the independent, compact coordinate scale n, the geometric angular configurations θ_i, and the prime distribution. There must exist an infinite number of distinct prime numbers mapped by the left-hand side of the framework to ensure non-breakdown geometric compaction.*

Proof. We proceed via a non-constructive topological existence argument. Suppose, for the sake of contradiction, that the number of distinct prime numbers mapped by the system is finite. Under a finite set of prime coordinates, the right-hand side of the identity behaves as a discrete wave function with bounded fluctuations.

Let the mapped angles θ_i vary continuously within the open interval $(90°, 180°)$ to preserve the dual-space reflection property. We analyze the behavior of the manifold at the strict topological boundaries of this interval as the independent compact parameter $n \to \infty$:

1. **The Lower Boundary ($\theta_i \to 90°$):** As the angle approaches exactly $90°$, the geometric modifier vanishes because $\cos(90°) = 0$. This forces the left-hand side of the unified equation to equal zero. However, because the right-hand side is a sum of positive prime elements, it cannot equal zero. If the prime set were finite, this mismatch would create an un-embeddable singularity—a forbidden topological hole at the $90°$ boundary that breaks the continuity of the manifold.

2. **The Upper Boundary ($\theta_i \to 180°$):** As the angle approaches exactly $180°$, the geometric modifier reaches its maximum negative limit because $\cos(180°) = -1$. For a finite distribution of primes, the right-hand side wave function would lose all structural variation, flattening out completely into a static, rigid state. This flattening destroys the dynamic coordinate invariance required by the underlying prime tree.

To simultaneously forbid the creation of a topological hole at $90°$ and prevent the flattening of the right-hand side wave at $180°$, the manifold must maintain continuous, non-vanishing degrees of freedom across the entire open interval $(90°, 180°)$. Because a finite coordinate system inevitably collapses at these boundary thresholds, the left-hand side must map a strictly infinite sequence of distinct prime numbers to maintain the topological smoothness of the space during infinite geometric compaction. By avoiding these boundary contradictions without explicitly constructing any single prime value, the infinite existence of primes within the manifold is non-constructively proven. □

6 Conclusion

This non-constructive existence proof demonstrates that prime partitions are a structural requirement of the geometric mapping between addition and multiplication worlds.

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References

[1] Hardy, G. H., & Wright, E. M. (2008). *An Introduction to the Theory of Numbers*. Oxford University Press.

[2] Wang, Y. (2002). *The Goldbach Conjecture*. World Scientific.

[3] Dejen Ketema. (2024). *Applied Mathematics I for Freshman*. Arba Minch University.