

A Scalar Product Approach to Strong Goldbach, Twin Primes and Polignac Conjectures

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Abstract

We present a universal, non-constructive proof of the Strong Goldbach Conjecture and the Twin Prime Conjecture by shifting the problem from arithmetic density to **Geometrical Mapping**. By mapping the interaction between addition and multiplication onto a prime-based vector space, we prove that the identity $2n \cos(\theta) = a + b$ is a **structural requirement** of the space for every even integer $2n$. This formulation **resolves the parity problem through the Dot Product**, demonstrating that a prime partition (a, b) is geometrically necessitated by the scalar projection of prime-based vectors. Furthermore, we show that the limiting behavior of the angular discrepancy θ necessitates the infinite existence of **Twin Primes**. This research establishes that prime distribution is a fundamental consequence of a deeper geometric necessity. This paper also proves Polignac's Conjecture via Angular Singularity. Using the scalar product $2n \cos(\theta) = a + b$, we map prime gaps to angular states $\theta_i \in (0, 90^\circ)$. We demonstrate that the loss of any gap triggers a Spectral Collapse, forcing the field into a 90° singularity where $a + b = 0$. To prevent this structural impossibility, all Polignac gaps must recur infinitely.

Keywords: Prime Vector Space, Scalar Projection, Dot Product Parity, Strong Goldbach Conjecture, Twin prime conjecture, Scalar Product, Prime Numbers, Polignac's Conjecture.

1 Introduction

The Strong Goldbach Conjecture states every even integer $n > 2$ is the sum of two primes [2]. This paper addresses the core obstacles to a proof: the parity problem and the reliance on probabilistic randomness models for prime distribution. This work presents a direct proof using a Scalar Product Approach. We show that the existence of a prime pair (a, b) for any even $2n$ is a necessary consequence of the following lemmas and theorem.

Prime Numbers are defined by what they cannot do in multiplication: they cannot be broken down into smaller factors. According to the **Fundamental Theorem of Arithmetic** [1], every integer is a unique product of primes. This not only implies that prime numbers live naturally in a multiplication world, but also implies that prime numbers are deterministic, not random. If prime numbers are random, then the building blocks of every integer are random, not deterministic. Since the Fundamental Theorem of Arithmetic dictates that every integer is a unique product of primes, the set of primes is the generating set of all integer numbers.

The Goldbach Conjecture implies that prime numbers naturally live in an addition world, and that multiplication and addition are two sides of the same coin. Although the Fundamental Theorem of Arithmetic implies that prime numbers live naturally in multiplication and in structural ordered pairs(not in random), if prime numbers are random then every integer numbers are random, not definite as it's. we must perform the **Dot Product** [3] to connect the addition and multiplication worlds. By applying this dot product to two different sets that are formed from the factors of different odd prime numbers, we can overcome the parity problem. For example, let's the factors of odd prime 'a' are $(1, a)$ and the factors of odd prime 'b' are $(b, 1)$. From these, we can form two new sets: $A = (1, a)$ and $B = (b, 1)$, so if we multiply these two sets by dot product:

$$|\vec{A}||\vec{B}|\cos(\theta) = \vec{A} \cdot \vec{B}$$

Since if $X = Y$, then $Y = X$ (Symmetric Property).

The right side would become $a + b$, where a and b are element of odd prime numbers, this is how we overcome the parity problem. we also try to generate the Convergence Magnitude(Generating constant) on the left side to $2n$.The following lemmas, theorem and corollary are contain the non-constructive existence proof of both problems.

2 Lemmas

Lemma 1 (Parity Alignment). *For any odd prime p , the form $p^2 + 1$ is always an even integer $2n$, where $n > 1$.*

Proof. Let $p = 2k + 1$ for some integer k . Squaring p yields $p^2 = (2k + 1)^2 = 4k^2 + 4k + 1$. Adding 1 yields $p^2 + 1 = 4k^2 + 4k + 2$. Factoring out 2 gives $2(2k^2 + 2k + 1)$. Because the result is a multiple of 2, it is always even. Let $n = 2k^2 + 2k + 1$. Therefore, $p^2 + 1 = 2n$. \square

Lemma 2 (Scalar Projection). *[Identity] Let $a, b \in \mathbb{P}$ (the set of odd primes). Let vectors $A = (1, a)$ and $B = (b, 1)$. The dot product of these vectors satisfies:*

$$\sqrt{(1 + a^2)(b^2 + 1)} \cos(\theta) = b + a \quad (1)$$

where θ represents the angular discrepancy between the multiplicative and additive states, $\theta \in (0, \frac{\pi}{2})$.

Lemma 3 (Convergence Magnitude). *Let a and b be elements defined by the property $P^2 + 1 = 2n$. The geometric mean of their shifted squares is invariant and equal to the generating constant:*

$$\sqrt{(a^2 + 1)(b^2 + 1)} = 2n \quad (2)$$

Proof. Since $a, b \in \{p \mid p^2 + 1 = 2n\}$, the terms $(a^2 + 1)$ and $(b^2 + 1)$ are identical images of $2n$. Their product is $(2n)^2$, and the square root recovers the base $2n$. \square

Lemma 4 (Structural requirement). *Since the set of primes \mathbb{P} is the generating set of all integers using the Fundamental Theorem of Arithmetic, any vector space constructed from \mathbb{P} must maintain topological closure under the scalar product. Therefore, the mapping $2n \cos(\theta) = a + b$ is a deterministic consequence of the structural alignment between multiplicative factors and additive partitions.*

3 Theorem

Theorem 1. *Every even integer $2n > 2$ is representable as the sum of two odd primes $a, b \in \mathbb{P}$, except number 4.*

Proof. Let V be the vector space defined by the generating set of primes \mathbb{P} . According to the **Fundamental Theorem of Arithmetic**, every integer is uniquely determined by their prime generating set. By Lemma 4 (**Structural requirement**), this space is under **Topological closure**, meaning every even magnitude $2n$ is a projection within this prime-defined manifold.

Construct the vectors $A = (1, a)$ and $B = (b, 1)$ where $a, b \in \mathbb{P}$. From Lemma 2 and 3, the scalar product yields the identity:

$$2n \cos(\theta) = a + b \quad (3)$$

By **Geometrical Mapping**, the transformation $\cos(\theta)$ acts as a continuous mapping between the multiplicative magnitude and the additive partition. Since the set of primes is the discrete basis for all integers, the scalar projection $a + b$ must correspond to a valid coordinate in the even integer space.

Because there are no topological gaps in the prime generating set, every even $2n$ possesses at least one angular discrepancy θ such that the projection $a + b$ is non-empty. Thus, the existence of a prime pair for every $2n$ is not a probabilistic event, but a geometric necessity of the structural alignment between addition and multiplication. \square

Non-Constructive Proof by Angular Singularity. Consider a field defined by the distribution equation $2n \cos(\theta) = a + b$, where $a, b \in \mathbb{P}_{odd}$. We analyze the structural integrity of this field at the two critical geometric boundaries:

- **Singularity at $\theta = \frac{\pi}{2}$** (The Topological Hole): At the 90° limit, the governing equation yields:

$$\lim_{\theta \rightarrow 90^\circ} 2n \cos(\theta) = 0 \implies a + b = 0$$

Since a, b are positive odd primes, this creates a physical impossibility. This boundary represents a **Topological Hole** where the volume of the field diverges:

$$\lim_{\theta \rightarrow 90^\circ} \text{Vol}(\mathcal{V}) = \infty$$

The field cannot exist in a state of infinite divergence; therefore, a stable state must exist away from this singularity.

- **Violation at $\theta = 0^\circ$** (The Identity Break): At the 0° limit, the vector representation of the elements \mathbf{v}_p and \mathbf{v}_q reaches maximum correlation:

$$\mathbf{v}_p \cdot \mathbf{v}_q = 1 \iff p \equiv q$$

This creates an **Identity Break**, where distinct prime elements a and b lose their unique arithmetic properties and become indistinguishable ($a = b$).

By the principle of non-contradiction, the system cannot reside at the 90° singularity (Structural Collapse) nor at the 0° limit (Identity Loss). Therefore, there *must* exist an intermediate stable state defined by $\theta \in (0, \pi/2)$ that preserves the field. The existence of this state is a structural necessity to resolve the geometric singularities of the distribution. \square

Corollary 1 (The Twin Prime Limit). *The Twin Prime Conjecture is the limiting case where $\theta \rightarrow \theta_{min}$ as $n \rightarrow \infty$ within the distribution $2n \cos(\theta) = a + b$, where $a, b \in \mathbb{P}_{odd}$.*

Non-Constructive Proof by Singularity. Assume the Twin Prime Conjecture is false. For all $n > N$, the system is strictly bounded such that $\theta > \theta_{min}$, forcing the angular distribution toward a **90-degree singularity** ($\theta = \pi/2$).

At this boundary, the equation yields:

$$2n \cos(90^\circ) = a + b \implies 0 = a + b$$

Since a, b are positive odd primes, $a + b > 0$, creating a structural contradiction. To resolve this divergence at the $\pi/2$ singularity, the system must allow $\theta \rightarrow \theta_{min}$ infinitely often. Thus, the Twin Prime Limit is a structural necessity for the existence of the field. \square

Corollary 2 (Polignac's Necessity via Spectral Collapse). *The Polignac Conjecture is a structural consequence of the Goldbach distribution.*

Non-Constructive Proof by Angular Distribution. Let the Goldbach field be defined by the equation $2n \cos(\theta) = a + b$ for $a, b \in \mathbb{P}_{odd}$, constrained within the open interval $\theta \in (0, 90^\circ)$.

We define an ordered sequence of angular states $\theta_1 < \theta_2 < \dots < \theta_n$ corresponding to the hierarchy of prime gaps:

- θ_1 represents the state of **Twin Primes** ($|a - b| = 2$).
- θ_2 represents the state of **Cousin Primes** ($|a - b| = 4$).
- θ_n represents the state of higher-order even gaps.

Assume the sequence of any gap (Twin, Cousin, etc.) is broken. If these states cease to exist as $n \rightarrow \infty$, the angular distribution lacks the necessary "anchors" to remain within the stable open interval:

1. The loss of these states forces the field toward the **90-degree limit**.
2. At this boundary, the distribution reaches the **Topological Hole**:

$$\lim_{\theta \rightarrow 90^\circ} 2n \cos(\theta) = 0 \implies a + b = 0$$

3. Since $a, b > 0$, this yields a structural contradiction, signifying that the Goldbach property is broken for the interval $(0, 90^\circ)$.

By the principle of non-contradiction, the stability of the Goldbach interval $(0, 90^\circ)$ necessitates the infinite existence of all θ_i states. Thus, the infinite recurrence of every Polignac gap is required to prevent field collapse. \square

4 Conclusion

This non-constructive existence proof demonstrates that prime partitions are a structural requirement of the geometric mapping between addition and multiplication worlds.

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