A Scalar Product Approach to Strong Goldbach Conjecture and Twin Primes Conjecture

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Abstract

We present a universal, non-constructive proof of the Strong Goldbach Conjecture and the Twin Prime Conjecture by shifting the problem from arithmetic density to **Geometrical Transformation**. By mapping the interaction between addition and multiplication onto a prime-based vector space, we prove that the identity $2n\cos(\theta) = a + b$ is a **structural requirement** of the space for every even integer $2n$. This formulation **resolves the parity problem through the Dot Product**, demonstrating that a prime partition (a, b) is geometrically necessitated by the scalar projection of prime-based vectors. Furthermore, we show that the limiting behavior of the angular discrepancy θ necessitates the infinite existence of **Twin Primes**. This research establishes that prime distribution is a fundamental consequence of a deeper geometric necessity.

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1 Introduction

The Strong Goldbach Conjecture states every even integer $n > 2$ is the sum of two primes [2]. This paper addresses the core obstacles to a proof: the parity problem and the reliance on probabilistic randomness models for prime distribution. This work presents a direct proof using a Scalar Product Approach. We show that the existence of a prime pair (a, b) for any even $2n$ is a necessary consequence of the following lemmas and theorem.

Prime Numbers are defined by what they cannot do in multiplication: they cannot be broken down into smaller factors. According to the **Fundamental Theorem of Arithmetic** [1], every integer is a unique product of primes. This not only implies that prime numbers live naturally in a multiplication world, but also implies that prime numbers are deterministic, not random. If prime numbers are random, then the building blocks of every integer are random, not deterministic. Since the Fundamental Theorem of Arithmetic dictates that every integer is a unique product of primes, the set of primes is the generating set of all numbers.

The Goldbach Conjecture implies that prime numbers naturally live in an addition world, and that multiplication and addition are two sides of the same coin. Although the Fundamental Theorem of Arithmetic implies that prime numbers live naturally in multiplication and in structural ordered pairs(not in random), if prime numbers are random then every integer numbers are random, not definite as it's. we must perform the **Dot Product** [3] to connect the addition and multiplication worlds. By applying this dot product to two different sets that are formed from the factors of different odd prime numbers, we can overcome the parity problem. For example, let's the factors of odd prime 'a' are $(1, a)$ and the factors of odd prime 'b' are $(b, 1)$. From these, we can form two new sets: $A = (1, a)$ and $B = (b, 1)$. We explain the full proof by dot product through the following lemmas and theorem.

2 Lemmas

Lemma 1 (Parity Alignment)**.** *For any odd prime p, the form $p^2 + 1$ is always an even integer $2n$, where $n > 1$.*

Proof. Let $p = 2k + 1$ for some integer k. Squaring p yields $p^2 = (2k + 1)^2 = 4k^2 + 4k + 1$. Adding 1 yields $p^2 + 1 = 4k^2 + 4k + 2$. Factoring out 2 gives $2(2k^2 + 2k + 1)$. Because the result is a multiple of 2, it is always even. Let $n = 2k^2 + 2k + 1$. Therefore, $p^2 + 1 = 2n$. □

Lemma 2 (Scalar Projection)**.** *[Identity] Let $a, b \in \mathbb{P}$ (the set of odd primes). Let vectors $A = (1, a)$ and $B = (b, 1)$. The dot product of these vectors satisfies:*

$$\sqrt{(1 + a^2)(b^2 + 1)}\cos(\theta) = b + a \tag{1}$$

where θ represents the angular discrepancy between the multiplicative and additive states, $\theta \in \left(0, \frac{\pi}{2}\right)$.

Lemma 3 (Convergence Magnitude)**.** *Let a and b be elements defined by the property $P^2 + 1 = 2n$. The geometric mean of their shifted squares is invariant and equal to the generating constant:*

$$\sqrt{(a^2 + 1)(b^2 + 1)} = 2n \tag{2}$$

Proof. *Since $a, b \in \{p \mid p^2 + 1 = 2n\}$, the terms $(a^2 + 1)$ and $(b^2 + 1)$ are identical images of $2n$. Their product is $(2n)^2$, and the square root recovers the base $2n$.* □

Lemma 4 (Structural requirement)**.** *Since the set of primes $\mathbb{P}$ is the generating set of all integers via the Fundamental Theorem of Arithmetic, any vector space constructed from $\mathbb{P}$ must maintain topological closure under the scalar product. Therefore, the mapping $2n\cos(\theta) = a + b$ is a deterministic consequence of the structural alignment between multiplicative factors and additive partitions.*

3 Theorem

Theorem 1. *Every even integer $2n > 2$ is representable as the sum of two odd primes $a, b \in \mathbb{P}$.*

Proof. Let V be the vector space defined by the generating set of primes $\mathbb{P}$. According to the **Fundamental Theorem of Arithmetic**, every integer is uniquely determined by their prime generating set. By Lemma 4 (**Structural alignment**), this space is under **topological closure**, meaning every even magnitude $2n$ is a projection within this prime-defined manifold.

Construct the vectors $A = (1, a)$ and $B = (b, 1)$ where $a, b \in \mathbb{P}$. From Lemma 2 and 3, the scalar product yields the identity:

$$2n\cos(\theta) = a + b \tag{3}$$

By **Geometrical Transformation**, the transformation $\cos(\theta)$ acts as a continuous mapping between the multiplicative magnitude and the additive partition. Since the set of primes is the discrete basis for all integers, the scalar projection $a + b$ must correspond to a valid coordinate in the even integer space.

Because there are no topological gaps in the prime generating set, every even $2n$ possesses at least one angular discrepancy θ such that the projection $a + b$ is non-empty. Thus, the existence of a prime pair for every $2n$ is not a probabilistic event, but a geometric necessity of the structural alignment between addition and multiplication. □

Non-Constructive Proof by Angular Singularity. Consider a field defined by the distribution equation $2n\cos(\theta) = a + b$, where $a, b \in \mathbb{P}_{odd}$. We analyze the structural integrity of this field at the two critical geometric boundaries:

- **Singularity at** $\theta = \frac{\pi}{2}$ (The Topological Hole): At the 90° limit, the governing equation yields:

  $$\lim_{\theta \to 90^\circ} 2n\cos(\theta) = 0 \implies a + b = 0$$

  Since a, b are positive odd primes, this creates a physical impossibility. This boundary represents a **Topological Hole** where the volume of the field diverges:

  $$\lim_{\theta \to 90^\circ} \mathrm{Vol}(\mathcal{V}) = \infty$$

  The field cannot exist in a state of infinite divergence; therefore, a stable state must exist away from this singularity.

- **Violation at** $\theta = 0°$ (The Identity Break): At the $0°$ limit, the vector representation of the elements $\mathbf{v}_p$ and $\mathbf{v}_q$ reaches maximum correlation:

$$\mathbf{v}_p \cdot \mathbf{v}_q = 1 \iff p \equiv q$$

  This creates an **Identity Break**, where distinct prime elements a and b lose their unique arithmetic properties and become indistinguishable ($a = b$).

By the principle of non-contradiction, the system cannot reside at the $90°$ singularity (Structural Collapse) nor at the $0°$ limit (Identity Loss). Therefore, there *must* exist an intermediate stable state defined by $\theta \in (0, \pi/2)$ that preserves the field. The existence of this state is a structural necessity to resolve the geometric singularities of the distribution. □

Corollary 1 (The Twin Prime Limit)**.** *The Twin Prime Conjecture is the limiting case where* $\theta \to \theta_{min}$ *as* $n \to \infty$ *within the distribution* $2n\cos(\theta) = a + b$*, where* $a, b \in \mathbb{P}_{odd}$*.*

Non-Constructive Proof by Singularity. Assume the Twin Prime Conjecture is false. For all $n > N$, the system is strictly bounded such that $\theta > \theta_{min}$, forcing the angular distribution toward a **90-degree singularity** ($\theta = \pi/2$).

At this boundary, the equation yields:

$$2n\cos(90°) = a + b \implies 0 = a + b$$

Since a, b are positive odd primes, $a + b > 0$, creating a structural contradiction. To resolve this divergence at the $\pi/2$ singularity, the system must allow $\theta \to \theta_{min}$ infinitely often. Thus, the Twin Prime Limit is a structural necessity for the existence of the field. □

4 Conclusion

This non-constructive existence proof demonstrates that prime partitions are a structural requirement of the geometric mapping between addition and multiplication worlds.

References

[1] Hardy, G. H., & Wright, E. M. (2008). *An Introduction to the Theory of Numbers*. Oxford University Press.

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[3] Dejen Ketema. (2024). *Applied Mathematics I for Freshman*. Arba Minch University.