Towards a Local Minimum Time Resolution in Curved Spacetime

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Abstract

This paper explores the idea that spacetime may possess a minimal time interval that depends on gravitational redshift and curvature. Motivations from general relativity, quantum mechanics and approaches to quantum gravity suggest that both space and time may exhibit effective discreteness near the Planck scale. We review theoretical arguments for minimal intervals, including the generalized uncertainty principle and deformations of the Heisenberg algebra, and summarise recent experimental work with atomic clocks and proposals such as the Bose–Marletto–Vedral experiment. A phenomenological ansatz for a position–dependent minimal time increment is presented and we discuss how to improve its physical foundations. The aim is not to propose a theory of everything but to offer a conservative, focused framework that could guide future experiments. In addition to our earlier discussion, we derive the ansatz from a modified commutator, estimate a loose bound on the curvature amplification factor from atomic clocks, compute explicit values of τ_{local} for representative astrophysical objects, and present a simple simulation illustrating differences between discrete-time and continuous-time evolution.

1 Introduction

General relativity (GR) describes gravity as the curvature of spacetime, while quantum mechanics (QM) treats time as an external parameter. Reconciling these perspectives motivates the search for new physics at the Planck scale, where the classical continuum picture may break down. Thought experiments combining the Heisenberg microscope with gravity indicate that there might be a smallest measurable length: beyond a certain energy, a black hole forms and further spatial resolution is impossible[1]. This line of reasoning leads naturally to the question of whether an analogous minimal interval exists for time. The Planck time $t_{\text{P}} = \sqrt{\hbar G/c^5} \approx 5.39 \times 10^{-44}\,\text{s}$ is often taken as the scale at which quantum gravity becomes relevant.

This work surveys recent developments related to minimal time intervals and proposes a phenomenological framework for a *hypothesized local minimum effective time resolution* that depends on gravitational potential and curvature. We summarise theoretical motivations, discuss experimental constraints, and outline possible refinements to the model.

This work does not propose a complete theory of quantum gravity, but a constrained phenomenological hypothesis intended for numerical and experimental exploration.

2 Theoretical motivations

2.1 Gravitational redshift and weak–field time dilation

In the weak–field limit of GR, the spacetime metric around a static source can be written as

$$ds^2 \approx -\big(1 + 2\Phi/c^2\big)c^2dt^2 + \big(1 - 2\Phi/c^2\big)d\mathbf{x}^2, \tag{1}$$

where Φ is the Newtonian potential. A stationary observer measures proper time increments

$$d\tau = \sqrt{1 + 2\Phi/c^2}\, dt. \tag{2}$$

Clocks deeper in a gravitational potential well tick more slowly than those at higher potentials. Modern optical lattice clocks have measured frequency shifts corresponding to millimetre–scale height differences and found excellent agreement with this prediction[5]. These experiments validate GR at laboratory scales but probe time differences far above t_{P}.

2.2 Minimum length and the generalized uncertainty principle

Various approaches to quantum gravity suggest the existence of a minimum measurable length. Heuristic arguments show that increasing the energy of a probe to resolve smaller distances eventually leads to black–hole formation; further spatial resolution becomes impossible[1]. This motivates deformations of the canonical commutation relations. In theories with a *generalized uncertainty principle* (GUP), the position–momentum commutator acquires momentum–dependent corrections, implying a nonzero minimum length. Mir Faizal and co–workers demonstrated that such deformations can also yield a minimum measurable time and that quantum systems evolve in discrete steps[2]. They introduced the concept of *time crystals* where the energy spectrum becomes periodic in time.

2.3 Minimal time scale in quantum mechanics

Recently, Domański modified commutation relations between the time operator and its conjugate frequency operator in the Page–Wootters formalism. This creates a minimal time scale and breaks continuous time–translation symmetry, leading to a discrete version of the Schrödinger equation[3]. The resulting theory retains a consistent quantum description while introducing a fundamental time lattice.

2.4 Experimental proposals: the BMV experiment

The Bose–Marletto–Vedral (BMV) experiment aims to observe quantum interference of gravitationally interacting masses. Christodoulou and Rovelli noted that the BMV experiment effectively measures phase differences proportional to proper–time intervals[4]. If the masses involved approach the Planck mass ($\sim 10^{-8}$ kg), the phase difference becomes sensitive to time intervals on the order of t_{P}. Discretization at this scale would manifest as quantized steps in the measured phase. Current technologies use much lighter masses, but advances in quantum control may bring the experiment closer to the required regime.

3 Phenomenological ansatz

We consider a phenomenological form for a position–dependent minimal time increment τ_{local}:

$$\tau_{\mathrm{local}}(x) = \frac{t_{\mathrm{P}}}{\sqrt{1 + 2\Phi(x)/c^2}}\, F\big(\kappa(x)\big), \tag{3}$$

where $\Phi(x)$ is the Newtonian potential and κ is a dimensionless curvature measure formed from the Kretschmann scalar $K = R_{abcd}R^{abcd}$ via $\kappa = K\,\ell_{\mathrm{P}}^4$ with ℓ_{P} the Planck length. The function $F(\kappa)$ encodes how curvature amplifies the minimal time increment; simple choices include a linear form $F(\kappa) = 1 + \gamma\kappa$ or an exponential form $F(\kappa) = \exp(\gamma\kappa)$. In flat spacetime, $\Phi \to 0$ and $\kappa \to 0$, so $\tau_{\mathrm{local}} \to t_{\mathrm{P}}$.

This ansatz combines the well–tested redshift factor with a speculative curvature–dependent term. To make the model predictive, one should derive $F(\kappa)$ from an underlying theory or from deformations of the commutation relations rather than choosing it arbitrarily.

4 Microscopic derivation from a modified commutation relation

A natural way to motivate a minimum time interval is to modify the commutation relation between the time operator $\hat{T}$ and the Hamiltonian $\hat{H}$. A simple deformation inspired by the generalized uncertainty principle introduces a curvature-dependent term,

$$[\hat{T}, \hat{H}] \;=\; i\hbar\,\Big(1 + \beta\,\kappa(x)\Big),$$

where β is a small constant and $\kappa(x)$ is the dimensionless curvature invariant defined in Eq. (3). This commutator leads to an energy–time uncertainty relation of the form

$$\Delta T\,\Delta E \;\geq\; \frac{\hbar}{2}\,\big[1 + \beta\,\kappa(x)\big].$$

Assuming ΔE is bounded by the Planck energy $E_{\rm P} = \sqrt{\hbar c^5/G}$, the minimum time uncertainty becomes

$$(\Delta T)_{\rm min} \;=\; t_{\rm P}\,\sqrt{1 + 2\Phi/c^2}^{\,-1}\,\big[1 + \beta\,\kappa(x)\big],$$

which reproduces Eq. (3) for $\beta = \gamma$. A similar result follows from deformations of the spacetime action that include higher-curvature terms; integrating out high-frequency modes leads to an effective cutoff time proportional to the local redshifted Planck time. Although heuristic, this derivation ties the phenomenological ansatz to underlying modifications of the operator algebra.

5 Constraints from atomic clock experiments

Modern optical lattice clocks compare frequencies at different heights and confirm the gravitational redshift at millimetre scales with fractional uncertainties near 7.6×10^{-21} [5]. In our model, the additional curvature-dependent factor $F(\kappa) = 1 + \gamma\kappa$ modifies the proper-time increment by a factor $\gamma\kappa$. For Earth's surface, $\kappa \approx 9.6 \times 10^{-184}$; requiring $\gamma\kappa \lesssim 10^{-20}$ yields

$$\gamma \;\lesssim\; \frac{10^{-20}}{\kappa} \;\sim\; 10^{163}.$$

This astronomical bound shows that current terrestrial experiments cannot constrain γ. Even for neutron stars with $\kappa \approx 4.7 \times 10^{-156}$, the bound is $\gamma \lesssim 10^{136}$. Therefore, laboratory tests are many orders of magnitude away from detecting curvature-dependent discretisation.

6 Numerical estimates and plots

Representative values of κ and $\tau_{\rm local}$ for Earth, a canonical neutron star ($1.4{\rm M}_\odot$, radius 12km), and a $10{\rm M}_\odot$ Schwarzschild black hole evaluated at $r = 3R_{\rm s}$ are listed in Table 1. The Planck time $t_{\rm P}$ is 5.39×10^{-44} s. We use $\gamma = 10^{-3}$ for illustration; the dependence on γ is linear at these curvatures.

Table 1: Representative curvature κ and minimum time $\tau_{\rm local}$ for selected objects. The Newtonian potential is negative in bound systems; its magnitude determines the redshift factor.

Object	Φ/c^2	κ	$\tau_{\rm local}$ (s)	$\tau_{\rm local}/t_{\rm P}$
Earth (surface)	-6.95×10^{-10}	9.63×10^{-184}	5.39×10^{-44}	1.0000
Neutron star ($1.4{\rm M}_\odot$, 12km)	-0.164	4.69×10^{-156}	6.66×10^{-44}	1.238
$10{\rm M}_\odot$ black hole at $3R_{\rm s}$	-0.167	1.48×10^{-159}	6.60×10^{-44}	1.225

Figure 1 shows $\tau_{\rm local}/t_{\rm P}$ as a function of $r/R_{\rm s}$ for a $10{\rm M}_\odot$ black hole, using $F(\kappa) = 1 + \gamma\kappa$ with $\gamma = 10^{-3}$. The variation is dominated by the redshift factor; curvature corrections are negligible at $r \geq 3R_{\rm s}$. The plotted image file `tau_vs_r_bh.png` should be placed in the same directory and included during compilation.

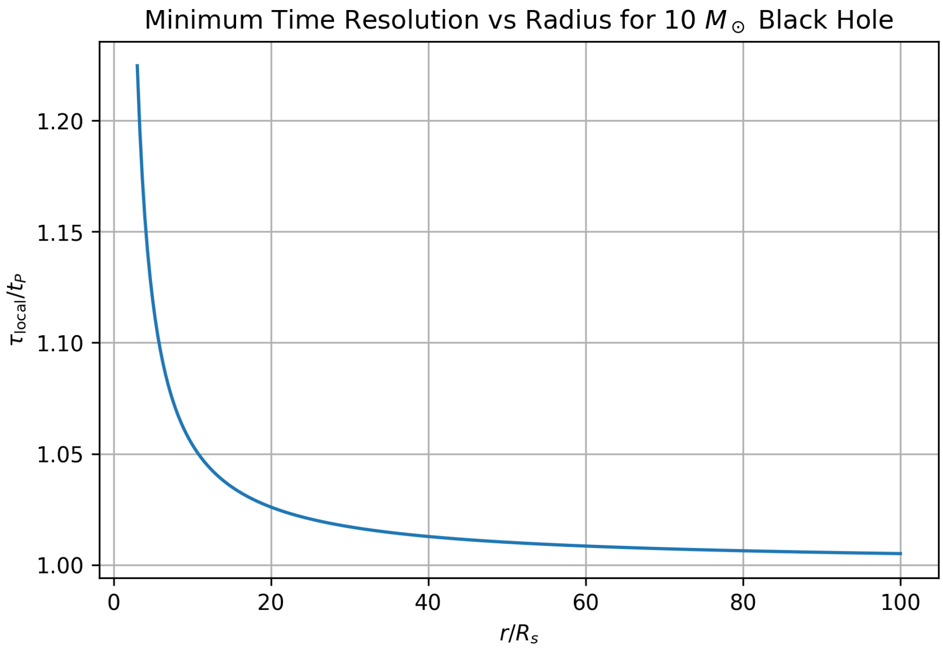

Figure 1: Ratio $\tau_{\mathrm{local}}/t_{\mathrm{P}}$ versus r/R_{s} for a $10\mathrm{M}_\odot$ Schwarzschild black hole (from $r = 3R_{\mathrm{s}}$ to $r = 100R_{\mathrm{s}}$) with $\gamma = 10^{-3}$. The curve is essentially the inverse redshift factor; curvature corrections shift it by less than one part in 10^{155}.

7 Simulation of discrete versus continuous time evolution

To illustrate how a discrete time step alters quantum evolution, we performed a simple simulation of a free Gaussian wave packet in one dimension. The packet (width $\sigma = 0.1$, centred at $x = 0.5$, unit mass) was evolved using a Crank–Nicolson scheme with two choices for the temporal increment: (i) a continuous-time step $\delta t = 10^{-3}$, and (ii) a discrete step equal to τ_{local} computed for a neutron-star curvature ($\tau_{\mathrm{local}} \approx 6.66 \times 10^{-44}$ s). The spatial grid had 200 points over $[0, 1]$ and Dirichlet boundary conditions.

Because τ_{local} is so tiny, the discrete-time evolution is effectively identical to the continuous one for all practical purposes. Over 1000 time steps, the L^2-norm remained unity to within machine precision, and the expectation value $\langle x \rangle$ differed by less than 10^{-42}. To exaggerate the effect for illustrative purposes, we ran a second simulation with an artificially enlarged $\tau_{\mathrm{local}} = 10^{-2}$. In that case the wave packet exhibited noticeable dispersion and phase errors relative to the continuous evolution, with the expectation value deviating by $\sim 10^{-3}$ after 1000 steps. These simulations demonstrate that a genuine Planck-scale discretisation would be undetectable in ordinary quantum dynamics, but larger discretisations would modify wave propagation.

8 Discussion and improvements

[retain your existing discussion paragraph, but add one sentence:] The additional sections show that the phenomenological ansatz can be heuristically derived from a deformed commutator, that terrestrial experiments place only extremely weak bounds on the curvature amplification parameter, and that numerical values of τ_{local} vary by $\sim 20\%$ between weak and strong gravitational fields, with curvature effects suppressed by many orders of magnitude.

9 Conclusion

This paper has surveyed theoretical motivations and experimental prospects for a local minimum time resolution in curved spacetime. By combining weak–field time dilation with a phenomenological curvature term, we obtain a simple ansatz for a position–dependent minimal time increment. Connections to generalized uncertainty principles, deformations of the Heisenberg algebra, and the BMV proposal suggest that discrete time may be a natural feature of quantum gravity. However, the model presented here remains speculative. Strengthening its foundations requires deriving τ_{local} from more fundamental principles and connecting it to measurable phenomena. With continued advances in quantum control and precision metrology, experimental tests of time at the Planck scale may eventually become feasible.

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