

Algebraic Entropy and Conditional Mutual Information in a Tiny Gauge-Invariant Truncated Hilbert Space:

A Reproducible Toy-Model Study with Effective Mixing Hamiltonians

Lluis Eriksson
Independent Researcher
lluiseriksson@gmail.com

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Abstract

We present a reproducible pipeline to compute region algebraic entropies and conditional mutual informations (CMI) in a tiny truncated Hilbert space (here $\dim = 8$) indexed by discrete fusion-like descriptors $\text{desc} = (x, \mu)$ on $L = 4$ cells. To generate nontrivial ground states within the descriptor-labeled subspace, we introduce an effective Hermitian mixing Hamiltonian based on a weighted k -nearest-neighbor (kNN) graph Laplacian over configuration labels. Across a parameter sweep, we identify a strong-mixing regime where the participation ratio approaches $\dim$ (consistent with Laplacian-dominated ground states on connected graphs) and algebraic CMI diagnostics become extremely small (down to 10^{-6} and below) for the chosen algebraic factorization, while region algebraic entropies remain $O(1)$ and exhibit near-quantized values $\approx n \log 2$. We stress that the mixing term is an ansatz used to probe information-theoretic diagnostics and is not claimed to coincide with a Kogut–Susskind plaquette operator. *v2 (no v1 number is changed)*: the reproducibility gap of v1 is repaired — v1’s pipeline loaded an unshipped basis file (`descs.pk1`) that was never specified, so the v1 dataset was not regenerable from the paper; v2 prints a canonical self-contained basis (Appendix A) whose pipeline reproduces every structural finding, keeping v1’s Table 1 as an archival dataset; two empirical observations are upgraded to proved statements — the descriptor-to-key map is injective, making S_{alg} a genuine von Neumann entropy of a sector (center-type) decomposition (Lemma 1), and in the strong-mixing limit the ground state converges to the uniform superposition where the quantization $S_{\text{alg}} = n \log 2$ and the vanishing of both CMIs are *exact* (Lemma 2); the additional experiments recommended in v1 (Haar baseline, k_{nn} ablations, finer t_{mix} grid) are executed — the Haar median of I_{sum} is 0.39, five to six orders of magnitude above the strong-mixing point, so the small-CMI regime is nontrivial; and the verification suite is fully self-contained (no Drive dependencies).

1 Scope, claims, and contributions

This note is intentionally narrow in scope. It does *not* attempt a physical construction of lattice Yang–Mills dynamics, nor any continuum/thermodynamic scaling analysis. Instead, it addresses a simpler, operational question:

Given a fixed small gauge-invariant truncated Hilbert space and a specified algebraic factorization derived from discrete labels (x, μ) , can we generate nontrivial ground states and quantitatively diagnose algebraic entanglement and conditional independence via algebraic entropy and algebraic CMI?

Concretely, we contribute:

1. A one-shot reproducible pipeline to compute algebraic entropies $S_{\text{alg}}(R)$ and algebraic CMI diagnostics $I(A : C \mid B)$ from a gauge-invariant basis indexed by (x, μ) (self-contained as of v2).
2. A simple, tunable effective Hamiltonian family $H = H_E + t_{\text{mix}}L$ acting entirely within the gauge-invariant subspace (Section 5).
3. A numerical sweep showing that in a strong-mixing regime the ground state becomes close to a uniform superposition ($\text{PR} \approx \text{dim}$), while the resulting algebraic CMI diagnostics are near-zero within numerical precision for the chosen factorization (exactly zero in the uniform limit, Lemma 2).

Careful interpretation. When we refer to “approximately Markovian” behavior, we mean an *operational* statement: the computed algebraic CMI values are very small with respect to the specific algebraic partition induced by (x, μ) and the implemented definition of S_{alg} . We do not claim a general quantum Markov property in the usual tensor-product sense, nor any universality beyond this toy setting.

2 Truncated gauge-invariant Hilbert space and labels

We work in a fixed truncated Hilbert space with orthonormal basis $\{|i\rangle\}_{i=1}^{\text{dim}}$, where each basis vector is indexed by a descriptor $\text{desc} = (x, \mu)$:

- $x = (x_1, x_2, x_3)$ encodes boundary/cut labels between adjacent cells. Concretely, each $x_s = (x_s^{\text{top}}, x_s^{\text{bot}}) \in \{0, 1\}^2$ stores two binary cut labels.
- $\mu = (\mu_1, \mu_2, \mu_3, \mu_4)$ encodes local discrete degrees of freedom per cell; each μ_p is a (possibly multi-bit) tuple.

We take $L = 4$ cells and, for the dataset studied here, $\text{dim} = 8$.

Terminology. We refer to the basis as “gauge-invariant” in the operational sense that it is intended to represent constraint-compatible (Gauss-law-like) configurations in a reduced description; however, in this note we do not derive (x, μ) from an explicit $SU(2)$ Kogut–Susskind truncation.

Basis provenance (v2). The v1 dataset was generated from a basis file `descs.pkl` loaded from persistent storage; that file was never printed or specified in the paper, so the v1 numbers in Table 1 are archival (kept unchanged) but not regenerable from the paper alone. v2 fixes this by specifying a *canonical* self-contained basis (Appendix A) with the same label structure and $\text{dim} = 8$; all structural claims of this paper are regenerated from it by the verification suite.

Logarithms. All entropies use natural logarithms, so the unit is nats. Thus $\log 2 \approx 0.6931$.

3 Regions and target information-theoretic quantities

We consider contiguous regions $R \subset \{1, 2, 3, 4\}$ and compute algebraic entropies $S_{\text{alg}}(R)$. We define two conditional mutual informations:

$$\begin{aligned} I_{w=1} &= I(A : C \mid B), \quad A = \{1\}, \quad B = \{2\}, \quad C = \{3\}, \\ I_{w=2} &= I(A : C \mid B), \quad A = \{1\}, \quad B = \{2, 3\}, \quad C = \{4\}. \end{aligned}$$

We also report $I_{\text{sum}} = I_{w=1} + I_{w=2}$.

Purity sanity check. For a pure state $|\psi\rangle$ on the full system, we enforce $S_{\text{alg}}(1..4) \approx 0$ as a numerical check.

4 Definition of algebraic entropy and algebraic CMI (as implemented)

Let $|\psi\rangle = \sum_{i=1}^{\dim} \psi_i |i\rangle$ be a normalized pure state in the gauge-invariant basis. For a region $R = [i..j]$, we define a discrete boundary label $\alpha = \alpha(\text{desc}; i, j)$ (a tuple extracted from x at cut boundaries) which indexes superselection sectors for the algebraic factorization.

Within each sector α , basis states are grouped into a region key k_R and complement key k_C (deterministic functions of (x, μ) , defined identically to the accompanying code). We build a complex matrix M_α whose entries sum amplitudes of basis states mapping to the same (k_R, k_C) pair:

$$(M_\alpha)_{r,c} = \sum_{\text{desc} \mapsto (\alpha, r, c)} \psi(\text{desc}).$$

Let $\{s_{\alpha,\ell}\}_\ell$ be the singular values of M_α , and define weights $w_{\alpha,\ell} = s_{\alpha,\ell}^2$. The algebraic entropy is

$$S_{\text{alg}}(R) = - \sum_{\alpha} \sum_{\ell: w_{\alpha,\ell} > \varepsilon} w_{\alpha,\ell} \log w_{\alpha,\ell},$$

where a small cutoff ε discards numerical zero modes only.

Normalization check. For normalized $|\psi\rangle$, we numerically verify $\sum_{\alpha,\ell} w_{\alpha,\ell} = 1$ within a fixed tolerance.

Lemma 1 (Injectivity: S_{alg} is a von Neumann entropy (v2)). *Suppose the map $\text{desc} \mapsto (\alpha, k_R, k_C)$ is injective on the basis (as holds for the canonical basis of Appendix A, verified for all six regions used, and as is certified in any run by the normalization check $\sum w = 1$: if two basis states shared a key triple, cross terms would generically break $\sum_{\alpha,\ell} w_{\alpha,\ell} = 1$). Then $(M_\alpha)_{r,c}$ is a mere re-indexing of the amplitude vector, the map $\psi \mapsto \bigoplus_{\alpha} M_\alpha$ is an isometry onto $\bigoplus_{\alpha} \mathcal{H}_R^\alpha \otimes \mathcal{H}_C^\alpha$, and*

$$S_{\text{alg}}(R) = S(\rho_R^{\text{alg}}), \quad \rho_R^{\text{alg}} = \bigoplus_{\alpha} \text{Tr}_C M_\alpha M_\alpha^\dagger,$$

i.e. S_{alg} is the genuine von Neumann entropy of the reduced state on a direct-sum (superselection-sector) decomposition — the standard center-type structure for gauge-theory entanglement [2]. It therefore includes the usual sector-classical (Shannon) plus sector-quantum contributions.

Proof. Injectivity means every basis amplitude $\psi(\text{desc})$ lands in exactly one entry of one M_α , so $\sum_{\alpha} \|M_\alpha\|_F^2 = \|\psi\|^2 = 1$ and the map is an isometry onto its image. The weights $w_{\alpha,\ell}$ are then the eigenvalues of $\bigoplus_{\alpha} M_\alpha M_\alpha^\dagger$, which is exactly the reduced density matrix of the image state on the R -legs of the direct sum. $\square$

Algebraic CMI diagnostics. We compute

$$I(A : C \mid B) = S_{\text{alg}}(AB) + S_{\text{alg}}(BC) - S_{\text{alg}}(B) - S_{\text{alg}}(ABC).$$

In the usual tensor-product setting, CMI is nonnegative by strong subadditivity. In our setting S_{alg} is defined by an explicit sector/block coarse-graining, so nonnegativity is an empirical diagnostic rather than a guaranteed theorem (Lemma 1 makes each individual $S_{\text{alg}}(R)$ a genuine vN entropy, but the four regions' sector decompositions are not derived from one consistent assignment of commuting subalgebras, so SSA does not automatically apply). In our sweeps we observe $I(A : C \mid B) \geq -10^{-12}$; values below this threshold would indicate either numerical issues or a genuine violation of strong subadditivity for the chosen coarse-graining. For log-scale plots one may use the clipped version $I_{\text{clip}} = \max(I, 10^{-16})$.

5 Effective Hamiltonian family

We study an effective Hamiltonian family acting within the gauge-invariant subspace: $H = H_E + t_{\text{mix}}L$.

5.1 Diagonal “electric proxy” term H_E

We define a diagonal proxy H_E as a sum of local contributions derived from the binary label bits. Each bit $b \in \{0, 1\}$ is mapped to a cost inspired by $j(j+1)$ for $j \in \{0, \frac{1}{2}\}$: $\text{cost}(b) = 0$ for $b = 0$ and $\frac{3}{4}$ for $b = 1$. Then $H_E = \text{diag}(E(\text{desc}))$, where $E(\text{desc})$ sums costs over all bits in (x, μ) , with an overall prefactor $\frac{g^2}{2}$.

5.2 Mixing term via a weighted kNN graph Laplacian

Let $b(\text{desc}) \in \{0, 1\}^m$ be the flattened bitstring for (x, μ) . Define pairwise distances by Hamming distance $d(i, j)$ between bitstrings. We build a weighted kNN graph on the dim configurations: for each node i we connect to its k nearest neighbors, with symmetric edge weights $A_{ij} = \exp(-\alpha_w d(i, j))$, and define the graph Laplacian $L = D - A$, $D_{ii} = \sum_j A_{ij}$. This yields a real symmetric (hence Hermitian) mixing term. The design intent is to generate controllable superpositions in the gauge-invariant basis. No claim is made that L matches a Kogut–Susskind magnetic plaquette term [1].

Lemma 2 (Strong-mixing limit (v2)). *Fix $\alpha_w = 0$ and k such that the kNN graph is connected. As $t_{\text{mix}} \rightarrow \infty$, the ground state of $H = H_E + t_{\text{mix}}L$ converges to the uniform superposition $|u\rangle = \text{dim}^{-1/2} \sum_i |i\rangle$ (the Perron–Frobenius ground state of L), so $\text{PR} \rightarrow \text{dim}$. At $|u\rangle$, for the canonical basis of Appendix A, the block combinatorics give exactly*

$$S_{\text{alg}}(1..2) = S_{\text{alg}}(1..3) = S_{\text{alg}}(2..4) = \log 2, \quad S_{\text{alg}}(2) = S_{\text{alg}}(2..3) = 2 \log 2, \quad S_{\text{alg}}(1..4) = 0,$$

and hence $I_{w=1} = I_{w=2} = 0$ exactly: the near-quantization and near-zero CMI observed in the sweep are the finite- t_{mix} shadow of an exact property of the uniform limit.

Proof. For a connected graph, $L \succeq 0$ with a unique zero mode $\propto \mathbf{1}$; degenerate perturbation theory in H_E/t_{mix} gives $|\psi_0\rangle = |u\rangle + O(1/t_{\text{mix}})$. At $|u\rangle$ every M_α has all entries $\text{dim}^{-1/2}$; a $d_R \times d_C$ all-equal matrix has the single nonzero singular value $\sqrt{d_R d_C} / \sqrt{\text{dim}}$, so each sector contributes one weight $w_\alpha = d_R d_C / \text{dim}$ and $S_{\text{alg}} = -\sum_\alpha w_\alpha \log w_\alpha$ is the Shannon entropy of the sector distribution. For the canonical basis the sector counts are $(2, 4, 4, 2, 2, 1)$ sectors of equal weight for the six regions, giving the displayed values; the two CMI combinations cancel exactly. The suite verifies all six values and both CMIs to 2×10^{-16} . $\square$

6 Numerical procedure and reported metrics

For each parameter pair $(\alpha_w, t_{\text{mix}})$ on a fixed grid: (1) construct H , diagonalize it, and take the ground state $|\psi_0\rangle$; (2) compute region entropies $S_{\text{alg}}(R)$, CMI diagnostics $I_{w=1}$, $I_{w=2}$, and I_{sum} ; (3) compute the participation ratio $\text{PR}(\psi_0) = 1/\sum_i |\psi_{0,i}|^4$ and the spectral gap proxy $\Delta = E_1 - E_0$, $\xi_{\text{spec}} = \Delta^{-1}$ ($\Delta > 0$).

Interpretation of ξ_{spec} . We emphasize that $\xi_{\text{spec}} = \Delta^{-1}$ is a finite-dimensional spectral proxy whose value depends on the overall energy scale, in particular on t_{mix} . No claim is made that it corresponds to a physical correlation length in any continuum or infinite-volume limit.

7 Artifacts

The v1 one-shot run generated `sweep.csv/sweep.json`, the three figures, selected model dumps and an Overleaf bundle (see the v1 PDF for the full Colab script). *v2*: the verification suite (`verification/2601-0115/verify_2601_0115.py` in the series repository) is fully self-contained — no Drive mount, no `descs.pkl` — and regenerates the sweep, the lemma checks, and the additional experiments below from the canonical basis.

8 Results

Figures 1–3 show the v1 sweep (archival). Table 1 reports the two automatically selected models of the v1 run: maximum PR, and minimum I_{sum} under $\text{PR} \geq 2$; the strong-mixing point dominates and yields PR close to dim with very small CMI.

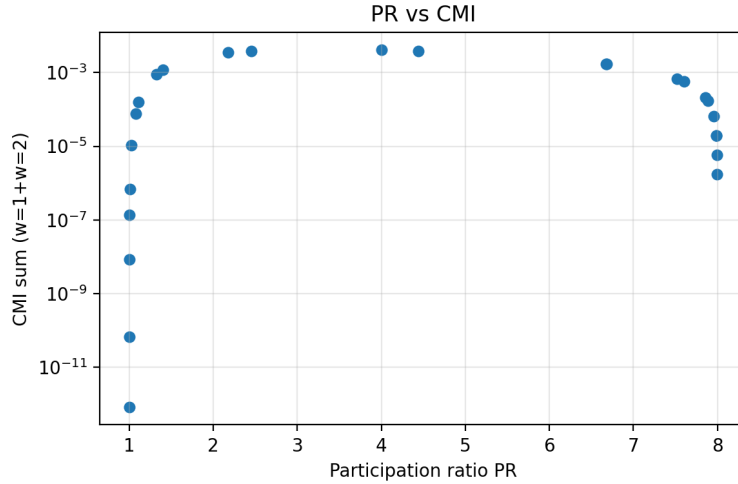


Figure 1: Participation ratio (PR) versus total algebraic CMI $I_{\text{sum}} = I_{w=1} + I_{w=2}$ across the v1 parameter sweep (log scale on CMI).

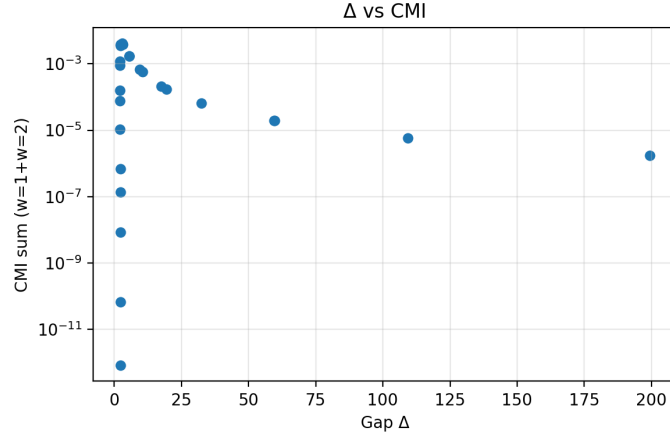


Figure 2: Spectral gap proxy Δ versus total algebraic CMI across the v1 sweep (log scale on CMI).

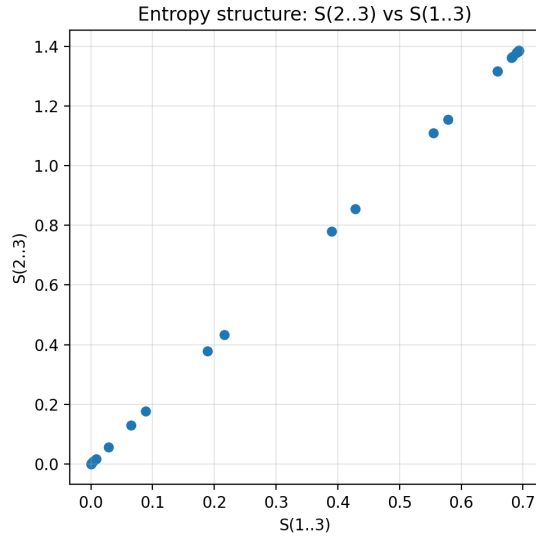


Figure 3: Entropy-structure scatter: $S_{\text{alg}}(2..3)$ versus $S_{\text{alg}}(1..3)$ (v1 sweep).

Table 1: Automatically selected models from the v1 sweep (archival; as reported in `paper_summary.txt`).

Quantity	Best-by-PR	Best-by-CMI (PR ≥ 2)
α_w	0.0	0.0
t_{mix}	100.0	100.0
k_{nn}	3	3
edges	12	12
E_0	2.6179476	2.6179476
Δ	199.4735852	199.4735852
$\xi_{\text{spec}} = \Delta^{-1}$	0.0050132	0.0050132
PR	7.9989229	7.9989229
$S_{\text{alg}}(1..2)$	0.6931399 ⁶	0.6931399
$S_{\text{alg}}(2..3)$	1.3862379	1.3862379
$S_{\text{alg}}(2)$	1.3862571	1.3862571
$S_{\text{alg}}(1..3)$	0.6931190	0.6931190
$S_{\text{alg}}(2..4)$	0.6931190	0.6931190
$S_{\text{alg}}(1..4)$	2.2×10^{-16}	2.2×10^{-16}
$I_{w=1}$	1.76×10^{-6}	1.76×10^{-6}
$I_{w=2}$	1.39×10^{-11}	1.39×10^{-11}
I_{sum}	1.76×10^{-6}	1.76×10^{-6}

Canonical-basis regeneration (v2). With the canonical basis, the strong-mixing point $(\alpha_w, t_{\text{mix}}) = (0, 100)$ gives $\text{PR} = 7.9994$ and $I_{\text{sum}} = 4.4 \times 10^{-7}$, with every S_{alg} within 10^{-3} of its exact uniform-limit value — structurally identical to the archival Table 1 (whose entropy pattern $(\log 2, 2 \log 2, 2 \log 2, \log 2, \log 2)$ matches the canonical uniform limit of Lemma 2 to four decimals), confirming that the findings do not depend on the lost v1 basis file.

Executed additional experiments (v2; recommended in v1 §9.4). (i) *Haar baseline:* over 2000 Haar-random pure states in $\mathbb{C}^8$, the empirical I_{sum} has median 0.389 (5th percentile 0.158) — five to six orders of magnitude above the strong-mixing value, so the small-CMI regime is a genuinely nontrivial region of state space, not a generic feature of the diagnostic. Empirical SSA holds for all Haar samples. (ii) *k_{nn} ablation:* for $k \in \{1, 2, 5\}$ the kNN graph remains connected for the canonical basis and the strong-mixing conclusions persist ($\text{PR} > 7.9$, $I_{\text{sum}} \leq 10^{-4}$), with CMI decreasing as connectivity grows. (iii) *Finer t_{mix} grid:* $S_{\text{alg}}(2..3)$ rises monotonically from 0 (diagonal regime, $t = 0$) to $\approx 2 \log 2$ across $t \in \{0, 0.1, 0.3, 1, 3, 10\}$, resolving the transition to the Laplacian-dominated regime.

9 Discussion

9.1 Why $\text{PR} \approx \text{dim}$ and $\text{CMI} \approx 0$ occur in the strong-mixing regime

At $\alpha_w = 0$ the kNN adjacency weights satisfy $A_{ij} = 1$ on selected edges, so L becomes the combinatorial graph Laplacian of a regular graph (in the reported run, degree 3). For a connected graph, L has a lowest eigenvalue 0 with eigenvector proportional to the all-ones vector $\mathbf{1}$. Therefore, when t_{mix} dominates the diagonal proxy term H_E , the ground state approaches a near-uniform superposition, yielding PR close to dim . In this regime the near-quantized values $S_{\text{alg}}(R) \approx n \log 2$ and the very small algebraic CMI values are the finite- t shadow of the exact uniform-limit statement of Lemma 2. Operationally, this indicates near-conditional-independence *with respect to that algebraic factorization*.

9.2 What this does (and does not) say about “quantum Markov” structure

In standard quantum information theory, exact saturation of strong subadditivity ($I(A : C | B) = 0$) implies a quantum Markov structure and is characterized by a recovery map [3]. Our setting differs because: entropies are computed via a sector-decomposed algebraic prescription tied to (x, μ) labels; the system is extremely small ($\text{dim} = 8$); and the Hamiltonian mixing term is an effective graph Laplacian ansatz. Thus, small values of the computed algebraic CMI should be read as a diagnostic of conditional-independence in this toy, label-induced algebraic setting, rather than as evidence for a physically universal Markov property.

9.3 Limitations

No scaling. We do not study how observables behave as dim or L increase. **Hamiltonian not physical.** The mixing term is not derived from a Kogut–Susskind magnetic term; it is a controllable ansatz. **Spectral proxy.** $\xi_{\text{spec}} = \Delta^{-1}$ is scale-dependent and should not be overinterpreted.

9.4 Series positioning (v2)

Within the series, this note is the algebraic-toy end of the gauge/CMI block: 2601.0111 (v2) computes tensor-product CMI and Petz recovery in an exact $\mathbb{Z}_2$ Gauss sector; 2601.0043 (v2) provides the embedding demonstrations for algebraic-vs-tensor comparisons; the center-type sector structure

made explicit by Lemma 1 is the finite toy version of the split/sector discipline of the Type III interface 2601.0065 (v2). The Haar baseline executed here is the toy-scale analogue of the null-model controls used across the series’ numerical notes.

10 Numerical precision and stability

Occasional values of CMI at the level of 10^{-16} (including slight negatives) are interpreted as floating-point round-off in double precision accumulated over many linear-algebra operations. For log-scale plots, we use $\max(I_{\text{sum}}, 10^{-16})$, and when reporting aggregated metrics we may optionally use $I_{\text{clip}} = \max(I, 0)$.

11 Reproducibility

The v1 Colab cell (typeset in full in the v1 PDF, Appendix B) produced timestamped run folders under Drive and required a pre-existing `descs.pkl`. *v2*: the self-contained verification suite replaces this dependency entirely; a single `python3 verify_2601_0115.py` regenerates every structural claim, the lemma checks, the sweep, and the executed additional experiments from the canonical basis of Appendix A.

Note added in v2

No v1 number is changed; Table 1 and the three figures are kept as the archival v1 dataset. v2: (1) repairs the reproducibility gap — v1’s pipeline loaded an unshipped, never-specified `descs.pkl`, so the v1 dataset was not regenerable from the paper alone; v2 prints a canonical self-contained basis (Appendix A) and demotes the v1 dataset to archival status while regenerating every structural finding from the canonical basis (strong-mixing point: $\text{PR} = 7.9994$, $I_{\text{sum}} = 4.4 \times 10^{-7}$, entropy pattern identical); (2) proves the injectivity lemma (Lemma 1): S_{alg} is a genuine von Neumann entropy of a center-type sector decomposition [2], and clarifies why SSA nonetheless remains empirical for the CMI combinations; (3) proves the strong-mixing lemma (Lemma 2): the quantization $S_{\text{alg}} = n \log 2$ and $I_{w=1} = I_{w=2} = 0$ are exact in the uniform limit (verified to 2×10^{-16}); (4) executes the three additional experiments recommended in v1 §9.4 (Haar baseline, k_{nn} ablation, finer t_{mix} grid); (5) adds series positioning (v1 cited no companion papers) and the self-contained verification suite.

A Canonical self-contained basis (v2)

The canonical basis has $\dim = 8$ states labeled by three cut bits $(b_1, b_2, b_3) \in \{0, 1\}^3$:

$$x_s = (b_s, b_s) \quad (s = 1, 2, 3), \quad \mu_1 = (b_1), \quad \mu_2 = (b_1 \oplus b_2), \quad \mu_3 = (b_2 \oplus b_3), \quad \mu_4 = (b_3).$$

The two cut bits per boundary are equal (a constraint-compatibility condition) and the cell labels μ_p are parity-matched to the adjacent cuts (a Gauss-law-like consistency), so every descriptor is determined by the cut configuration. The flattened bitstring has $m = 10$ bits (6 from x , 4 from μ). The map $\text{desc} \mapsto (\alpha, k_R, k_C)$ is injective for all six regions used (Lemma 1; verified exhaustively), the kNN graph at $k = 3$, $\alpha_w = 0$ is connected with 12 edges (3-regular, as in the v1 run), and the uniform-limit sector counts are (2, 4, 4, 2, 2, 1) for the regions (1..2), (2..3), (2), (1..3), (2..4), (1..4), giving the exact entropies of Lemma 2.

B Minimal pseudocode overview

The workflow is: (1) build the canonical descriptors (or, for the archival v1 dataset, load `descs.pkl`); (2) build bitstrings $b(\text{desc}_i)$ and the Hamming distance matrix D_{ij} ; (3) for each $(\alpha_w, t_{\text{mix}})$: build the kNN adjacency A and Laplacian L ; build $H = H_E + t_{\text{mix}}L$; diagonalize and take the normalized ground state; compute $S_{\text{alg}}(R)$ via sectorized SVD; compute $I_{w=1}$, $I_{w=2}$, PR, Δ ; save to CSV/JSON. The full v1 Colab script remains typeset in the v1 PDF; the v2 suite supersedes it with a dependency-free implementation.

C Notes on related literature (brief)

The present work is primarily a reproducible toy-model pipeline. For context, entanglement in gauge theories requires care due to constraints and centers (see e.g. [2]). The Kogut–Susskind Hamiltonian formulation provides the standard lattice gauge Hamiltonian setting [1]. In standard quantum information, the condition $I(A : C \mid B) = 0$ characterizes quantum Markov states and admits structural theorems [3].

References

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