Emergence of Spacetime and a Big-Bang–Like Singularity from Octonionic Algebra

Abstract

We present a conceptual framework in which the octonionic division algebra is taken as a pre-geometric fundamental structure. We demonstrate that the intrinsic non-associativity of the octonions excludes the existence of a fundamental global time and enforces a symmetry reduction in order to admit physically interpretable dynamics. This reduction occurs through an algebraically singular transition, naturally interpreted as a Big-Bang–like origin. Four-dimensional spacetime emerges as a stable associative subalgebra, while cosmological expansion arises as a necessary consequence of residual non-associative degrees of freedom. The framework is mathematically consistent but remains speculative and currently lacks experimental support.

1 Introduction

Modern cosmology is formulated on the assumption that spacetime geometry is a fundamental structure. Cosmological singularities, most notably the Big Bang, signal the breakdown of this description and suggest that spacetime itself may be emergent.

This work explores a framework in which geometry is not postulated but arises from a more primitive algebraic structure. The octonionic division algebra is a natural candidate, being the largest real division algebra and the only one that is fundamentally non-associative. We investigate whether the emergence of spacetime and a Big-Bang–like origin follow necessarily from algebraic consistency alone.

2 Octonionic Algebra

The octonions $\mathbb{O}$ form an eight-dimensional real division algebra

$$\mathbb{O} = \mathrm{span}_{\mathbb{R}}\{e_0, e_1, \dots, e_7\}, \quad e_0 = 1, \tag{1}$$

with multiplication

$$e_i e_j = -\delta_{ij} + C_{ijk} e_k, \tag{2}$$

where C_{ijk} is totally antisymmetric and encoded by the Fano plane.

Unlike $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{H}$, the octonions are non-associative. The associator is defined as

$$[a, b, c] := (ab)c - a(bc), \tag{3}$$

and is generically non-vanishing.

3 Absence of Fundamental Time

Associativity is a prerequisite for defining a globally consistent composition of operations. Due to the non-vanishing associator, the octonionic algebra admits no canonical ordering of products.

Proposition. Within the full octonionic algebra, no globally defined notion of time evolution exists.

Consequence. Time cannot be fundamental in an octonionic framework. Any physically meaningful temporal structure must emerge from associative subalgebras.

4 Automorphisms and Symmetry Reduction

The automorphism group of the octonions is the exceptional Lie group

$$\mathrm{Aut}(\mathbb{O}) = G_2. \tag{4}$$

A fully G_2-symmetric configuration preserves all non-associative degrees of freedom. However, physically interpretable dynamics require stable associative composition laws. This enforces a symmetry reduction of the form

$$G_2 \longrightarrow SU(3) \longrightarrow SU(2) \times U(1). \tag{5}$$

This reduction is not imposed externally but arises as a consistency requirement for dynamics.

5 The Big Bang as an Algebraic Singularity

We introduce an order parameter measuring non-associativity,

$$\Phi := \langle [e_i, e_j, e_k] \rangle. \tag{6}$$

In a maximally symmetric phase, $\Phi \neq 0$. As associative subalgebras become dynamically selected,

$$\Phi \to 0 \tag{7}$$

on the emergent physical sector.

The transition is singular in the sense that

$$\frac{d\Phi}{dt} \to \infty, \tag{8}$$

even though no spacetime metric has yet been defined.

Interpretation. The Big Bang corresponds to an algebraic singularity: the transition from maximal non-associativity to effective associativity.

6 Emergence of Spacetime

Any triple of compatible imaginary units generates a quaternionic subalgebra

$$\mathbb{H} \subset \mathbb{O}, \tag{9}$$

on which the associator vanishes identically.

The octonionic norm

$$N(x) = x\bar{x} \tag{10}$$

induces a quadratic form on $\mathbb{H}$, which may be identified with a Lorentzian metric,

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2. \tag{11}$$

Thus, four-dimensional spacetime emerges as a stable associative subalgebra rather than being fundamental.

7 Octonionic Action Principle

Since spacetime geometry is not assumed a priori, the fundamental action must be purely algebraic. The associator provides the natural dynamical measure.

We introduce an octonionic field

$$\Psi(\tau) \in \mathbb{O}, \tag{12}$$

where τ is a pre-geometric ordering parameter.

The fundamental action is defined as

$$S_{\mathbb{O}} = \int d\tau \left[\frac{1}{2} \langle D_\tau \Psi, D_\tau \Psi \rangle - \frac{\lambda}{4} \langle [\Psi, \Psi, \Psi], [\Psi, \Psi, \Psi] \rangle \right], \tag{13}$$

with $\lambda > 0$.

Variation yields the equation of motion

$$D_\tau^2 \Psi + \lambda \frac{\delta}{\delta \Psi} \langle [\Psi, \Psi, \Psi]^2 \rangle = 0. \tag{14}$$

The action is minimized when

$$[\Psi, \Psi, \Psi] = 0, \tag{15}$$

forcing the system into associative subalgebras.

8 Cosmological Expansion

Residual non-associativity acts as a positive effective energy density. An effective stress-energy contribution may be written as

$$T_{\mu\nu}^{(\text{assoc})} \sim \langle [e_\mu, e_\alpha, e_\beta][e_\nu, e^\alpha, e^\beta] \rangle. \tag{16}$$

This leads to an effective Friedmann equation,

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho_{\text{assoc}}, \tag{17}$$

implying unavoidable expansion as long as non-associative degrees of freedom relax.

9 Discussion

Within this framework:

- time is emergent rather than fundamental,
- consistent dynamics enforce symmetry reduction,
- the Big Bang is an algebraic singularity,
- spacetime arises as an associative subalgebra,
- expansion follows from residual non-associativity.

These features are not independent assumptions but consequences of the octonionic structure.

10 Conclusion

Assuming octonionic algebra as a pre-geometric foundation naturally leads to an emergent spacetime and a Big-Bang–like origin. The results follow from algebraic consistency alone. While speculative, the framework provides a coherent foundations-based perspective on the origin of spacetime and cosmology.

Disclaimer. This work is purely theoretical. No experimental verification is currently known.

A Mathematical Appendix

A.1 Basic Identities of the Octonionic Associator

Let $\mathbb{O}$ denote the octonionic division algebra. The associator is defined as

$$[a, b, c] := (ab)c - a(bc). \tag{18}$$

The associator satisfies the following fundamental identities:

Total Antisymmetry. For all $a, b, c \in \mathbb{O}$,

$$[a, b, c] = -[b, a, c] = -[a, c, b]. \tag{19}$$

Alternativity. The octonions are alternative, implying

$$[a, a, b] = [b, a, a] = 0. \tag{20}$$

As a consequence, any subalgebra generated by two elements is associative.

Norm Compatibility. Let $\bar{x}$ denote octonionic conjugation. The associator satisfies

$$\langle [a, b, c], 1 \rangle = 0, \tag{21}$$

and hence is purely imaginary. Moreover,

$$\langle [a, b, c], [a, b, c] \rangle \geq 0. \tag{22}$$

This ensures that quadratic expressions constructed from the associator define positive semi-definite scalar quantities.

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A.2 Quaternionic Subalgebras

For any triple of imaginary units (e_i, e_j, e_k) satisfying

$$e_i e_j = e_k, \tag{23}$$

and closed under cyclic permutation, the generated subalgebra

$$\mathbb{H}_{ijk} = \text{span}\{1, e_i, e_j, e_k\} \tag{24}$$

is isomorphic to the quaternion algebra $\mathbb{H}$.

Proposition. On $\mathbb{H}_{ijk}$, the associator vanishes identically:

$$[a, b, c] = 0 \quad \forall a, b, c \in \mathbb{H}_{ijk}. \tag{25}$$

Proof. Quaternionic multiplication is associative. Since $\mathbb{H}_{ijk}$ is closed under multiplication, the associator vanishes by restriction. □

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A.3 The Exceptional Lie Group G_2

The automorphism group of the octonions is the exceptional Lie group

$$\mathrm{Aut}(\mathbb{O}) = G_2. \tag{26}$$

An automorphism $\phi \in G_2$ preserves multiplication:

$$\phi(ab) = \phi(a)\phi(b), \tag{27}$$

and therefore preserves both the norm and the associator:

$$\phi([a, b, c]) = [\phi(a), \phi(b), \phi(c)]. \tag{28}$$

Dimension. The Lie algebra $\mathfrak{g}_2$ has dimension 14 and is the stabilizer of the associative 3-form

$$\varphi = \frac{1}{6} C_{ijk}\, e^i \wedge e^j \wedge e^k. \tag{29}$$

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A.4 Associative 3-Form and G_2 Structure

The octonionic structure constants define a canonical 3-form

$$\varphi_{ijk} := C_{ijk}, \tag{30}$$

which satisfies

$$\nabla\varphi = 0 \quad \text{iff the connection has } G_2 \text{ holonomy.} \tag{31}$$

The associator can be written in index notation as

$$[a, b, c]^m = \varphi^m{}_{ij}\, \varphi^j{}_{kl}\, a^i b^k c^l, \tag{32}$$

up to normalization.

This expression makes explicit that non-associativity is controlled by the G_2-invariant tensor φ.

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A.5 Symmetry Reduction and Stabilizers

Choosing a quaternionic subalgebra $\mathbb{H} \subset \mathbb{O}$ breaks G_2 to the stabilizer subgroup

$$G_2 \longrightarrow SU(2). \tag{33}$$

More generally, selecting a preferred imaginary unit induces the chain

$$G_2 \supset SU(3) \supset SU(2) \times U(1). \tag{34}$$

This reduction corresponds to suppressing non-associative directions orthogonal to the chosen subalgebra.

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A.6 Justification of the Action Functional

The scalar

$$\langle [\Psi, \Psi, \Psi], [\Psi, \Psi, \Psi] \rangle \tag{35}$$

is:

- real,
- positive semi-definite,
- invariant under G_2.

Therefore, it is the lowest-order nontrivial invariant available for constructing a purely algebraic action.

In the associative limit, this term vanishes identically, justifying its interpretation as a non-associative energy density.

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A.7 Summary of Mathematical Structure

- Non-associativity is encoded in the octonionic associator.
- G_2 preserves both the norm and the associator.
- Quaternionic subalgebras are maximally associative sectors.
- Symmetry reduction corresponds to selecting associative directions.
- The action principle is uniquely fixed at lowest order.

These results support the interpretation of the Big-Bang–like transition as an algebraic singularity driven by symmetry reduction.

B Projection to Effective Einstein Equations

B.1 Associative Projection Map

Let $\mathbb{O}$ denote the octonionic algebra and $\mathbb{H} \subset \mathbb{O}$ a selected quaternionic subalgebra. We define a projection operator

$$\mathcal{P}_{\mathbb{H}} : \mathbb{O} \longrightarrow \mathbb{H}, \tag{36}$$

such that for any $x \in \mathbb{O}$,

$$x = x_{\parallel} + x_{\perp}, \quad x_{\parallel} := \mathcal{P}_{\mathbb{H}}(x), \quad x_{\perp} := x - x_{\parallel}. \tag{37}$$

By construction,

$$[a, b, c] = 0 \quad \forall a, b, c \in \mathbb{H}. \tag{38}$$

All non-associative contributions are therefore contained in $x_{\perp}$.

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B.2 Decomposition of the Octonionic Field

We decompose the fundamental octonionic field as

$$\Psi = \psi + \chi, \tag{39}$$

where

$$\psi := \mathcal{P}_{\mathbb{H}}(\Psi), \qquad \chi := \Psi - \psi. \tag{40}$$

Here $\psi \in \mathbb{H}$ describes the emergent spacetime sector, while χ parametrizes non-associative fluctuations.

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B.3 Projected Action

Inserting the decomposition into the fundamental action,

$$S_{\mathbb{O}} = \int d\tau \left[\frac{1}{2}\langle D_\tau \Psi, D_\tau \Psi\rangle - \frac{\lambda}{4}\langle [\Psi, \Psi, \Psi], [\Psi, \Psi, \Psi]\rangle \right], \tag{41}$$

and projecting onto $\mathbb{H}$ yields

$$S_{\text{eff}} = \int d\tau \left[\frac{1}{2}\langle D_\tau \psi, D_\tau \psi\rangle - V_{\text{eff}}(\chi) \right], \tag{42}$$

where the effective potential is

$$V_{\text{eff}}(\chi) = \frac{\lambda}{4}\langle [\chi, \chi, \chi], [\chi, \chi, \chi]\rangle + \mathcal{O}(\chi^4). \tag{43}$$

To lowest order, the associative sector ψ is dynamically decoupled from non-associative fluctuations.

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B.4 Emergent Metric and Covariant Derivative

On the associative subalgebra $\mathbb{H}$, the octonionic norm induces a Lorentzian metric

$$g_{\mu\nu} = \langle e_\mu, e_\nu \rangle, \qquad \mu, \nu = 0, 1, 2, 3. \tag{44}$$

The derivative D_τ induces an effective spacetime covariant derivative

$$D_\mu \psi = \partial_\mu \psi + \Gamma_\mu \psi, \tag{45}$$

where Γ_μ arises from variations of the projection $\mathcal{P}_{\mathbb{H}}$ along τ.

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B.5 Effective Gravitational Action

At leading order, the projected action can be rewritten as

$$S_{\text{grav}} = \frac{1}{16\pi G_{\text{eff}}} \int d^4x \sqrt{-g}\, R + \int d^4x \sqrt{-g}\, \mathcal{L}_\chi, \tag{46}$$

where:

- R is the Ricci scalar constructed from $g_{\mu\nu}$,
- G_{eff} is an emergent gravitational coupling,
- $\mathcal{L}_\chi$ encodes non-associative corrections.

The Einstein–Hilbert structure arises from the kinetic term of the associative projection ψ.

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B.6 Emergent Einstein Equations

Variation with respect to the metric yields

$$G_{\mu\nu} = 8\pi G_{\text{eff}} \left(T^{(\chi)}_{\mu\nu} + T^{(\text{matter})}_{\mu\nu} \right), \tag{47}$$

where the non-associative contribution is given by

$$T^{(\chi)}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} \int d^4x \sqrt{-g}\, \mathcal{L}_\chi. \tag{48}$$

To leading order,

$$T^{(\chi)}_{\mu\nu} \sim \langle [\chi, \chi, \chi]^2 \rangle \, g_{\mu\nu}, \tag{49}$$

which acts as an effective cosmological energy density.

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B.7 Cosmological Limit

In a homogeneous and isotropic sector, the projected Einstein equations reduce to

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G_{\text{eff}}}{3} \rho_\chi, \tag{50}$$

where

$$\rho_\chi \propto \langle [\chi, \chi, \chi]^2 \rangle. \tag{51}$$

As $\chi \to 0$, the universe approaches a standard Einstein regime.

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B.8 Interpretational Remarks

- Einstein gravity emerges as a low-energy, associative limit of octonionic dynamics.
- The Big-Bang–like transition corresponds to χ-dominated dynamics.
- Non-associativity provides a natural source for early-universe expansion.

No additional geometric postulates are required.