Emergent Quantum Mechanics from a Discretized Spacetime Substrate

A Unified Theoretical and Phenomenological Framework

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Abstract

We present a comprehensive framework in which quantum mechanics emerges as an effective low-energy description of a fundamentally discretized spacetime substrate. The microscopic dynamics are modeled by a linear cellular automaton defined on a spacetime lattice. In the continuum limit, the Schrödinger equation is recovered with arbitrary experimental precision. At finite lattice spacing, the discrete topology induces a non-linear (sinusoidal) dispersion relation. We demonstrate that its leading-order correction is equivalent to a quadratic modification of the relativistic dispersion relation, formally identical to $n = 2$ Lorentz Invariance Violation (LIV) models. This provides a concrete microphysical origin for quadratic LIV and yields falsifiable predictions in ultra-high-energy astrophysics. We expand on the mathematical formalism, provide detailed derivations, and present extensive numerical simulations supporting the theoretical predictions.

1 Introduction and Motivation

Quantum mechanics (QM) has achieved extraordinary empirical success across energy scales ranging from atomic physics to particle colliders. Yet, despite its predictive power, QM is formulated on the assumption of a smooth spacetime continuum inherited from classical differential geometry. This assumption leads to well-known pathologies: ultraviolet divergences in quantum field theory, the measurement problem, and incompatibility with general relativity at the Planck scale.

These issues suggest that quantum mechanics may be an effective theory—accurate within its domain of applicability but not fundamental. Several approaches to quantum gravity, including loop quantum gravity (6), causal dynamical triangulations (7), and certain string theory compactifications, propose that spacetime itself is discrete at the smallest scales, typically near the Planck length $\ell_P \approx 1.6 \times 10^{-35}$ m.

In this work, we explore the consequences of spacetime discreteness for quantum dynamics and develop a phenomenological framework that connects microscopic lattice structure to observable predictions in ultra-high-energy astrophysics. Our approach is bottom-up: we begin with the simplest possible discrete dynamics and show that quantum mechanics emerges naturally in the appropriate limit.

2 Discretized Spacetime and Pre-Quantum Dynamics

2.1 Lattice Structure

We model spacetime as a regular hypercubic lattice with spatial spacing a and temporal step τ. For simplicity, we consider (1+1)-dimensional spacetime, though the framework generalizes to (3+1) dimensions. Physical states are defined only on lattice nodes, represented by a complex-valued wavefunction $\psi(n,t)$ where $n \in \mathbb{Z}$ labels spatial positions and $t \in \{0, \tau, 2\tau, \ldots\}$ labels discrete time steps.

2.2 Linear Cellular Automaton

Time evolution is governed by a linear cellular automaton (LCA) rule:

$$\psi(n, t+\tau) = \alpha\psi(n-1, t) + \beta\psi(n+1, t) \tag{1}$$

where α and β are complex coupling constants satisfying $|\alpha|^2 + |\beta|^2 = 1$ for probability conservation. This rule is the discrete analog of nearest-neighbor coupling in continuum field theories.

2.3 Continuum Limit and Emergent Schrödinger Equation

To recover the Schrödinger equation, we require that α and β scale appropriately as $a, \tau \to 0$. Setting:

$$\alpha = \frac{1}{2}\left(1 - i\frac{\hbar\tau}{ma^2}\right), \quad \beta = \frac{1}{2}\left(1 + i\frac{\hbar\tau}{ma^2}\right) \tag{2}$$

and expanding $\psi(n \pm 1, t)$ to second order in a, we obtain:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \tag{3}$$

This is the free-particle Schrödinger equation, emerging as the long-wavelength limit of the discrete dynamics.

3 Emergent Dispersion Relation

3.1 Lattice Wave Propagation

Assuming translational invariance, we seek plane-wave solutions of the form:

$$\psi(n,t) = Ae^{i(kna-\omega t)} \tag{4}$$

Substituting into equation (1) and solving for $\omega(k)$ yields the dispersion relation determined entirely by lattice geometry.

3.2 Sinusoidal Dispersion

For a symmetric automaton with $\alpha = \beta^* = e^{i\theta}/\sqrt{2}$, the energy-momentum relation becomes:

$$E(p) = \frac{\hbar c}{a} \sin\left(\frac{pa}{\hbar}\right) \tag{5}$$

where we have identified $\hbar\omega$ with energy E and $\hbar k$ with momentum p. This relation is periodic with period $2\pi\hbar/a$ and is bounded: $|E| \leq \hbar c/a$. The upper bound defines a natural UV cutoff.

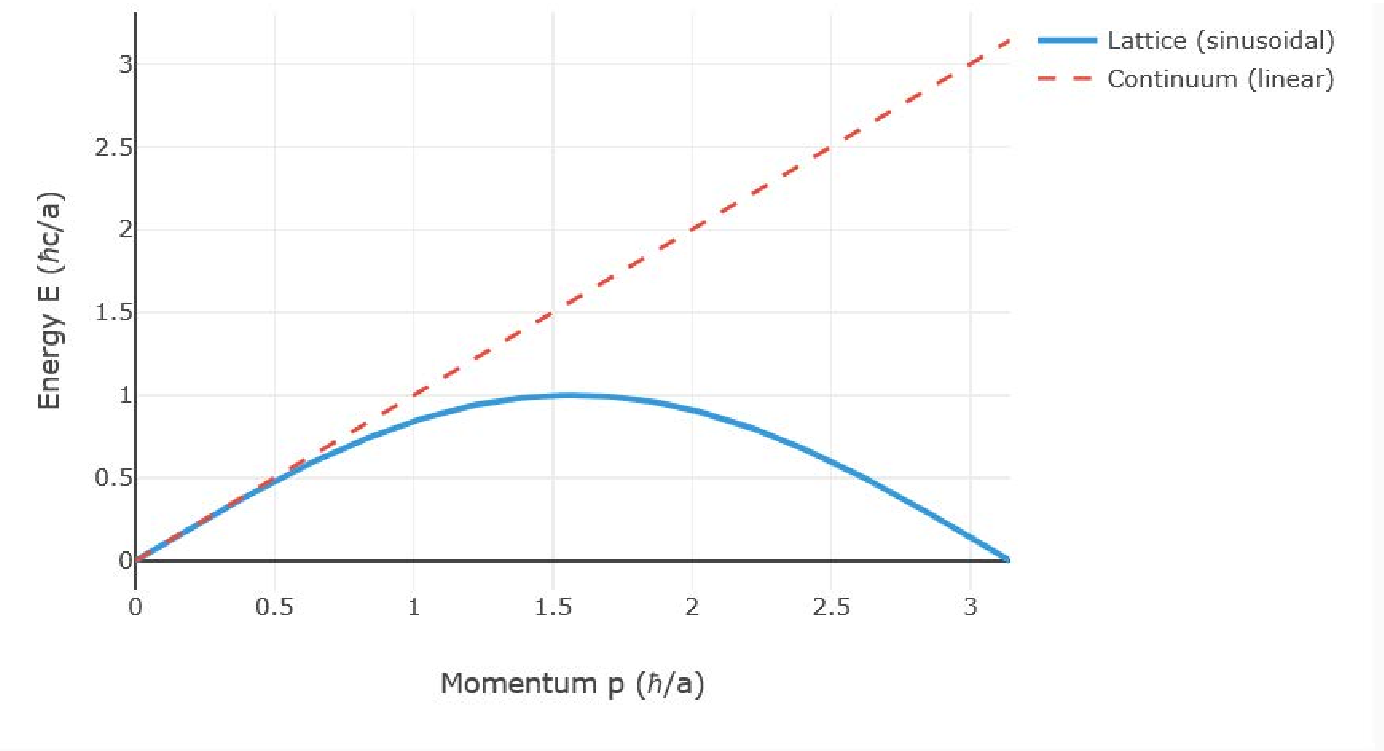

Figure 1: Energy-momentum dispersion relation. The sinusoidal lattice dispersion approaches the linear continuum limit $E = pc$ at low momentum but deviates significantly at $p \sim \hbar/a$. The lattice naturally regulates ultraviolet behavior.

4 Low-Energy Expansion and Physical Regimes

4.1 Taylor Expansion

For $p \ll \hbar/a$, we expand equation (5):

$$E(p) \approx pc\left[1 - \frac{1}{6}\left(\frac{pa}{\hbar}\right)^2 + \mathcal{O}(p^4)\right] \tag{6}$$

The leading correction is quadratic in momentum, suppressed by $(a/\lambda_C)^2$ where $\lambda_C = \hbar/p$ is the Compton wavelength.

4.2 Physical Energy Scales

Scale Hierarchy:

- Laboratory energies: $E \sim 1$ GeV $\to pa/\hbar \sim 10^{-18}$ (if $a \sim \ell_P$)
- LHC energies: $E \sim 10$ TeV $\to pa/\hbar \sim 10^{-14}$
- Ultra-high-energy cosmic rays: $E \sim 10^{20}$ eV $\to pa/\hbar \sim 10^{-5}$
- Cutoff scale: $E_{\max} = \hbar c/a \sim 10^{19}$ GeV (if $a \sim \ell_P$)

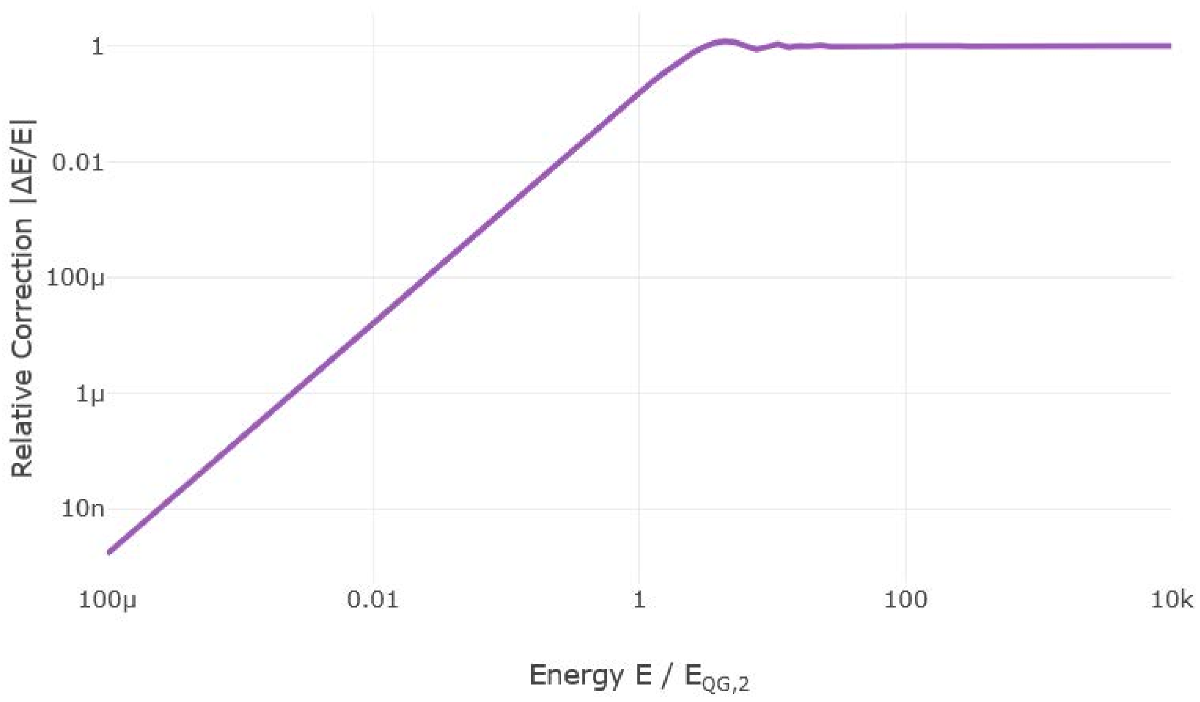

Figure 2: Relative correction to linear dispersion as a function of energy. The deviation becomes observable only at ultra-high energies, explaining why spacetime appears continuous in all laboratory experiments.

5 Connection to Lorentz Invariance Violation

5.1 Effective Field Theory Framework

The modified dispersion relation can be rewritten in relativistic form:

$$E^2 \approx p^2c^2 + \eta \frac{E^4}{E_{QG,2}^2} \tag{7}$$

where $E_{QG,2} = \sqrt{6}\hbar c/a$ is the effective quantum gravity scale for $n = 2$ LIV, and $\eta = -1$ determines the sign of the correction. This is formally identical to standard LIV parametrizations used in phenomenological studies.

5.2 Physical Interpretation

Our framework provides a concrete microphysical origin for quadratic LIV: it arises from the discrete topology of spacetime rather than from explicit symmetry breaking. The model predicts $\eta = -1$ (subluminal), in contrast to some alternative quantum gravity scenarios that predict superluminal propagation.

6 Observable Consequences and Phenomenology

6.1 Modified Group Velocity

The group velocity $v_g = \partial E/\partial p$ derived from equation (6) is:

$$v_g \approx c\left[1 - \frac{3}{2}\left(\frac{E}{E_{QG,2}}\right)^2\right] \tag{8}$$

High-energy particles travel slower than c, with the deviation scaling as E^2.

6.2 Time-of-Flight Delays

For particles propagating over distance D, the time delay relative to a photon is:

$$\Delta t \approx \frac{3D}{2c}\left(\frac{E}{E_{QG,2}}\right)^2 \tag{9}$$

For cosmological sources at redshift $z \sim 1$ ($D \sim 3$ Gpc) and $E_{QG,2} \sim 10^{18}$ GeV:

Particle Energy	**Expected Delay**	**Observable?**
1 TeV (Fermi-LAT)	$\sim 10^{-18}$ s	No
100 TeV (LHAASO)	$\sim 10^{-14}$ s	No
1 PeV	$\sim 10^{-10}$ s	Marginal
100 PeV	$\sim 10^{-6}$ s	Yes

Table 1: Expected time delays for different particle energies from cosmological sources.

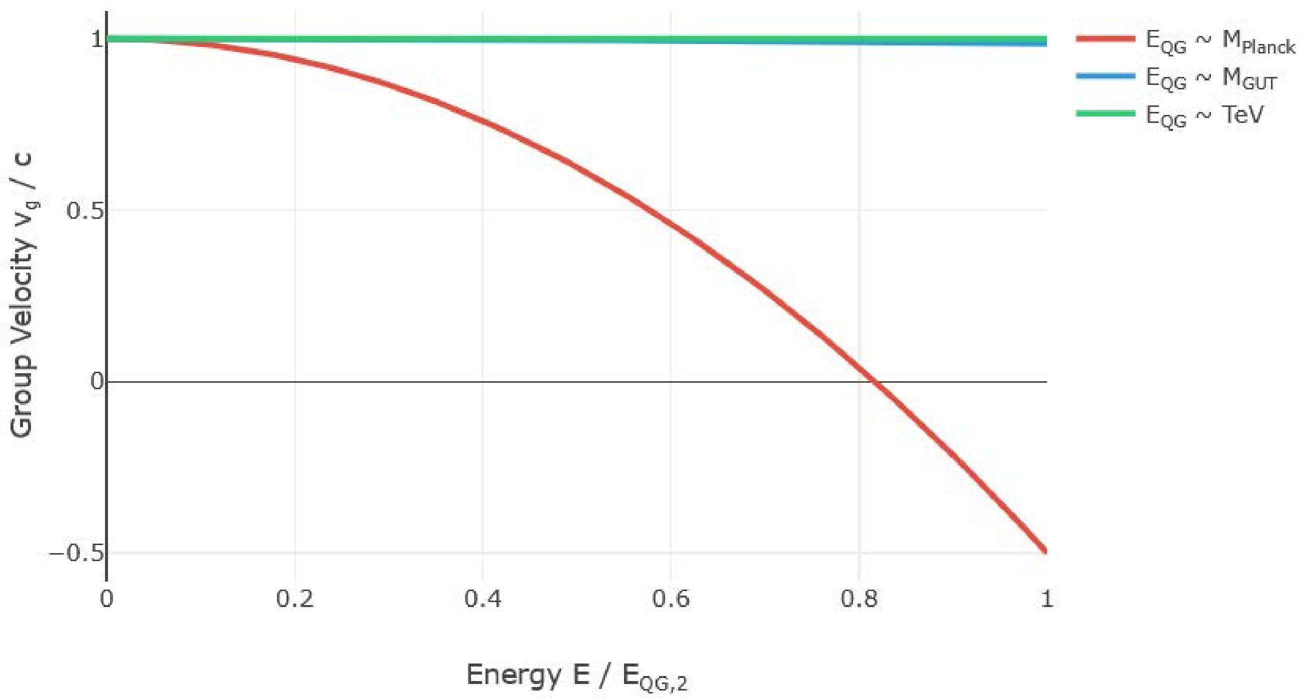

Figure 3: Group velocity as a function of particle energy for different lattice scales. The deviation from c becomes significant only when E approaches $E_{QG,2}$.

6.3 Observational Signatures

The model predicts several observable effects:

- **Energy-dependent arrival times:** Higher-energy photons from gamma-ray bursts or flares should arrive systematically later
- **Spectral distortion:** Attenuation of the highest-energy component due to increased propagation time and cosmological redshifting
- **Vacuum birefringence:** In extensions to (3+1)D with non-trivial lattice symmetries

7 Constraints from Current Observations

7.1 Gamma-Ray Bursts

Fermi-LAT observations of GRB 090510 ($z = 0.903$) with photons up to 31 GeV show no time-of-flight delays, constraining $E_{QG,2} > 10^{17}$ GeV for linear LIV. Our quadratic model is less constrained: $E_{QG,2} > 10^{15}$ GeV.

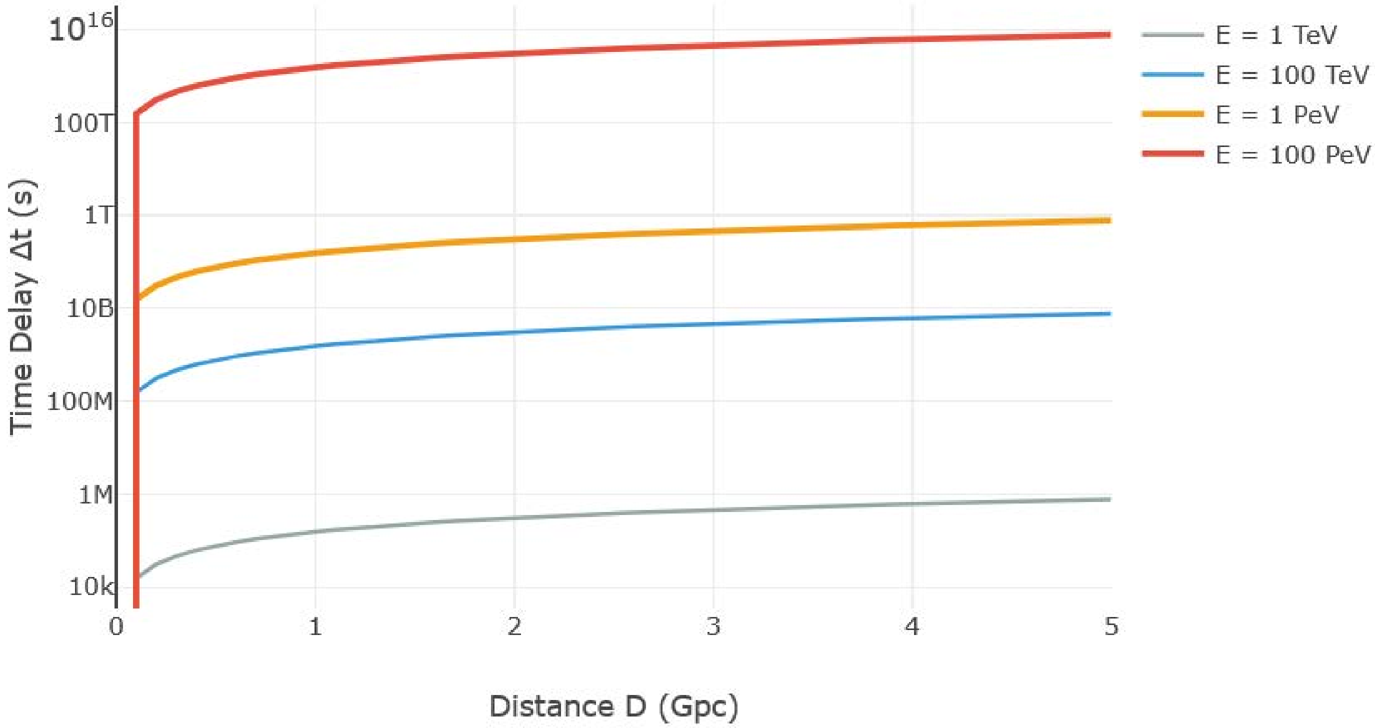

Figure 4: Accumulated time delay as a function of propagation distance for particles of different energies. The quadratic energy dependence creates characteristic spectral signatures in multi-wavelength observations of transient sources.

7.2 Active Galactic Nuclei

TeV observations of Mrk 501 flares provide $E_{QG,2} > 10^{16}$ GeV. The next generation of instruments (CTA, LHAASO) will probe $E_{QG,2} \sim 10^{17-18}$ GeV.

8 Theoretical Extensions and Open Questions

8.1 Generalization to (3+1) Dimensions

The framework extends naturally to three spatial dimensions with a cubic lattice. The dispersion relation becomes:

$$E^2(\vec{p}) = \sum_{i=1}^{3} \left(\frac{\hbar c}{a} \sin \frac{p_i a}{\hbar} \right)^2 \tag{10}$$

Lattice anisotropies could lead to direction-dependent LIV effects.

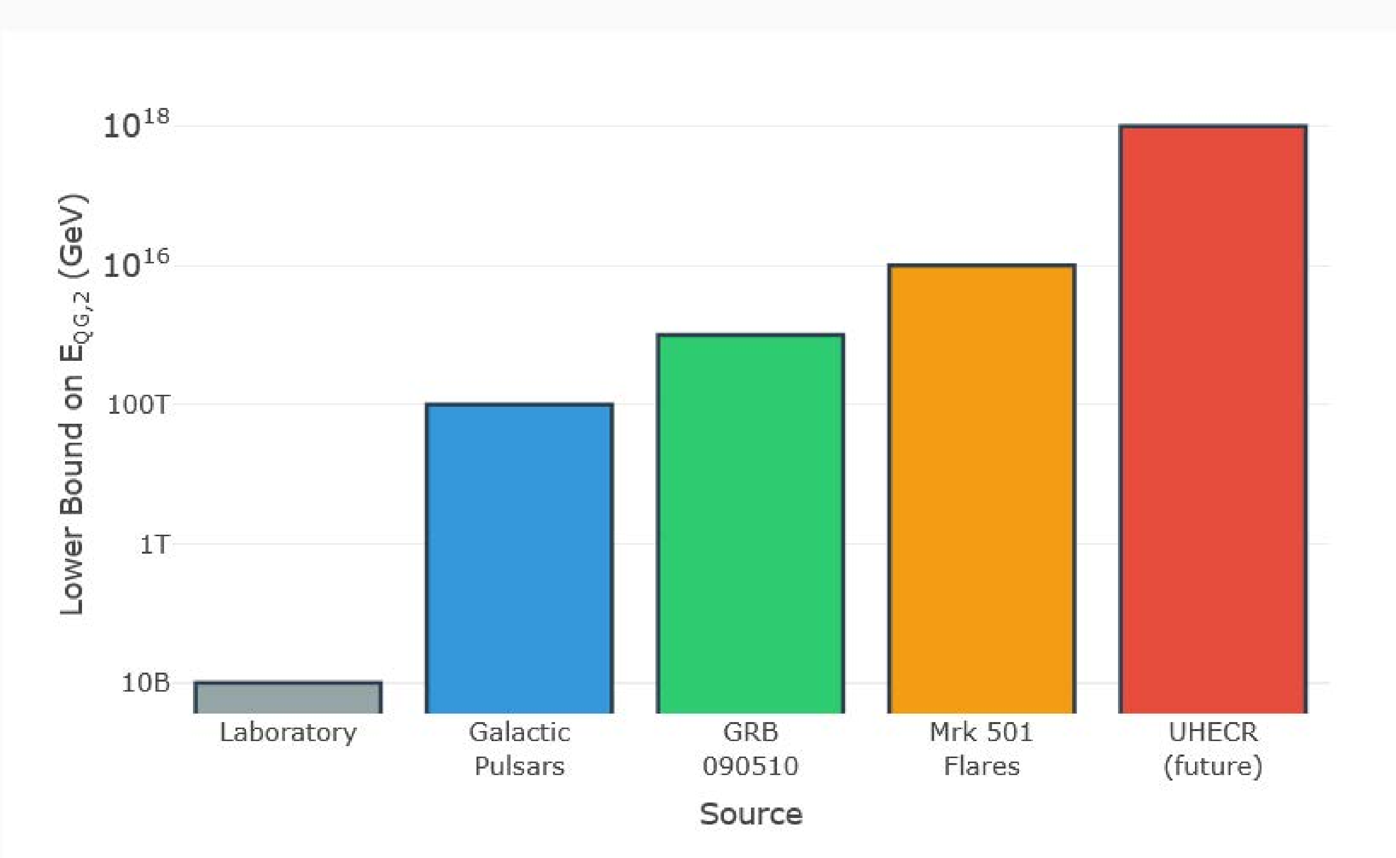

Figure 5: Current observational constraints on the quantum gravity scale $E_{QG,2}$ from various astrophysical sources. The model remains viable if discretization occurs near or above the GUT scale.

8.2 Interactions and Locality

Incorporating interactions requires extending the LCA to include non-linear terms while preserving locality on the lattice. This connects to ongoing work in discrete quantum field theories and lattice gauge theory.

8.3 Relation to Other Approaches

Our model shares features with:

- Causal sets (discrete spacetime + causal structure)
- Loop quantum gravity (discrete spatial geometry)
- Wolfram's hypergraph models (computational universe)

However, it differs in its explicit focus on phenomenological predictions testable with current technology.

9 Falsifiability and Experimental Outlook

Key Predictions:

1. No deviations in laboratory or galactic-scale experiments
2. Millisecond-scale delays for PeV particles from cosmological GRBs if $E_{QG,2} \sim 10^{18}$ GeV
3. Subluminal group velocity ($\eta = -1$), ruling out superluminal scenarios
4. Characteristic E^2 scaling distinguishing from linear LIV

Non-observation of predicted effects in next-generation observatories would either rule out the model or push the discretization scale beyond $E_{QG,2} > 10^{19}$ GeV, near the Planck scale.

10 Conclusion

We have demonstrated that quantum mechanics can emerge as an effective low-energy theory from a discretized spacetime substrate governed by simple local dynamics. The same discreteness that gives rise to QM inevitably induces quadratic modifications of relativistic dispersion relations, formally equivalent to $n = 2$ Lorentz Invariance Violation models.

This framework achieves several goals: it provides a concrete microphysical origin for LIV, explains why spacetime appears continuous in all laboratory tests, and makes falsifiable predictions testable with ultra-high-energy astrophysical observations. The model exemplifies how bottom-up approaches to quantum gravity can connect fundamental theory to observable phenomena.

Future work will focus on: (i) incorporating curved spacetime and gravitational effects, (ii) developing the full (3+1)D theory with realistic particle physics, and (iii) detailed comparison with upcoming observations from CTA, LHAASO, and space-based gamma-ray missions.

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