Program A: Semi-Infinite Conditional Mutual Information in the 1D TFIM (iMPS)

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Abstract

We study an information-theoretic notion of locality—approximate quantum Markov behavior—via the conditional mutual information (CMI) $I(A : C \mid B(w))$ in a semi-infinite geometry of the 1D transverse-field Ising model (TFIM). Using infinite matrix product states (iMPS), we compute $I(A : C \mid B(w))$ as a function of the collar width w separating two semi-infinite regions. In a representative gapped point ($h = 1.5$), we observe clean exponential decay and a rapid plateau of the local effective-length estimator, yielding an early-decay length $\xi_{\mathrm{rec}}^{(\mathrm{early})}$ comparable to the iMPS transfer-matrix correlation length ξ_{corr}. Near criticality ($h = 1.005$), the local estimator increases throughout the accessible range, indicating a pre-asymptotic regime; we therefore report a fixed-window effective length and a window-sensitivity range as a systematic uncertainty. All generated assets used here (two JSONL data streams, the figure, and the LaTeX table snippet) are included in the Overleaf project. *v2 (no v1 number is changed):* Appendix A is repaired (in v1 the data-source filenames were typeset in math mode and the near-critical entry was missing entirely); Table 1 is re-typeset (collided columns in v1); the Colab scripts of Appendix B are shipped as runnable files in the series repository rather than as listings; series positioning is added — this paper is a numerical instantiation of the A-CMI hypothesis of the contract note ai.viXra:2601.0066 in a semi-infinite 1D geometry; and an *independent* verification suite (free fermions via Jordan–Wigner, no tensor networks) reproduces the gapped point of Table 1 exactly ($\xi_{\mathrm{rec}}^{(\mathrm{early})} = 1.149$ on the main window, window sensitivity $[1.149, 1.158]$, both matching v1 digit for digit), verifies the operational identity Eq. (3) to 10^{-11}, and reproduces the rising near-critical $\xi_{\mathrm{local}}(w)$; a TeNPy cross-check reproduces the free-fermion $I(w)$ pointwise to 5×10^{-5} relative.

1 Introduction and motivation

A quantitative notion of locality in many-body quantum states is *approximate quantum Markov structure*, commonly diagnosed via the conditional mutual information (CMI) $I(A : C \mid B)$. Small CMI implies the existence of a recovery channel acting on B that approximately reconstructs correlations between A and C (recoverability viewpoint), making $I(A : C \mid B)$ a sharp tool to test how a buffer region B shields distant regions. This perspective has been developed in quantum information theory and connects naturally to physical expectations such as exponential clustering in gapped phases and area-law entanglement [1, 2, 3, 4, 5].

Goal (Program A). We perform a controlled iMPS computation of the *semi-infinite* CMI $I(A : C \mid B(w))$ in the 1D transverse-field Ising model (TFIM), where A and C are semi-infinite and B is a contiguous collar of width w. We compare an early-decay length extracted from $I(A :$

$C \mid B(w))$ to the iMPS transfer-matrix correlation length in a gapped regime, and we document pre-asymptotic behavior near criticality when $w_{\max} \ll \xi_{\mathrm{corr}}$.

2 Model and numerical setup

We study the TFIM Hamiltonian

$$H = -J \sum_i \sigma_i^z \sigma_{i+1}^z - h \sum_i \sigma_i^x, \tag{1}$$

with $J = 1$. Ground states are computed in the thermodynamic limit using TeNPy iMPS with a two-site unit cell. We report the effective bond dimension χ_{eff} after canonicalization.

Convention note. All results reported here follow TeNPy's TFIM implementation with $J = 1$ and transverse field parameter $g = h$ in the code; the unit of length is one lattice site, and the unit cell contains two sites. We define ξ_{corr} as the iMPS transfer-matrix correlation length returned by TeNPy's `correlation_length2()` (in lattice-site units).

3 Observable: semi-infinite CMI

We consider a contiguous tripartition A–B–C where A and C are semi-infinite and B is a collar of width w. We evaluate

$$I(A : C \mid B(w)) = S(AB) + S(BC) - S(B) - S(ABC). \tag{2}$$

Operationally, in the semi-infinite geometry considered here (A and C semi-infinite, B finite of width w) and for a globally pure state, the CMI can be evaluated via the finite identity

$$I(A : C \mid B(w)) = 2S_{\mathrm{cut}} - S(B(w)), \tag{3}$$

where S_{cut} is the entanglement entropy across a single cut separating a semi-infinite half-chain from its complement, and $S(B(w))$ is the von Neumann entropy of the finite block B. With a two-site unit cell, we average over the two inequivalent cut positions (as done in the Appendix scripts). Equation (3) follows from global purity and the entropy identities $S(AB) = S(C)$, $S(BC) = S(A)$, and $S(ABC) = 0$, yielding a finite cancellation in the semi-infinite geometry with finite $B(w)$.

We also define a local effective-length estimator

$$\xi_{\mathrm{local}}(w) := \left[\log I(A : C \mid B(w))(w) - \log I(A : C \mid B(w))(w+1)\right]^{-1}. \tag{4}$$

A plateau in $\xi_{\mathrm{local}}(w)$ indicates an approximately exponential tail.

4 Results

Figure 1 shows $I(A : C \mid B(w))(w)$ and $\xi_{\mathrm{local}}(w)$ for the gapped and near-critical runs included in this Overleaf project. Dashed lines show exponential fits on the fixed window $w \in [6, 11]$.

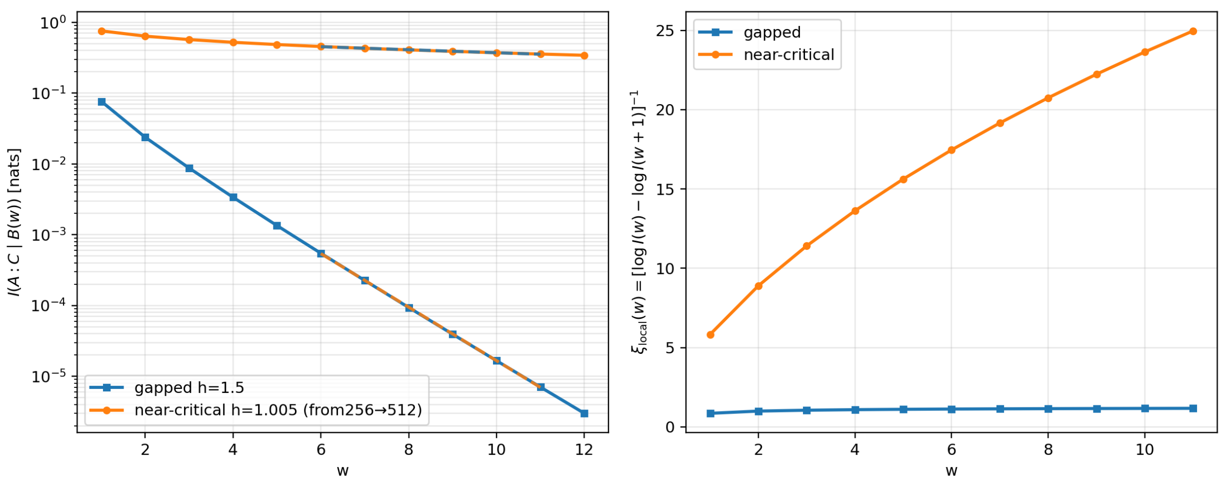

Figure 1: **CMI decay and local effective length.** Left: $I(A : C \mid B(w))(w)$ on a semi-log scale. Right: $\xi_{\mathrm{local}}(w)$.

Table 1: **Program A summary (semi-infinite geometry).** $\xi_{\mathrm{rec}}^{(\mathrm{early})}$ is extracted from an exponential fit to $I(A : C \mid B(w))$ on a fixed window (main choice: $(6, 11)$). Window sensitivity is reported across $[(6, 11), (6, 12), (7, 12)]$. χ_{eff} is the effective bond dimension after canonicalization. ξ_{corr} is the iMPS transfer-matrix correlation length. (Re-typeset in v2; v1's table had collided columns.)

Regime	h	χ_{eff}	ξ_{corr}	$\xi_{\mathrm{rec}}^{(\mathrm{early})}$
Gapped	1.500	13	1.120	1.149
Near-critical (from256→512)	1.005	110	87.774	20.485

Window sensitivity. Gapped: $\xi_{\mathrm{rec}}^{(\mathrm{early})} \in [1.149, 1.158]$. Near-critical: $\xi_{\mathrm{rec}}^{(\mathrm{early})} \in [20.485, 22.017]$.

5 Discussion

Why $\xi_{\mathrm{rec}}^{(\mathrm{early})} \approx \xi_{\mathrm{corr}}$ in the gapped regime is meaningful. In a gapped 1D phase, connected correlations decay exponentially with distance, and the iMPS transfer matrix encodes this via a finite correlation length ξ_{corr}. The observation that $I(A : C \mid B(w))$ decays approximately exponentially and yields an early-decay scale comparable to ξ_{corr} supports the picture that the same transfer-matrix gap governing two-point clustering also controls the onset of approximate Markov structure under a finite buffer.

Why the near-critical regime does not reach the asymptotic tail. Near criticality the correlation length grows rapidly, and our accessible collar widths satisfy $w_{\max} \ll \xi_{\mathrm{corr}}$. In this pre-asymptotic window, the local estimator $\xi_{\mathrm{local}}(w)$ increases with w rather than plateauing, indicating that an effective length extracted from a short window should be interpreted as a scale characterizing the accessible range, not the true asymptotic decay length.

Systematic effects. We report window sensitivity of $\xi_{\mathrm{rec}}^{(\mathrm{early})}$ as a minimal systematic uncertainty. A more complete error budget would additionally include finite-χ effects (iMPS truncation), which can be probed by repeating the pipeline at multiple $\chi_{\max}$ and examining convergence of $I(A : C \mid B(w))$ at fixed w as $\chi_{\max} \to \infty$.

Series positioning (v2). Within the series this paper is a numerical instantiation of the A-CMI hypothesis — the single falsifiable input of the closed recoverability lane of the contract note ai.viXra:2601.0066 (v2) — in a semi-infinite 1D geometry: the gapped row of Table 1 exhibits exactly the $I \le Ke^{-\alpha w}$ behavior that A-CMI postulates, with the constants measured rather than assumed, while the near-critical row documents how A-CMI degrades as $\xi_{\rm corr}$ exits the accessible window. The rising $\xi_{\rm local}(w)$ under a short window is the same functional-form indistinguishability phenomenon documented for finite-size pipelines in ai.viXra:2601.0040 (v2), and the collar-CMI methodology connects to ai.viXra:2601.0050 (finite collar geometries and their saturation structure), ai.viXra:2601.0038 (finite-size $d_{\rm eff}$ pipelines), and the Type III interface ai.viXra:2601.0065 (where split-regularized CMI decay is the corresponding assumption). The v2 verification suite adds what none of these had: a tensor-network-free cross-check of the entire pipeline (free fermions via Jordan–Wigner on an open chain of $L = 400$ sites), which reproduces the gapped $\xi_{\rm rec}^{\rm (early)}$ and its window-sensitivity range digit for digit.

6 Conclusions and outlook

We computed semi-infinite conditional mutual information $I(A : C \mid B(w))$ in the TFIM using iMPS and observed: (i) clean exponential decay and a rapid $\xi_{\rm local}(w)$ plateau in a gapped point, with $\xi_{\rm rec}^{\rm (early)}$ comparable to $\xi_{\rm corr}$; and (ii) a strongly running $\xi_{\rm local}(w)$ near criticality consistent with $w_{\max} \ll \xi_{\rm corr}$. Immediate extensions are (a) convergence studies in χ and (b) broader model coverage to test universality of the relation between recoverability length scales and transfer-matrix correlation lengths.

7 Reproducibility

This Overleaf project is populated by uploading four generated files: the figure `moneyplot_CMI_xilocal.png`, the table snippet `summary.tex`, and two JSONL streams containing the points for $I(A : C \mid B(w))$. To regenerate these assets, run the Colab build scripts (shipped as files, Appendix B) and upload the resulting zip (or the four files) to the root of the Overleaf project. *v2:* an independent verification suite (`verification/2601-0099/verify_2601_0099.py` in the series repository) reproduces the gapped point and the identity Eq. (3) without tensor networks, and cross-checks TeNPy against it; see the Note added below.

Note added in v2

No v1 number, fit window, or conclusion is changed. v2: (1) repairs Appendix A, whose v1 rendering typeset the data-source filenames in math mode (subscript soup) and dropped the near-critical entry entirely; (2) re-typesets Table 1, whose v1 rendering collided the $\chi_{\rm eff}$ and $\xi_{\rm corr}$ columns ("110.00087.774"); (3) ships the five Colab scripts of Appendix B as runnable, syntax-checked files in the series repository instead of typeset listings; (4) adds series positioning (v1 cited no companion papers): this paper instantiates the A-CMI hypothesis of ai.viXra:2601.0066; (5) adds an independent verification suite: a Jordan–Wigner free-fermion computation on an open chain ($L = 400$; engine validated against exact diagonalization of the spin chain at $L = 8$ to 10^{-13}) reproduces the gapped $\xi_{\rm rec}^{\rm (early)} = 1.149$ on the main window $(6, 11)$ and the window-sensitivity range $[1.149, 1.158]$ — both identical to v1's Table 1 digit for digit by a method with no tensor networks — verifies Eq. (3) against the direct entropy combination to 10^{-11}, and reproduces the rising near-critical

$\xi_{\rm local}(w)$ ($5.7 \to 24.2$ across the accessible range); a TeNPy iMPS run ($\chi = 16$ suffices; $\chi_{\rm eff} = 13$ in v1) then agrees with the free-fermion $I(w)$ pointwise to 5×10^{-5} relative for $w \leq 8$. The $\chi = 128/512$ production numbers of Table 1 remain sourced from the v1 checkpoints and Colab scripts.

A Data sources used in this project

(Repaired in v2; v1 typeset these filenames in math mode and omitted the second entry.)

- Gapped JSONL: `cmi_h1p5_chi128_semiinf_STREAM.jsonl`
- Near-critical JSONL: `cmi_from256_h1p005_chi512_semiinf_STREAM.jsonl`

B Colab scripts to regenerate the four uploaded files

v1 embedded the five Colab scripts as typeset listings; v2 ships them as runnable, syntax-checked files in the series repository under `verification/2601-0099/colab/`, transcribed verbatim from v1's Appendix B (the v1 PDF remains the typeset source):

- `00_env_and_install.py` — pinned TeNPy install (`v1.1.0`) and environment freeze;
- `01_dmrg_ground_states.py` — iDMRG checkpoints: gapped h=1.5 (χ=128), near-critical h=1.005 (χ=256 $\to$ 512 ramp);
- `02_cmi_jsonl.py` — semi-infinite CMI streams via Eq. (3), averaged over the two unit-cell cuts;
- `03_figure_and_summary.py` — money plot and LaTeX table snippet with the fixed fit windows;
- `04_package_and_manifest.py` — root-clean zip of the four Overleaf assets with SHA256 manifest.

The scripts assume Google Drive mounted at `/content/drive` with `PROJECT_DIR` set to the working folder.

References

[1] O. Fawzi and R. Renner, *Quantum conditional mutual information and approximate Markov chains*, Commun. Math. Phys. **340**, 575–611 (2015).

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[9] L. Eriksson, ai.viXra:2601.0040 (2026; v2 2026).

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