Dynamical Generation of Spacetime Torsion from Quantum Geometric Charge: A Testable Extension of Einstein-Cartan Gravity

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The Standard-Model Extension (SME) parameterizes Lorentz violation using fixed background fields. We propose a dynamical alternative where the antisymmetric SME background $(k_G)_{\mu\nu}$ emerges as a propagating spacetime torsion field in Einstein-Cartan theory. Its source is the "Berry curvature" of composite particles, which acts as a *Quantum Geometric Charge*. The coupling is mediated by a species-dependent *Geometric Susceptibility* η_a, a calculable property of nuclear and atomic states. This Quantum Geometric Backreaction (QGB) establishes a genuine "two-way street": matter's quantum geometry sources local spacetime structure, which in turn modifies matter's dynamics via an effective four-fermion interaction mediated by the torsion field. The dynamical origin yields unique, falsifiable signatures absent in static background models: (i) a linear scaling of interferometric phases with source matter density, (ii) quadratic η_a^2 self-energy corrections revealed in atomic spectroscopy via a *Torsional King Plot*, and (iii) the potential for generating detectable fields using macroscopic topological materials with broken time-reversal symmetry. We address theoretical consistency and provide first heuristic estimates of η_a for ^{85,87}Rb. The theory presents a clear experimental roadmap for tests using atom interferometers and optical lattice clocks at the 10^{-18}–10^{-19} precision level, directly probing the feedback loop between quantum matter and emergent spacetime.

I. INTRODUCTION: FROM STATIC BACKGROUND TO DYNAMICAL FEEDBACK

The search for a unified description of quantum matter and spacetime often treats geometry as a fixed stage. In General Relativity (GR), spacetime responds to mass-energy but not to quantum internal structure. The Standard-Model Extension (SME) systematically parameterizes potential Lorentz violations but typically treats its background fields $(k_G)_{\mu\nu}$ as immutable constants [1–3]. This represents a "one-way" imposition of geometry upon matter. We propose a conceptual inversion: the local structure of spacetime—specifically, its torsional degrees of freedom—is dynamically generated by the *quantum geometric properties* of matter itself. This establishes a *two-way feedback loop*. Our central hypothesis, the **Quantum Geometric Backreaction (QGB)**, posits that the Berry curvature—a measure of quantum phase anholonomy in momentum space—acts as a source for spacetime torsion. In this view, the SME coefficient $(k_G)_{\mu\nu}$ is not a fundamental constant but a dynamical field $\mathcal{T}_{\mu\nu}$, a solution to a field equation sourced by the Berry curvature of particles. This framework integrates three well-established domains: 1. **Einstein-Cartan (EC) Gravity** [4, 5], which incorporates torsion as a geometric field related to spin density. 2. **Modern SME Phenomenology** [11], which provides the experimental language for high-precision tests. 3. **Berry Phase Physics** [8], which describes how the geometry of quantum states manifests in physical responses. We extend minimal EC theory by promoting torsion to a *propagating* Kalb-Ramond field [6, 7]. Each particle species a couples to this field with strength η_a, its *Geometric Susceptibility*, derived from its internal Berry curvature distribution. This leads to a new, testable physical picture: isotopes with different nuclear structures (e.g., ^{85}Rb vs. ^{87}Rb) possess different η_a, and thus differentially "sculpt" the local torsional vacuum. Precision comparisons of such isotopes become direct probes of the matter-spacetime feedback loop.

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II. THEORETICAL FRAMEWORK: SOURCING TORSION FROM QUANTUM GEOMETRY

A. Dynamical Action and Field Equations

We work in the Palatini formalism where the metric $g_{\mu\nu}$ and the torsionful connection are independent, with the kinetic term for torsion chosen to propagate its trace-free component [6, 18]. The total action is:

$$S_{\text{total}} = S_{\text{EH}} + S_{\text{kin}}[\mathcal{T}] + S_{\text{int}}[\mathcal{T},\psi] + S_{\text{matter}}[\psi], \tag{1}$$

$$S_{\text{EH}} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R, \tag{2}$$

$$S_{\text{kin}}[\mathcal{T}] = -\frac{1}{12\lambda^2}\int d^4x\sqrt{-g}\,H_{\mu\nu\rho}H^{\mu\nu\rho}, \tag{3}$$

$$S_{\text{int}}[\mathcal{T},\psi] = \int d^4x\sqrt{-g}\,\mathcal{T}_{\mu\nu}J^{\mu\nu}_{\text{Berry}}. \tag{4}$$

Here, $H_{\mu\nu\rho} = \partial_\mu\mathcal{T}_{\nu\rho} + \partial_\nu\mathcal{T}_{\rho\mu} + \partial_\rho\mathcal{T}_{\mu\nu}$ is the field strength for $\mathcal{T}$, λ is a coupling constant with dimensions of inverse mass, and $J^{\mu\nu}_{\text{Berry}}$ is the *Berry curvature current* source term (defined below, containing η_a). The matter action for a Dirac fermion ψ_a of species a includes the minimal coupling:

$$S_{\text{matter}} = \int d^4x\sqrt{-g}\,\bar{\psi}_a\left[i\gamma^\mu\left(D_\mu - i\eta_a\mathcal{T}_{\mu\rho}\gamma^\rho\right) - m_a\right]\psi_a, \tag{5}$$

where D_μ is the standard gravitational covariant derivative (including the spin connection). Variation with respect to $\mathcal{T}_{\mu\nu}$ yields the dynamical field equation:

$$\boxed{\nabla_\rho H^{\mu\nu\rho} = \lambda^2 J^{\mu\nu}_{\text{Berry}}.} \tag{6}$$

This is the core equation of the QGB mechanism. The formal solution can be written as $\mathcal{T}_{\mu\nu} = \lambda^2(1/(\Box - m_T^2))J^{\mu\nu}_{\text{Berry}}$ with $m_T \propto 1/\lambda$. Substituting this back into the interaction terms yields an effective four-fermion interaction that closes the feedback loop:

$$\mathcal{L}_{\text{eff}} \sim \lambda^2\eta_a\eta_b\,(\bar{\psi}_a\Sigma^{\alpha\beta}\psi_a)\,\frac{1}{\Box - m_T^2}\,(\bar{\psi}_b\Sigma_{\alpha\beta}\psi_b), \tag{7}$$

where $\Sigma^{\alpha\beta} = \frac{i}{4}[\gamma^\alpha, \gamma^\beta]$. For light torsion (long-range effects relevant to lab scales), $1/(\Box - m_T^2) \approx 1/\Box$, so the experimental estimates hold. This structure explicitly shows how the geometric charge of one particle (η_a) influences the phase evolution of another (η_b) via the mediating torsion field.

B. The Berry Curvature Current Source

The source term $J^{\mu\nu}_{\text{Berry}}$ bridges momentum-space quantum geometry to position-space field theory. For a particle with quantum states labeled by band index n and momentum $\mathbf{k}$, the Berry curvature $\Omega^n_{ij}(\mathbf{k}) = -2\,\text{Im}\langle\partial_{k_i}u^n(\mathbf{k})|\partial_{k_j}u^n(\mathbf{k})\rangle$ acts as an effective magnetic field in momentum space [8]. To construct a macroscopic classical source, we must connect the microscopic Berry curvature to a coarse-grained observable. In the long-wavelength, low-energy limit appropriate for our effective field theory, the natural candidate is the spin density. This is physically motivated: the integrated Berry curvature over occupied bands is known to generate equilibrium spin and orbital magnetizations in condensed matter systems [8]. For a composite particle like a nucleus, the net spin polarization is a direct, measurable manifestation of the integrated geometric phase of its constituent fermions. Therefore, we adopt the following ansatz for the effective low-energy source:

$$J^{\mu\nu}_{\text{Berry}}(x) \equiv g_0\sum_a \eta_a\,\langle S^{\mu\nu}_a(x)\rangle, \tag{8}$$

where $S^{\mu\nu}_a = \bar{\psi}_a\Sigma^{\mu\nu}\psi_a$ is the spin density operator, g_0 is a universal coupling, and η_a is the species-specific **Geometric Susceptibility**. The expectation value $\langle\cdots\rangle$ denotes a suitable average over microscopic quantum and statistical fluctuations. This modeling assumption identifies the macroscopic spin density, rescaled by the efficiency factor η_a, as the natural classical source for spacetime torsion [4]. The geometric susceptibility η_a is not a free phenomenological parameter, nor merely a bookkeeping device. It is a low-energy effective response coefficient, analogous in status to a magnetic susceptibility or axial charge renormalization factor, arising from integrating out the internal quantum degrees of freedom of a composite particle. Formally, η_a appears as a Wilson coefficient in the matching between a microscopic theory defined on the internal Hilbert space of the bound state and the infrared effective field theory governing its center-of-mass and spin dynamics.

More precisely, η_a is defined operationally by the linear response of the expectation value of the antisymmetric spin density operator $\langle S^{\mu\nu}\rangle$ to an adiabatic deformation of the particle's internal collective quantum state along a closed loop in its configuration space. In this sense, η_a encodes the integrated Berry curvature flux associated with the occupied internal modes of the composite system. Its value is therefore fixed, in principle, by the microscopic wavefunction and is independent of the external experimental environment.

C. Calculating the Geometric Susceptibility

The susceptibility η_a is defined as the integrated Berry phase response over the particle's internal degrees of freedom. It is a calculable property of nuclear and atomic states. For rigorous prediction, η_a is calculated via one of two concrete pathways: 1. **Shell-Model Computation:** Using large-scale codes (e.g., NuShellX [17] with the JUN45 interaction), compute the Berry phase γ for the ground-state wavefunction as a function of an adiabatically varied quadrupole deformation parameter. The geometric susceptibility is proportional to the derivative $d\gamma/d\beta$ at the spherical point. 2. **Empirical Proxy Relation:** Correlate η_a with measurable nuclear observables sensitive to valence nucleon wavefunction geometry:

$$\eta_a \approx c_1\delta\langle r^2\rangle + c_2 Q_s + c_3 g_{\text{spin}}, \tag{9}$$

where $\delta\langle r^2\rangle$ is the mean-square charge radius difference, Q_s is the spectroscopic quadrupole moment, and g_{spin} is the spin g-factor. The coefficients c_i can be calibrated from a few shell-model benchmarks.

D. Calculating the Geometric Susceptibility

In practice, the geometric susceptibility η_a can be determined by controlled matching procedures. These include (i) direct many-body Berry phase calculations within nuclear or atomic structure models, and (ii) indirect extraction through calibrated relations to experimentally measurable observables sensitive to the same underlying geometric structure, such as charge radius differences, quadrupole moments, and spin g-factors. Once determined for a given species, η_a enters the effective action as a fixed property of that particle, in the same sense that the axial charge g_A characterizes nucleons.

The predictive content of the theory therefore lies not in treating η_a as adjustable, but in its species dependence and sign structure across isotope chains, which lead to sharply defined differential signatures in interferometric and spectroscopic experiments.

Table I provides first-order estimates, highlighting the sizable difference $\Delta\eta_{85-87} \approx -5.0\, g_0$, which motivates sensitive differential tests.

E. Mass Scale and Range of the Torsion Field

The kinetic term in Eq. (3) introduces a characteristic mass scale for the propagating torsion field $\mathcal{T}_{\mu\nu}$. Writing the quadratic action in momentum space, the torsion propagator takes the Yukawa form

$$D_{\mu\nu,\alpha\beta}(q) \sim \frac{P_{\mu\nu,\alpha\beta}}{q^2 + m_T^2}, \tag{10}$$

where $P_{\mu\nu,\alpha\beta}$ is the antisymmetric tensor projector and

$$m_T \equiv \lambda^{-1}. \tag{11}$$

The inverse mass sets the interaction range $\ell_T = m_T^{-1}$. For the laboratory-scale experiments considered in this work, sensitivity is maximal when the torsion field is effectively long-range over atomic and mesoscopic distances. We therefore focus on the regime

$$m_T \lesssim 10^{-3}\,\text{eV}, \qquad \ell_T \gtrsim 0.1\,\text{mm}, \tag{12}$$

for which the propagator reduces to its massless form over the relevant length scales and the effective interaction is well approximated by $1/\Box$ in the transverse sector.

This choice is conservative and consistent with existing constraints on weakly coupled antisymmetric tensor fields. Heavier torsion modes would introduce Yukawa suppression and rapidly diminish laboratory sensitivity, while lighter modes remain compatible with current astrophysical and cosmological bounds due to the extreme smallness of the effective coupling α_{geo}.

Importantly, all proposed experimental signatures—the linear matter-density scaling, the η_a^2 self-energy shifts, and the torsional King plot nonlinearity—persist in the limit $m_T \to 0$, making the long-range regime both theoretically clean and experimentally optimal.

F. Recovery of the Static Limit and Experimental Benchmark

If a universe-averaged, homogeneous Berry charge density $\bar{J}^{\mu\nu}_{\text{Berry}}$ dominates, Eq. (4) admits a constant solution: $\mathcal{T}_{\mu\nu} \approx (\lambda^2/\Box)\bar{J}^{\mu\nu}_{\text{Berry}} \to \text{const}$. In this limit, our dynamical field $\mathcal{T}_{\mu\nu}$ mimics the static SME background $(k_G)_{\mu\nu}$. Explicitly, the SME coefficients are sourced as

$$(k_G)^{(\text{SME})}_{\mu\nu} \equiv \frac{\lambda^2}{M_{\text{Pl}}^2}\bar{J}^{\mu\nu}_{\text{Berry}}, \tag{13}$$

providing a microphysical origin for a subset of SME coefficients. The dimensionless coupling governing all phenomena is $\alpha_{\text{geo}} = g_0\lambda$. A naive naturalness argument suggests $\alpha_{\text{geo}} \sim \Lambda_{\text{low}}/\Lambda_{\text{Planck}}$, where Λ_{low} could be the QCD scale ($\sim$ 200 MeV), yielding $\alpha_{\text{geo}} \sim 10^{-20}$. Existing SME constraints on static $(k_G)_{\mu\nu}$ [11] translate to $|g_0\mathcal{T}| \lesssim 10^{-27}$ GeV, consistent with extreme suppression. However, specific model-building mechanisms (e.g., infrared amplification from many particle species or low-scale quantum gravity) could yield a measurably larger effective coupling in the low-energy limit. To define a concrete target for next-generation experiments, we adopt a conservative, phenomenologically interesting *benchmark value*. We ask: what coupling strength would produce a signal at the threshold of detectability in the proposed Class I and II experiments? As shown in Sections 3.1 and 3.2, a value of

$$\alpha_{\text{geo}} \sim 10^{-10}\,\text{GeV}^{-1}$$

produces phase shifts $\Delta\Phi \sim 10^{-4}$ rad in matter-density interferometry and fractional frequency shifts $\delta\nu/\nu \sim 10^{-19}$ in clock comparisons. These are within the projected sensitivity of advanced experiments [13, 14]. Therefore, we use this benchmark to illustrate experimental feasibility. A null result in the proposed experiments would constrain α_{geo} to be smaller than this value.

III. EXPERIMENTAL SIGNATURES: DISTINGUISHING DYNAMICS FROM STATIC BACKGROUNDS

The dynamical nature of the QGB mechanism produces experimental consequences that are *qualitatively distinct* from those of a static SME background. We outline three classes of decisive tests.

A. Class I: Matter-Density Scaling in Interferometry

In a static background model, the phase shift in an atom interferometer is independent of the density of a nearby source mass. In the QGB model, the source mass itself generates the local $\mathcal{T}_{\mu\nu}$ field. For a probe atom of species B near a source ensemble of species A with number N_A, the induced phase shift scales as:

$$\boxed{\Delta\Phi_B \propto \alpha_{\text{geo}}^2\, \eta_A\, \eta_B\, N_A\, \mathcal{F}(\text{geometry}).} \tag{14}$$

A linear scaling $\Delta\Phi \propto N_A$ is a smoking-gun signature of dynamical sourcing. For a benchmark experiment with a source cloud of $N_A = 10^8$ ^{87}Rb atoms ($\eta_A \approx +1$)

TABLE I. Heuristic estimates of the Geometric Susceptibility η_a for Rubidium isotopes, based on the single-particle shell model scaling $\eta_a^{(\text{s.p.})} \propto \ell(\ell+1)$ derived in Appendix A. The sign difference arises from full many-body wavefunction considerations. Rigorous many-body values require the shell-model calculation outlined in Sec. 2.3.

Isotope	Nuclear Spin I^π	Valence Subshell	Single-Particle Estimate $\eta_a^{(\text{s.p.})}/g_0$
Rubidium-85 (^{85}Rb)	$5/2^-$	$1f_{5/2}$	≈ -4.0
Rubidium-87 (^{87}Rb)	$3/2^-$	$2p_{3/2}$	$\approx +1.0$

and a probe ^{85}Rb interferometer ($\eta_B \approx -4$), taking $\alpha_{\text{geo}} \sim 10^{-10}$ GeV^{-1} and a characteristic geometric factor $\mathcal{F} \sim r_s/d^2$ for source radius r_s and source-probe distance d, we estimate:[1] $\Delta\Phi$ can reach the microradian scale. This is within reach of advanced Bose-condensed source interferometers with single-shot phase sensitivities at the 10^{-3} rad level [13].

B. Class II: Berry Self-Energy and the Torsional King Plot

A particle with geometric susceptibility η_a sources a field that acts back on itself, leading to a small shift of its internal energy levels:

$$\boxed{\frac{\delta\nu_a}{\nu_a} \sim \alpha_{\text{geo}}^2\, \eta_a^2.} \tag{15}$$

For clock transitions, this manifests as an isotope-specific frequency shift. In standard atomic theory, isotope shifts between two atomic transitions are analyzed via a *King plot*, where shifts in one transition are plotted linearly against shifts in another to cancel dominant volume effects. The QGB's quadratic self-energy term introduces a specific nonlinearity. A **Torsional King Plot**—plotting the residual shift after subtracting the standard linear model $\delta\nu_a = \mu K + F\delta\langle r^2\rangle_a$ against η_a^2—should reveal a linear relationship:

$$\delta\nu_a^{(\text{residual})} = \delta\nu_a - (\mu K + F\delta\langle r^2\rangle_a) \approx A\,\eta_a^2, \tag{16}$$

where A is a common coefficient. For $\alpha_{\text{geo}} \sim 10^{-10}$ GeV^{-1} and $\eta_a \sim 4$, the fractional shift is $\delta\nu/\nu \sim 10^{-19}$, within reach of next-generation optical lattice clocks boasting uncertainties of 10^{-18} [14]. This provides a concrete, nuclear-structure-dependent mechanism for subtle nonlinearities in isotope shift spectra [12].

C. Class III: Engineered Macroscopic Sources

Materials with large, coherent Berry curvature fluxes, such as magnetic Weyl semimetals (e.g., $Co_3Sn_2S_2$), could act as concentrated sources of the $\mathcal{T}_{\mu\nu}$ field [9]. While likely subdominant to atomic tests initially, this direction connects the QGB mechanism directly to condensed matter physics and could offer complementary constraints on spin- or geometry-dependent forces.

IV. THEORETICAL CONSISTENCY AND OUTLOOK

The chosen kinetic term H^2 ensures the theory is ghost-free and propagates a single massive spin-1 degree of freedom (the "torsionon"). This extends minimal EC theory, where torsion is non-propagating, a standard approach in generalized gravity [6, 7]. The coupling in Eq. (3) respects local Lorentz invariance; apparent violations arise only from the solution $\mathcal{T}_{\mu\nu}$ in a given frame. The theory is power-counting renormalizable, with standard heat-kernel techniques applicable for fermions in a torsion background [5]. The QGB mechanism offers a novel microphysical origin for SME coefficients and connects to broader ideas of emergent spacetime in quantum gravity. It suggests that $\mathcal{T}_{\mu\nu}$ could be an effective low-energy field arising from the collective dynamics of more fundamental quantum geometric degrees of freedom. This perspective places laboratory tests of Lorentz symmetry within a larger narrative of how classical spacetime geometry condenses from quantum information. The QGB mechanism explored here, linking Berry curvature to propagating torsion, represents one specific realization of quantum-geometric backreaction. Other theoretically consistent possibilities exist, such as coupling to entanglement entropy gradients (linking to holographic emergence) or a massive Proca-type torsion field (introducing a new scale). The experimental signatures outlined here—particularly the linear density scaling—are unique to the dynamical, long-range torsion model and will serve to distinguish between these possibilities. The fundamental action defined in Eqs. (1)–(5) is strictly invariant under local Lorentz transformations and diffeomorphisms. No explicit Lorentz-violating terms are introduced at the level of the Lagrangian. Apparent Lorentz violation arises only at the level of particular classical solutions when a nonzero expectation value of the torsion field $\mathcal{T}_{\mu\nu}$ is generated by matter with nonvanishing geometric charge density. This distinction mirrors that in the Standard-Model Extension between observer Lorentz invariance, which remains exact, and particle Lorentz in-

[1] The phase shift estimate follows from $\Delta\Phi \sim (\alpha_{\text{geo}}^2 \eta_A \eta_B N_A \hbar c/(4\pi d^2)) \times r_s$. With $r_s \sim 10$ μm, $d \sim 1$ mm, $N_A = 10^8$, and $\alpha_{\text{geo}} = 10^{-10}$ GeV^{-1}, we find $\Delta\Phi \sim 10^{-4}$ rad.

variance, which may be spontaneously broken by background field configurations.

The identification of Berry curvature as a source for torsion proceeds through controlled coarse-graining. Berry curvature is fundamentally a property of the parameter space of quantum states and does not itself reside in spacetime. However, when internal quantum degrees of freedom are adiabatically slaved to the center-of-mass motion of a composite particle, the integrated Berry curvature manifests as a spacetime-local contribution to conserved currents. In relativistic field theory, the unique antisymmetric local current associated with internal angular structure is the spin density $S^{\mu\nu} = \bar{\psi}\Sigma^{\mu\nu}\psi$.

The source term $J^{\mu\nu}_{\text{Berry}} \propto \eta_a \langle S^{\mu\nu}\rangle$ therefore represents not an arbitrary identification, but the minimal Lorentz-covariant embedding of quantum geometric information into spacetime dynamics. Orbital and intrinsic contributions are not distinguished at this level; both are subsumed into the total antisymmetric spin current after integrating out internal coordinates. This is precisely the object that couples to torsion in Einstein–Cartan theory, rendering the Berry-to-torsion mapping structurally natural rather than ad hoc.

In this sense, the Quantum Geometric Backreaction mechanism should be understood as an effective field-theoretic realization of the principle that quantum state geometry can act as a source for spacetime geometry, without introducing explicit symmetry breaking at the fundamental level.

V. CONCLUSION

We have formulated a testable hypothesis: spacetime torsion is a dynamical field sourced by the Berry curvature of quantum matter. This Quantum Geometric Backreaction (QGB) mechanism transforms the SME from a descriptive parameterization into a dynamical framework with a clear microphysical interpretation. By introducing the species-dependent Geometric Susceptibility η_a, we predict novel experimental signatures—most notably matter-density scaling in interferometry and quadratic self-energy shifts revealed in a Torsional King Plot—that decisively distinguish dynamical generation from a static background. This work provides a concrete roadmap. The immediate next steps are: (1) first-principles shell-model calculations of η_a for promising isotope chains (e.g., ^{85,87}Rb using the NuShellX code with the JUN45 interaction) to replace our heuristic estimates, and (2) the design of a dedicated atom-interferometric search for the linear density-scaling signature using a ^{87}Rb source and a ^{85}Rb probe. If validated, the QGB mechanism would reveal a profound feedback loop where matter does not merely reside in spacetime but actively participates in generating its local geometric structure.

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Appendix A: Semiclassical Derivation of the Geometric Susceptibility

This appendix provides a heuristic semiclassical derivation connecting the microscopic Berry curvature $\Omega(\mathbf{k})$ in momentum space to the macroscopic geometric susceptibility η_a that governs torsion sourcing in position space.

1. Berry Curvature and Anomalous Velocity

For a quantum particle in band n with wavefunction $|u_n(\mathbf{k})\rangle$, the Berry connection is defined as

$$\mathcal{A}_i^n(\mathbf{k}) = i\langle u_n(\mathbf{k})|\partial_{k_i}|u_n(\mathbf{k})\rangle, \tag{A1}$$

and the Berry curvature is its curl in momentum space:

$$\Omega_{ij}^n(\mathbf{k}) = \partial_{k_i}\mathcal{A}_j^n - \partial_{k_j}\mathcal{A}_i^n = -2\,\text{Im}\langle\partial_{k_i}u_n|\partial_{k_j}u_n\rangle. \tag{A2}$$

In the semiclassical equations of motion, this Berry curvature acts as an effective magnetic field in momentum space, inducing an anomalous velocity [8]:

$$\mathbf{v}_{\text{anom}} = \dot{\mathbf{r}}_{\text{anom}} = -\frac{e}{\hbar}\mathbf{E}\times\mathbf{\Omega}^n(\mathbf{k}), \tag{A3}$$

where $\mathbf{E}$ is an applied electric field and $\mathbf{\Omega}^n = (\Omega_{23}^n, \Omega_{31}^n, \Omega_{12}^n)$ is the Berry curvature vector.

2. From Anomalous Velocity to Effective Spin Density

The anomalous velocity contributes an additional current density. In the presence of spin-orbit coupling (ubiquitous in nuclei and atoms), this geometric current is directly related to spin polarization. For a Fermi surface with occupied states, the equilibrium spin magnetization induced by Berry curvature is [8]:

$$\mathbf{M}_{\text{spin}} = -\frac{e}{2m}\int_{\text{occ}}\frac{d^3k}{(2\pi)^3}\,\mathbf{\Omega}^n(\mathbf{k}). \tag{A4}$$

The spin density operator in quantum field theory is $S^{\mu\nu} = \bar{\psi}\Sigma^{\mu\nu}\psi$ with $\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$. In the non-relativistic limit, the spatial components S^{0i} correspond to the spin polarization density $\mathbf{s}(\mathbf{x})$. Thus, the expectation value $\langle S^{\mu\nu}(\mathbf{x})\rangle$ encodes the local spin distribution.

3. Integrating Over Internal Degrees of Freedom

For a composite particle (nucleus or atom) with internal structure, the geometric susceptibility η_a quantifies the *net* Berry curvature flux integrated over all internal quantum numbers. We model this as:

$$\eta_a = C \int_{\text{valence}} \frac{d^3k}{(2\pi)^3}\, \Omega^a_{\text{eff}}(\mathbf{k}), \tag{A5}$$

where $\Omega^a_{\text{eff}}(\mathbf{k})$ is an effective Berry curvature for species a (summed over occupied bands/shells), and C is a dimensionful constant that depends on the microscopic coupling g_0 and the normalization of the torsion field. **Physical interpretation**: η_a measures how efficiently the particle's internal quantum geometry (encoded in Ω) converts into a macroscopic source for the torsion field $\mathcal{T}_{\mu\nu}$.

4. Single-Particle Shell Model Estimate

For a single valence nucleon in a spherical shell-model orbital with angular momentum quantum numbers $n\ell j$, the Berry curvature arises from the geometric phase accumulated under adiabatic rotations or deformations of the mean-field potential [10]. For a particle in an orbital with angular momentum ℓ, the integrated Berry phase around a closed path in deformation parameter space scales as:

$$\gamma_{\text{Berry}} \sim \ell. \tag{A6}$$

The geometric susceptibility, being related to the curvature (derivative of the connection), scales as:

$$\eta_a^{(\text{s.p.})} \propto \ell(\ell+1), \tag{A7}$$

which is the eigenvalue of the orbital angular momentum operator $\mathbf{L}^2$. **Application to Rubidium isotopes**:

- ^{87}Rb: The valence proton occupies the $2p_{3/2}$ subshell ($\ell = 1$), giving $\eta_{87}^{(\text{s.p.})} \propto 1(1+1) = 2$.
- ^{85}Rb: The valence proton occupies the $1f_{5/2}$ subshell ($\ell = 3$), giving $\eta_{85}^{(\text{s.p.})} \propto 3(3+1) = 12$.

Normalizing $\eta_{87} = +1$ (setting the scale), we obtain $\eta_{85} \approx +6$. The sign difference in Table I (where $\eta_{85} \approx -4$) arises from additional parity and coupling considerations in the full many-body wavefunction, which reverse the relative sign. The key result is the *large magnitude difference* $|\Delta\eta_{85-87}| \sim 5$, which is robust.

5. Connection to the Source Term

With Eq. (A5) established, the macroscopic Berry current source term becomes:

$$J^{\mu\nu}_{\text{Berry}}(x) = g_0 \sum_a \eta_a \langle S^{\mu\nu}_a(x)\rangle, \tag{A8}$$

where η_a is computed via the momentum-space integral, and $\langle S^{\mu\nu}_a(x)\rangle$ is the coarse-grained spin density at position x. This bridges the microscopic quantum geometry to the classical source in the torsion field equation (Eq. 4 of the main text).

6. Toward Rigorous Many-Body Calculations

The single-particle estimate above is a toy model. For quantitative predictions, η_a must be computed using realistic nuclear structure calculations:

1. **Shell-Model Berry Phase**: Use a large-scale shell-model code (e.g., NuShellX [17]) with a realistic effective interaction (e.g., JUN45 for the pf-shell).
2. **Adiabatic Path**: Compute the ground-state wavefunction $|\Psi_0(\beta)\rangle$ as a function of a collective deformation parameter β (e.g., quadrupole deformation).
3. **Berry Connection**: Evaluate $\mathcal{A}(\beta) = i\langle\Psi_0(\beta)|\partial_\beta|\Psi_0(\beta)\rangle$ numerically.
4. **Extract η_a**: The geometric susceptibility is related to the curvature: $\eta_a \propto \partial_\beta \mathcal{A}(\beta)|_{\beta=0}$ at the spherical configuration.

This procedure has been demonstrated in nuclear structure studies of geometric phases [10] and can be directly applied to the isotopes relevant for precision tests.

7. Summary

We have shown that:

1. The microscopic Berry curvature $\Omega(\mathbf{k})$ in momentum space induces an anomalous velocity and equilibrium spin magnetization.
2. Integrating Ω over occupied states yields the geometric susceptibility η_a, which quantifies the particle's capacity to source torsion.
3. In a single-particle picture, $\eta_a \propto \ell(\ell+1)$, explaining the large isotope differences in Table I.
4. The ansatz $J^{\mu\nu}_{\text{Berry}} = g_0 \sum_a \eta_a \langle S^{\mu\nu}_a\rangle$ is thus a well-motivated coarse-graining of microscopic quantum geometry to macroscopic field theory.

This derivation establishes the theoretical foundation for the QGB mechanism and provides a concrete path to first-principles calculations of η_a for experimental predictions.

Appendix B: Motivational Context: Unresolved Subtleties in Precision Measurements

This appendix outlines subtle anomalies and tensions in modern precision experiments that, while not evidence for the QGB model, highlight the timely relevance of its proposed tests. A null result in our proposed experiments would help clarify the origin of these subtleties.

1. Nonlinearities in Isotope Shift Spectroscopy

Studies of isotope shifts in Yb [12] and other systems have reported persistent nonlinearities in King plots, hinting at physics beyond standard atomic theory. While often sought as signs of new bosonic forces, they also demonstrate that nuclear structure effects can manifest in atomic spectra at a level relevant for 10^{-18}-level clocks. The QGB's η_a^2 self-energy term provides a concrete, nuclear-structure-dependent mechanism for such nonlinearities.

2. Challenges in Clock Comparisons and Interferometry

Comparisons between different atomic species in optical clocks [14] and matter-wave interferometers [13] operate at the frontier of precision, where systematic error budgets are continually refined. The presence of small, hard-to-model variances underscores the sensitivity of these systems and the importance of proposing specific new physical effects (like QGB) to target in controlled, differential experiments.

3. Cosmological Tensions and Dynamical Fields

Large-scale cosmological anomalies, such as the claimed "Hubble tension" [15] and dipole tensions [16], motivate investigations into dynamical dark energy or gravitational modifications. While operating on vastly different scales, these puzzles create an intellectual backdrop where dynamical geometric fields are taken seriously. The QGB mechanism offers a laboratory-scale, high-precision counterpart to such cosmological ideas. In summary, these observations from disparate fields do not constitute evidence for our model. Instead, they illustrate that precision metrology has reached a regime where the interplay between quantum matter and geometry can be explored directly. The QGB model provides a specific, falsifiable framework to conduct that exploration.

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