RIP-U and the $\omega = 0$ obstruction in Davies dynamics

Upper envelopes, witness floors, and falsification protocols for separation-dependent decoherence rates

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Abstract

We isolate the dynamic hinge in typed separation→rate→power pipelines within the Davies (weak-coupling, Markovian) setting in finite dimension. First, we formulate an upper-envelope statement (RIP-U): under an explicit factorized bath-correlation envelope with separation-dependent amplitude $f(\epsilon)$ and integrable time profile, Fourier-transformed Davies rates inherit an $O(f(\epsilon))$ envelope. Under additional regularity assumptions preventing trivial degeneracies, this yields a worst-case bound $\kappa^{\uparrow}(\epsilon) \leq Cf(\epsilon)$ for the instantaneous relative loss rate of a coherence-like functional. Second, we isolate a structural obstruction to lower-envelope statements: we prove an exact identity for the $\omega = 0$ contribution to the Davies Dirichlet form,

$$\mathcal{E}_{\sigma}^{(0)}(O) = \frac{\gamma(0)}{2} \, \|[S(0), O]\|_{2,\sigma}^2 \, ,$$

and derive a witness mechanism showing how $\omega = 0$ channels can enforce a non-vanishing dissipation contribution for suitable observable families. We emphasize directionality: RIP-U (upper) does not imply a positive lower envelope $\kappa^{\downarrow}(\epsilon)$ without additional family-qualified input. All assumptions are explicit and accompanied by falsification routes.

1 Scope and conventions

What this paper does.

- Proves a Fourier-envelope inheritance lemma for rate coefficients under a factorized correlator bound.
- States a clean upper-envelope implication (RIP-U) under explicit additional bridge assumptions.
- Proves an exact $\omega = 0$ Dirichlet identity in standard Davies form and a witness corollary.

What this paper does not do. We do not claim a Hamiltonian spectral gap and we do not claim RIP-U implies a family-wide true floor $\kappa^{\downarrow}(\epsilon) \geq \kappa_0$.

Parameter convention. ϵ is geometric separation/buffer width; δ is tolerance/regularization. They are never interchanged.

2 Davies setting and typed envelopes

We work in finite dimension: system Hilbert space $\mathcal{H}$ with $\dim \mathcal{H} = d < \infty$.

Assumption 2.1 (DAVIES: weak-coupling Markovian dynamics). **[ASSUMED]**.
The system dynamics is governed by a CPTP semigroup $\mathcal{T}_t = e^{t\mathcal{L}}$ with a Davies-type GKLS generator $\mathcal{L}$ and a faithful stationary state σ (typically $\sigma \propto e^{-\beta H}$). We assume the standard Bohr-frequency decomposition and GKLS form used in Davies theory.

2.1 A coherence-like functional and instantaneous loss

Definition 2.2 (D1: Δ-track coherence). Fix a dephasing (pinching) CPTP map Δ on $\mathcal{B}(\mathcal{H})$. Define

$$C_\Delta(\rho) := S(\rho \| \Delta[\rho]),$$

where $S(\rho\|\tau) := \mathrm{Tr}(\rho(\log \rho - \log \tau))$ when well-defined.

Definition 2.3 (D2: instantaneous loss). Define the instantaneous change of C_Δ under $\mathcal{T}_t$ by

$$\dot{C}_{\Delta,\mathrm{loss}}(\rho) := -\left.\frac{d}{dt}\right|_{t=0} C_\Delta(\mathcal{T}_t(\rho)),$$

whenever the derivative exists.

Assumption 2.4 (Δ-MONO: sign convention for "loss"). **[ASSUMED]** (family restriction).
For the chosen Δ and semigroup $\mathcal{T}_t$, we restrict attention to families $\mathcal{F}_\epsilon$ such that

$$\dot{C}_{\Delta,\mathrm{loss}}(\rho) \geq 0 \quad \text{for all } \rho \in \mathcal{F}_\epsilon.$$

Definition 2.5 (D3: ratios and envelopes). Whenever $C_\Delta(\rho) > 0$, define

$$r(\rho) := \frac{\dot{C}_{\Delta,\mathrm{loss}}(\rho)}{C_\Delta(\rho)}.$$

For a specified family $\mathcal{F}_\epsilon$ define

$$\kappa^\uparrow(\epsilon) := \sup_{\rho\in\mathcal{F}_\epsilon:\ C_\Delta(\rho)>0,\ \dot{C}_{\Delta,\mathrm{loss}}(\rho)\ \text{exists}} r(\rho), \qquad \kappa^\downarrow(\epsilon) := \inf_{\rho\in\mathcal{F}_\epsilon:\ C_\Delta(\rho)>0,\ \dot{C}_{\Delta,\mathrm{loss}}(\rho)\ \text{exists}} r(\rho).$$

Remark 2.6 (Directionality). RIP-U targets $\kappa^\uparrow(\epsilon)$ (worst-case upper envelope). Minimum-maintenance-power lower bounds require $\kappa^\downarrow(\epsilon)$; RIP-U alone does not provide them.

3 Correlator envelope ⇒ Fourier rate envelope

Assumption 3.1 (CORR: factorized correlator envelope). **[ASSUMED]**.
For each ϵ, the relevant bath correlators satisfy

$$|G_{\alpha\beta}(t;\epsilon)| \leq g_{\alpha\beta}(t)\, f(\epsilon),$$

where $g_{\alpha\beta} \in L^1(\mathbb{R})$ and $f(\epsilon) \geq 0$ captures separation dependence.

Definition 3.2 (D4: Fourier rate coefficients)**.** Define the (formal) Fourier rate coefficients by

$$\gamma_{\alpha\beta}(\omega;\epsilon) := \int_{\mathbb{R}} dt\, e^{i\omega t}\, G_{\alpha\beta}(t;\epsilon),$$

whenever the integral exists (or under the regularity hypotheses used in Davies theory).

Lemma 3.3 (Fourier envelope inheritance)**.** ***[PROVED]*** *(under Definition 3.1 and integrability). Under Definition 3.1, for all ω and ϵ for which Definition 3.2 is well-defined,*

$$|\gamma_{\alpha\beta}(\omega;\epsilon)| \le \|g_{\alpha\beta}\|_{L^1}\ f(\epsilon).$$

Proof. By the triangle inequality and Definition 3.1,

$$|\gamma_{\alpha\beta}(\omega;\epsilon)| \le \int_{\mathbb{R}} dt\, |G_{\alpha\beta}(t;\epsilon)| \le \int_{\mathbb{R}} dt\, |g_{\alpha\beta}(t)|\, f(\epsilon) = \|g_{\alpha\beta}\|_{L^1}\ f(\epsilon).$$

□

4 RIP-U: an upper envelope on instantaneous loss

Assumption 4.1 (REG: non-degeneracy for relative-entropy ratios)**. [ASSUMED]** (family restriction).
The family $\mathcal{F}_\epsilon$ is restricted so that:

(i) $C_\Delta(\rho) \ge c_{\min}(\delta) > 0$ uniformly over $\rho \in \mathcal{F}_\epsilon$;

(ii) $\Delta[\rho] \succeq p_{\min}(\delta)\mathbf{1}$ uniformly over $\rho \in \mathcal{F}_\epsilon$;

(iii) when needed, ρ stays a fixed distance (within tolerance δ) away from $\mathrm{Fix}(\mathcal{T}_t)$.

Assumption 4.2 (BRIDGE: rate envelope $\Rightarrow$ loss envelope)**. [ASSUMED].**
Assume Definitions 2.1, 3.1 and 4.1. Suppose the Davies generator is built from Fourier rate coefficients $\gamma_{\alpha\beta}(\omega;\epsilon)$ in such a way that the envelope from Definition 3.3 implies: there exists a constant $C_{\mathrm{RIP}} > 0$ such that for all $\rho \in \mathcal{F}_\epsilon$,

$$\dot{C}_{\Delta,\mathrm{loss}}(\rho) \le C_{\mathrm{RIP}}\, f(\epsilon)\, C_\Delta(\rho).$$

Theorem 4.3 (T1: RIP-U (upper envelope))**.** ***[PROVED]*** *(given Definitions 2.1, 2.4, 3.1, 4.1 and 4.2).*
Under Definitions 2.1, 2.4, 3.1, 4.1 and 4.2, for all $\rho \in \mathcal{F}_\epsilon$,

$$r(\rho) \le C_{\mathrm{RIP}}\, f(\epsilon),$$

and hence

$$\kappa^{\uparrow}(\epsilon) \le C_{\mathrm{RIP}}\, f(\epsilon).$$

Proof. By Definition 4.2, $\dot{C}_{\Delta,\mathrm{loss}}(\rho) \le C_{\mathrm{RIP}} f(\epsilon) C_\Delta(\rho)$ for all $\rho \in \mathcal{F}_\epsilon$ with $C_\Delta(\rho) > 0$, hence $r(\rho) \le C_{\mathrm{RIP}} f(\epsilon)$. Taking the supremum yields the stated envelope. □

Remark 4.4 (RIP-U does not imply a floor). RIP-U controls worst-case *upper* loss rates. It is compatible with $\kappa^{\downarrow}(\epsilon)$ being arbitrarily small on a large family. Thus RIP-U alone does not imply minimum maintenance power lower bounds in pipelines requiring a positive lower envelope.

5 The $\omega = 0$ obstruction: an exact Dirichlet identity

Definition 5.1 (D5: σ-weighted 2-norm)**.** For a faithful state σ, define

$$\|X\|_{2,\sigma} := \sqrt{\mathrm{Tr}\big(\sigma^{1/2} X^\dagger \sigma^{1/2} X\big)}.$$

Definition 5.2 (D6: Dirichlet form (one convenient convention))**.** Let $\mathcal{L}$ be a GKLS generator with faithful stationary state σ. Define the Dirichlet form on observables by

$$\mathcal{E}_\sigma(O) := -\mathrm{Tr}\Big(\sigma^{1/2} O^\dagger \sigma^{1/2}\, \mathcal{L}^\sharp(O)\Big)\,,$$

where $\mathcal{L}^\sharp$ denotes the Heisenberg-picture action of the generator on observables.

Assumption 5.3 (BOHR: frequency decomposition)**. [ASSUMED]** (standard in Davies theory). Let H be the system Hamiltonian with spectral projectors $\{\Pi_E\}$. For a system coupling operator S, define its Bohr components

$$S(\omega) := \sum_{E'-E=\omega} \Pi_E S \Pi_{E'}.$$

Assume the Davies generator uses dissipators built from $S(\omega)$ with scalar rates $\gamma(\omega) \geq 0$.

Lemma 5.4 (Commutation at zero frequency)**. *[PROVED].***
If $\sigma \propto e^{-\beta H}$ and $S(0)$ is the $\omega = 0$ Bohr component, then

$$[S(0), H] = 0 \quad \Rightarrow \quad [S(0), \sigma] = 0 \quad \textit{and} \quad [S(0), \sigma^{1/2}] = 0.$$

Proof. By construction, $S(0) = \sum_E \Pi_E S \Pi_E$ commutes with $H = \sum_E E\Pi_E$, hence with any function of H, in particular σ and $\sigma^{1/2}$. □

Theorem 5.5 (T2: $\omega = 0$ Dirichlet identity)**. *[PROVED]*** *(under Definitions 2.1 and 5.3 in standard Davies form).*
The zero-frequency contribution $\mathcal{E}_\sigma^{(0)}$ to the Dirichlet form satisfies

$$\mathcal{E}_\sigma^{(0)}(O) = \frac{\gamma(0)}{2}\, \|[S(0), O]\|_{2,\sigma}^2\,.$$

Proof. We give a direct expansion in Appendix A. □

Remark 5.6 (The $\gamma(0) = 0$ case). If $\gamma(0) = 0$ (e.g. baths with vanishing zero-frequency spectral density), then $\mathcal{E}_\sigma^{(0)} \equiv 0$ and this particular $\omega = 0$ obstruction mechanism disappears. Any lower-envelope floor must then come from other frequency sectors or from structural restrictions defining the family.

Corollary 5.7 (C1: $\omega = 0$ witness mechanism)**. *[PROVED].***
Assume $\gamma(0) > 0$. If there exists an observable family O_ϵ such that

$$\frac{\|[S(0), O_\epsilon]\|_{2,\sigma}^2}{\|O_\epsilon\|_{2,\sigma}^2} \geq c > 0,$$

then

$$\mathcal{E}_\sigma^{(0)}(O_\epsilon) \geq \frac{\gamma(0)}{2}\, c\, \|O_\epsilon\|_{2,\sigma}^2\,.$$

Proof. Immediate from Definition 5.5. □

Remark 5.8 (C1 is not RIP-L). C1 is a mechanism for a chosen observable family. Converting it into a uniform lower-envelope bound $\kappa^{\downarrow}(\epsilon) \geq \kappa_0$ over a state family requires additional typed bridges.

Assumption 5.9 (RIP-L: true floor (family-qualified))**. [CONJECTURED]**.
There exist $\epsilon_* > 0$ and $\kappa_0 > 0$ such that

$$\kappa^{\downarrow}(\epsilon) \geq \kappa_0 \quad \text{for all } \epsilon \geq \epsilon_*,$$

for a specified family $\mathcal{F}_\epsilon$ (with any regularity restrictions stated explicitly).

6 Falsification matrix

Item	Status	Verification proxy	Concrete falsifier
CORR	[ASSUMED]	compute $G_{\alpha\beta}(t;\epsilon)$	separation dependence not factorizable as $g(t)f(\epsilon)$
Fourier envelope lemma	[PROVED]	compute $\|g\|_{L^1}$	violate integrability/envelope inequality
Δ-MONO	[ASSUMED]	evaluate $\dot{C}_{\Delta,\text{loss}}(\rho)$	find ρ with $\dot{C}_{\Delta,\text{loss}}(\rho) < 0$ in family
REG	[ASSUMED]	lower-bound spectrum of $\Delta[\rho]$	sequence with $\Delta[\rho_n]$ nearly singular
BRIDGE	[ASSUMED]	compute $r(\rho)$ on the family	show $r(\rho)$ violates $O(f(\epsilon))$ despite CORR
RIP-U	[PROVED]	apply Definition 4.3	refute any upstream assumption
$\omega = 0$ identity	[PROVED]	direct Dirichlet-form expansion	refute Davies form/hypotheses
RIP-L	[CONJECTURED]	estimate $\kappa^{\downarrow}(\epsilon)$	construct ρ_ϵ with $r(\rho_\epsilon) \to 0$

Table 1: Falsification matrix for the dynamic layer.

A Line-by-line proof of the $\omega = 0$ identity

We give a direct expansion showing

$$\mathcal{E}_\sigma^{(0)}(O) = \frac{\gamma(0)}{2} \, \|[S(0), O]\|_{2,\sigma}^2 .$$

For clarity we present the Hermitian case $S(0)^\dagger = S(0)$; the general case replaces $S(0)^2$ by $S(0)^\dagger S(0)$ and yields the same commutator structure.

The $\omega = 0$ Heisenberg dissipator has the standard GKLS form

$$\mathcal{L}^{\sharp,(0)}(O) = \gamma(0)\Big(S(0)OS(0) - \tfrac{1}{2}\{S(0)^2, O\}\Big).$$

Using the Dirichlet form convention from Definition 5.2,

$$\mathcal{E}_\sigma^{(0)}(O) = -\mathrm{Tr}\Big(\sigma^{1/2} O^\dagger \sigma^{1/2}\, \mathcal{L}^{\sharp,(0)}(O)\Big)\,.$$

Expand:

$$\mathcal{E}_\sigma^{(0)}(O) = -\gamma(0)\mathrm{Tr}\Big(\sigma^{1/2} O^\dagger \sigma^{1/2} S(0) O S(0)\Big) + \frac{\gamma(0)}{2}\mathrm{Tr}\Big(\sigma^{1/2} O^\dagger \sigma^{1/2} S(0)^2 O\Big) + \frac{\gamma(0)}{2}\mathrm{Tr}\Big(\sigma^{1/2} O^\dagger \sigma^{1/2} O S(0)^2\Big)\,.$$

On the other hand,

$$\begin{aligned}\|[S(0),O]\|_{2,\sigma}^2 &= \mathrm{Tr}\Big(\sigma^{1/2}\big(S(0)O - OS(0)\big)^\dagger \sigma^{1/2}\big(S(0)O - OS(0)\big)\Big)\\ &= \mathrm{Tr}\Big(\sigma^{1/2}(O^\dagger S(0) - S(0)O^\dagger)\sigma^{1/2}(S(0)O - OS(0))\Big)\,.\end{aligned}$$

Using Definition 5.4 to commute $S(0)$ through $\sigma^{1/2}$ and expanding the product yields

$$\|[S(0),O]\|_{2,\sigma}^2 = \mathrm{Tr}\Big(\sigma^{1/2} O^\dagger \sigma^{1/2} S(0)^2 O\Big) - 2\,\mathrm{Tr}\Big(\sigma^{1/2} O^\dagger \sigma^{1/2} S(0) O S(0)\Big) + \mathrm{Tr}\Big(\sigma^{1/2} O^\dagger \sigma^{1/2} O S(0)^2\Big)\,.$$

Comparing with the expression for $\mathcal{E}_\sigma^{(0)}(O)$ gives

$$\mathcal{E}_\sigma^{(0)}(O) = \frac{\gamma(0)}{2}\,\|[S(0),O]\|_{2,\sigma}^2\,,$$

as claimed.

References

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