Petz Recoverability versus Wilson-Loop Diagnostics in 2+1D $\mathbb{Z}_2$ Lattice Gauge Theory:

benchmarks by exact diagonalization and tensor-network ladders

Version 2

Lluis Eriksson
Independent Researcher
lluiseriksson@gmail.com

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Abstract

We provide reproducible finite-size benchmarks testing whether a Petz-type recoverability proxy correlates with Wilson-loop confinement diagnostics in $\mathbb{Z}_2$ lattice gauge theory in 2+1 dimensions: an exact-diagonalization benchmark on 2×2 and 2×3 plaquette lattices (Gauss penalty, $\langle G_v \rangle \simeq 1$ verified), and TeNPy DMRG ladders $2 \times L$, $L \in \{4, 6, 8\}$, $\chi_{\max} = 96$, with a warm-start bond-dimension stability check ($\chi = 96 \to 192$ stable to all shown digits). In the tensor-network part, $E_{\rm rec}^{\rm Petz}$ is reported as a function of contiguous buffer size $|B|$ in MPS site ordering (a declared proxy), not the BFS collar of the ED part. *v2 (no v1 number is changed):* the broken Appendix A.1 of v1 – whose published PDF literally prints "Missing file" where the ED reproduction script should appear – is repaired by pointing to the series repository, where that script and a full verification suite for the ED benchmark already live with the companion paper; internal working titles leaked throughout v1's scripts and captions are removed; the MPS-ordering proxy is sharpened with new evidence – on an exact $\mathbb{Z}_2$ ladder the geometric admissible buffer decays cleanly ($5 \times 10^{-4} \to 3 \times 10^{-8}$) while the contiguous-ordering proxy starts orders of magnitude higher and saturates, and a from-scratch DMRG replication (reduced χ, $L_x = 4$) reproduces both the trend direction of the money plots and v1's own buffer inversion $E_{\rm rec}^{\rm Petz}(|B|{=}2) > E_{\rm rec}^{\rm Petz}(|B|{=}1)$, confirming it as a property of the contiguous proxy rather than a numerical accident; the confinement-vs-gap degeneracy established for the ED twin applies verbatim to the ladder money plots and is now stated; and series positioning is added. All conclusions remain finite-size benchmark statements.

1 Overview, model, diagnostics

As in v1 (unchanged): benchmark study, no thermodynamic-limit claims; $H(g) = -g \sum X_\ell - \frac{1}{g} \sum_p \prod Z_\ell + \Lambda \sum_v (1 - G_v)$; gauge validity a posteriori via $\min_v \langle G_v \rangle$; confinement proxy $\sigma_{\rm eff} := \overline{\chi(2,2)}$ (averaged over placements), length proxy $1/\sigma_{\rm eff}$; regularized, trace-renormalized Petz-type $E_{\rm rec}^{\rm Petz}$.

Series positioning (added in v2). The ED part of this paper is the benchmark of the companion 2601.0047 [7] (same run, same output files); its v2 carries the densified statistics, the spectral-gap covariate, and the prescription-dependence analysis, all of which apply here verbatim. The CMI twin is 2601.0050 [8]; the gauge protocol and ladder lessons are 2601.0044 [9]; collaring-rule semantics are 2601.0043 [10].

2 Exact diagonalization benchmark

Unchanged from v1 / 2601.0047 (Figures 1–3). The reproduction script that v1's Appendix A.1 failed to include (the published PDF prints "Missing file") is available in the series repository under `verification/2601-0047/`, together with a regenerable suite that reproduces σ_{eff}, ground energies and the Gauss sector to $< 1\%$ and documents the prescription-dependence of $E_{\text{rec}}^{\text{Petz}}$ magnitudes.

3 Tensor-network ladders (TeNPy DMRG)

Unchanged from v1: geometry note (contiguous MPS-ordering proxy, explicit); multi-L money plots at $|B| = 1, 2$; warm-start stability check at $(L, g) = (8, 1.0)$, $\chi = 96 \to 192$ (Table 2).

Remark 3.1 (The contiguous proxy is not a geometric collar – quantified in v2). The suite adds two pieces of evidence. (i) *Exact ladder comparison:* on the $\mathbb{Z}_2$ ladder of 2601.0044 (14 links, exact ground states), the geometric admissible buffer gives cleanly decaying profiles (e.g. $5.5\times10^{-4} \to 3.3\times 10^{-8}$ over three widths at $g = 0.6$), while the contiguous-ordering proxy on the same states starts two orders of magnitude higher and saturates flat – contiguous MPS segments cut flux lines rather than bounding them. (ii) *DMRG replication:* an independent reduced-cost run ($L_x = 4$, $\chi = 32$) reproduces v1's buffer inversion $E_{\text{rec}}^{\text{Petz}}(|B|{=}2) > E_{\text{rec}}^{\text{Petz}}(|B|{=}1)$ at moderate coupling (3.1×10^{-3} vs 2.3×10^{-3} at $g = 0.8$) together with exact Gauss invariance and the money-plot trend direction ($E_{\text{rec}}^{\text{Petz}}$ grows with $1/\sigma_{\text{eff}}$). The inversion in v1's Table 2 is thus a reproducible property of the contiguous proxy, not a numerical accident – and one more instance of the collaring-rule lesson of 2601.0043/0050 v2. Geometric BFS collars on ladders remain the natural next implementation step for the TN part.

Remark 3.2 (Confinement specificity (added in v2)). The ED twin's v2 established that on these small systems $1/\sigma_{\text{eff}}$ and the inverse spectral gap are monotonically locked, so recoverability trends cannot be attributed specifically to confinement. The ladder money plots inherit this caveat unchanged; the p-values of any rank statistics on the coupling sweeps quantify ordering strength in a deterministic sweep, not statistical independence.

4 Verification suite (added in v2)

`verification/2601-0051/verify_2601_0051.py`: Part A (dependency-free) runs the exact-ladder collar-semantics comparison of Remark 3.1(i); Part B (requires TeNPy, skipped gracefully otherwise) runs the reduced-cost DMRG replication of Remark 3.1(ii) with Gauss checks and trend-direction verification. The v1 TN scripts (multi-L scan, validator, warm-start check) are retained alongside, with internal working titles removed from headers; the ED script lives with 2601.0047.

5 Discussion

The two-tier benchmark stands: ED with strict Gauss checks, TN ladders with a solid bond-dimension stability check, and consistent money-plot trends. v2's contribution is to close the reproducibility hole, quantify what the contiguous proxy does and does not measure, and inherit the

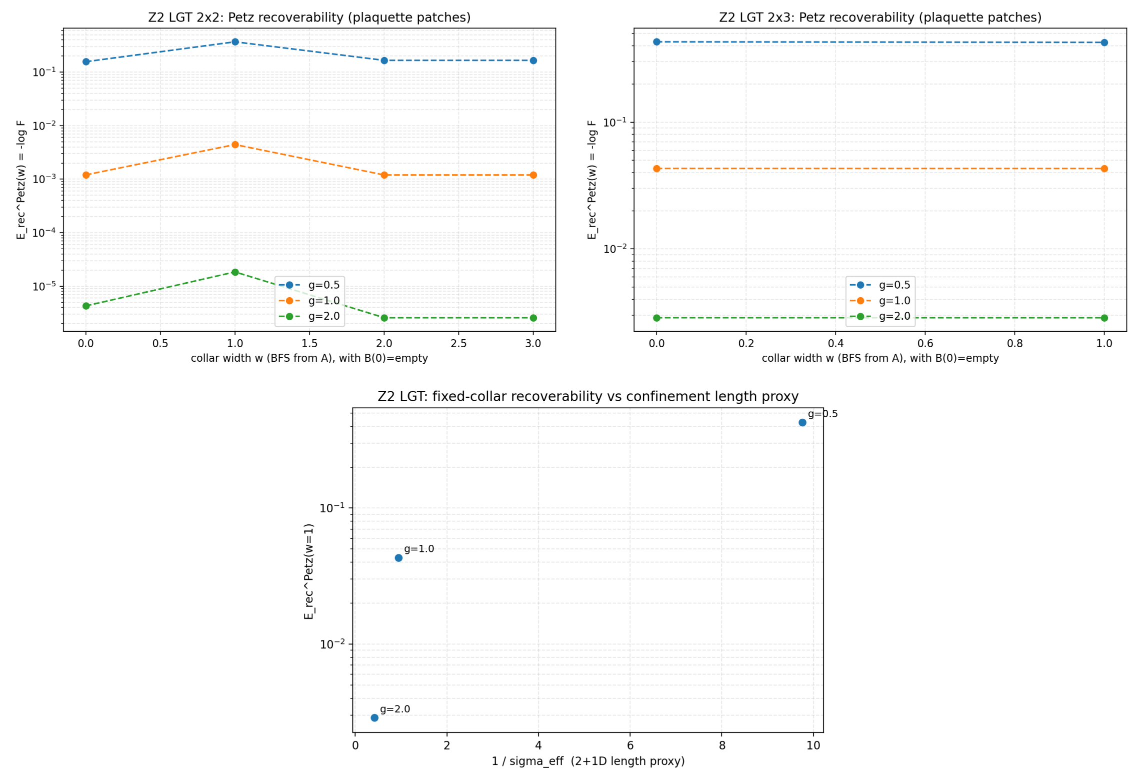

Figure 1: ED benchmark (v1 figures, unchanged): 2×2, 2×3 profiles and fixed-collar vs $1/\sigma_{\mathrm{eff}}$.

specificity caveat of the twin. Next steps: geometric BFS collars in the TN part, larger widths, and fixed-coupling geometry sweeps to break the σ–gap degeneracy.

Note added in v2

No v1 number is changed; all figures, Table 2 and the TN scripts stand. v2: (1) repairs Appendix A.1 (the published v1 PDF prints "Missing file" for the ED script; it now points to the repository, where the script and a full ED suite live with 2601.0047); (2) removes internal working titles leaked in v1's script headers, captions and filenames (the historical output filenames are kept for artifact compatibility); (3) adds the collar-semantics evidence and the DMRG replication of the buffer inversion (Remark 3.1); (4) states the confinement-vs-gap caveat inherited from the ED twin; (5) adds series positioning (v1 cited no companion papers) and the verification suite.

References

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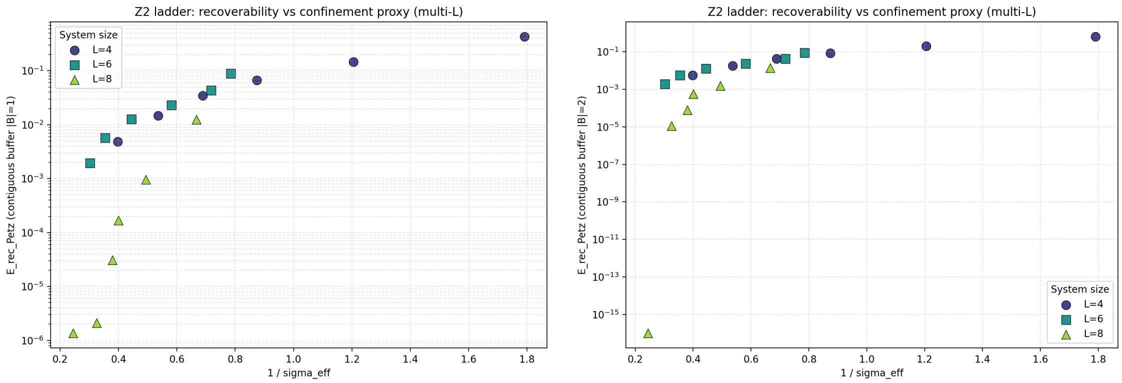

Figure 2: TN ladders (v1 figures, unchanged): fixed contiguous buffer $|B| = 1, 2$ (MPS-ordering proxy) vs $1/\sigma_{\text{eff}}$, colored by L. Read with Remark 3.1.

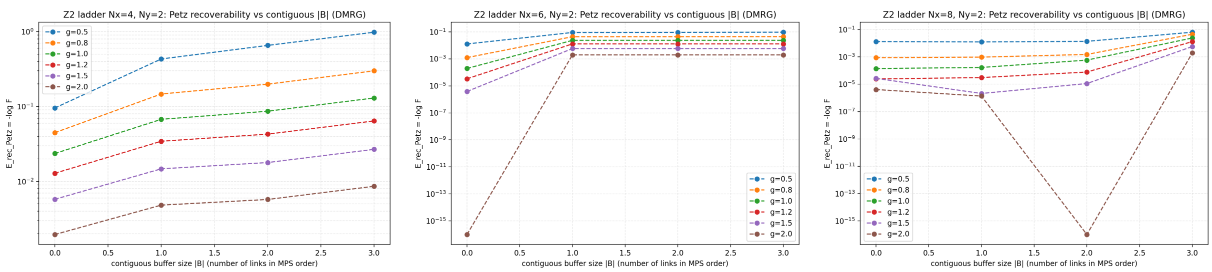

Figure 3: Recoverability profiles $E_{\text{rec}}^{\text{Petz}}(|B|)$ on ladders $2 \times L$, $L = 4, 6, 8$ (v1 figures, unchanged).

[6] J. Hauschild and F. Pollmann, SciPost Phys. Lect. Notes 5 (2018) [TeNPy].

[7] L. Eriksson, ai.viXra:2601.0047 (2026; v2 2026).

[8] L. Eriksson, ai.viXra:2601.0050 (2026; v2 2026).

[9] L. Eriksson, ai.viXra:2601.0044 (2026; v2 2026).

[10] L. Eriksson, ai.viXra:2601.0043 (2026; v2 2026).