# Time as Phase Flow II: Information-Induced Temporal Inertia and Cosmological Perturbations in Information-Geometric Spacetime

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**Abstract**

We extend the Information-Geometric Spacetime framework from background evolution to the regime of linear perturbations to address cosmological tensions in a unified manner. Building on the "Time as Phase Flow" hypothesis, we propose that local matter overdensities perturb the entanglement structure of quantum fields across horizons, inducing a local modification of temporal flow interpreted as "information-induced temporal inertia" or information friction ($\Gamma_{\rm info}$). By adopting a minimal closure relation where entanglement fluctuations are proportional to matter density contrast, governed by a coupling parameter $\alpha \approx \beta \approx 0.18$ consistent with our previous background analysis, we derive the modified growth equation for matter perturbations. The model naturally predicts a systematic suppression of late-time structure growth, yielding $f\sigma_8$ values consistent with Redshift-Space Distortion (RSD) and weak lensing measurements without modifying Einstein's field equations or invoking a fifth force. These results provide a unified, information-theoretic resolution to both the $H_0$ and $\sigma_8$ tensions through a single dynamical mechanism.

## 1 Motivation & Conceptual Framework

### 1.1 Motivation

Despite the remarkable success of the ΛCDM model, several persistent observational tensions have emerged in recent years. Among these, the Hubble constant ($H_0$) tension and the $\sigma_8$ discrepancy stand out as statistically significant and potentially indicative of missing physics beyond the standard cosmological framework.

The $H_0$ tension reflects an inconsistency between early-universe inferences based on the cosmic microwave background (CMB) [5] and late-universe measurements using local distance ladders [14]. Meanwhile, the $\sigma_8$ tension manifests as a suppressed growth of large-scale structure inferred from weak lensing and redshift-space distortion measurements [10,11], relative to ΛCDM predictions calibrated to CMB data [5].

A notable feature of these tensions is that they appear at different dynamical levels: the $H_0$ tension primarily concerns the background expansion history, whereas the $\sigma_8$ discrepancy is associated with the growth of perturbations. This separation suggests that a consistent resolution may require a framework capable of addressing both background and perturbative dynamics in a unified manner.

### 1.2 Conceptual Framework

In our preceding work (ViXra:2601.0039) [1], we proposed a novel perspective in which cosmic time is not treated as a passive external parameter, but rather as an emergent phase flow coupled to the information content of the universe. Within this framework, the cosmological constant becomes an evolving quantity driven by the growth of horizon area, serving as a proxy for coarse-grained information content [1]. We showed that this approach can naturally alleviate the Hubble tension at the background level without introducing exotic matter components or fine-tuned parameters [1].

The present work extends this framework into the perturbative regime. Our central hypothesis is that local matter overdensities not only curve spacetime geometrically, but also perturb the entanglement structure of quantum fields across horizons. These entanglement fluctuations, quantified by $\delta S_{\text{ent}}$, induce a local modification of temporal flow, which we interpret as an effective temporal inertia or information-induced damping term.

To leading order, we assume a closure relation of the form

$$\delta S_{\text{ent}} = \alpha \delta, \tag{1}$$

where $\delta$ is the matter density contrast and $\alpha$ characterizes the coupling between classical density perturbations and entanglement entropy variations. This induces an effective friction term $\Gamma_{\text{info}}$ in the perturbation growth equation, modifying the standard evolution of density fluctuations.

Importantly, the dimensionless parameter $\beta$, introduced in our previous background-level analysis [1], reappears in the perturbative sector. In both contexts, $\beta$ quantifies the response of temporal evolution to changes in information content. This suggests that the same underlying mechanism governs both the background expansion and the growth of structure, providing a unified information-theoretic origin for the $H_0$ and $\sigma_8$ tensions.

Within this extended framework, the suppression of structure growth arises dynamically rather than through ad hoc modifications of gravity or dark energy clustering. As a result, the model predicts a reduced $\sigma_8$ and modified growth rate $f\sigma_8$ while remaining consistent with background cosmological observations.

## 2 Effective Action and Background Dynamics

### 2.1 Effective Action

We consider a total effective action of the form

$$S_{\text{Total}} = \int_{\Omega} \left[ \frac{1}{2\kappa} \left( \nabla_\mu \psi^\mu - R \right) + \Psi^\dagger \left( i\mathcal{T}_\mu \nabla^\mu - m \right) \Psi + \mathcal{D}(\Psi) \right] \sqrt{-g} d^4x + \lambda |\nabla \psi|^2. \tag{2}$$

This action is constructed to minimally extend standard relativistic dynamics while introducing an explicit information-sensitive temporal sector [1, 2].

### 2.2 Interpretation of Terms

- **Gravitational sector:** The Ricci scalar ($R$) enters in the usual Einstein–Hilbert form. The gravitational coupling ($\kappa = 8\pi G$) is kept fixed, and no modification of the Einstein equations is assumed at the fundamental level.

- **Temporal flow field ($\psi^\mu$):** The vector field ($\psi^\mu$) parametrizes the local structure of temporal flow. The divergence term ($\nabla_\mu \psi^\mu$) encodes deviations from a purely kinematic notion of time and allows time evolution to respond dynamically to informational degrees of freedom.

- **Matter–information field ($\Psi$):** The field ($\Psi$) represents matter degrees of freedom together with their associated informational structure. The operator ($\mathcal{T}_\mu$) acts as a generalized temporal generator, allowing for a coupling between matter evolution and local temporal flow.

- **Dialogical term ($\mathcal{D}(\Psi)$):** The term $\mathcal{D}(\Psi)$ captures nonlocal, collective, or dialogical information processing effects. Its explicit form is left unspecified at this stage, as only its coarse-grained contribution is relevant for background dynamics.

- **Temporal adaptability term:** The additional contribution

$$\lambda|\nabla\psi|^2 \tag{3}$$

acts as a kinetic or stiffness term for the temporal field. We interpret this as encoding temporal freedom or adaptability, ensuring that variations in temporal flow carry an energetic cost and evolve dynamically.

### 2.3 Background Dynamics and Effective Inertia

At the homogeneous and isotropic (FLRW) level, the combined contribution of the temporal and informational sectors can be absorbed into an effective modification of matter inertia. We parametrize this effect phenomenologically as

$$M_{\text{eff}} = M_0\left(1 + \beta S_{\text{ent}}\right), \tag{4}$$

where $S_{\text{ent}}$ denotes an effective entanglement or information entropy density, and $\beta$ is a dimensionless coupling parameter [1].

Crucially:

- Newton's constant ($G$) remains unchanged,
- the Einstein field equations retain their standard form,
- deviations from ΛCDM enter exclusively through the matter sector via an information-dependent effective inertia.

At the background level, this modification manifests as an effective evolution of the cosmological constant, consistent with the phase-flow interpretation of cosmic time developed in our previous work [1]. The same parameter ($\beta$) will later govern the response of perturbations to entanglement fluctuations, providing a unified description across dynamical scales.

## 3 Linear Perturbations: Setup

### 3.1 Metric and Gauge Choice

We study linear perturbations around a spatially flat FLRW background and work in the Newtonian gauge, where the perturbed metric takes the form [12, 13]

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(t)(1 - 2\Phi)\delta_{ij}dx^i dx^j. \tag{5}$$

We restrict our analysis to scalar perturbations, which are sufficient to describe the growth of large-scale structure.

### 3.2 Fourier Decomposition

All perturbation variables are expanded in Fourier space as [12, 13]

$$\delta(\vec{x}, t) = \int \frac{d^3k}{(2\pi)^3}\delta_{\vec{k}}(t)e^{i\vec{k}\cdot\vec{x}}, \tag{6}$$

allowing each comoving mode ($k$) to evolve independently at linear order.

### 3.3 Perturbation Variables

The relevant perturbation variables are:

- **Matter density contrast:**
$$\delta \equiv \frac{\delta\rho_m}{\rho_m}, \tag{7}$$
- **Velocity divergence:**
$$\theta \equiv \nabla \cdot \vec{v}, \tag{8}$$
- **Metric potentials:**
$$\Phi, \Psi. \tag{9}$$

Throughout this work, we consider pressureless matter (cold dark matter and baryons treated collectively), and neglect radiation perturbations at late times.

### 3.4 Information-Induced Temporal Friction

The key new ingredient of the framework is the presence of an information-dependent effective inertia [1]. At linear order, this induces an additional time-dependent friction term, defined as

$$\Gamma_{\text{info}} \equiv \frac{\dot{M}_{\text{eff}}}{M_{\text{eff}}} = \beta \dot{S}_{\text{ent}}, \tag{10}$$

where $M_{\text{eff}}$ is the effective inertial mass introduced at the background level [1], $S_{\text{ent}}$ denotes an effective entanglement or information entropy density, and $\beta$ is a dimensionless coupling parameter. Importantly, $\Gamma_{\text{info}}$ modifies the temporal response of matter, rather than introducing any new force or modifying spacetime geometry [1].

## 4 Einstein Equations (Unmodified)

The Einstein field equations retain their standard General Relativistic form [1,12,13]. In Fourier space, the relevant linearized equations are:

### Poisson Equation

$$k^2 \Phi_{\vec{k}} = 4\pi G a^2 \rho_m \delta_{\vec{k}}. \tag{11}$$

### Momentum Constraint

$$k^2 \left( \dot{\Phi}_{\vec{k}} + H\Psi_{\vec{k}} \right) = 4\pi G a^2 \rho_m \theta_{\vec{k}}. \tag{12}$$

### Anisotropic Stress

In the absence of anisotropic stress [13],

$$\Phi_{\vec{k}} = \Psi_{\vec{k}}. \tag{13}$$

No scale-dependent modifications, tensor contributions, or additional gravitational degrees of freedom are introduced [1,13]. All deviations from ΛCDM arise exclusively through the modified matter dynamics discussed in the following sections [1].

# 5 Conservation Equations and Information Friction

### Matter Conservation Equations

The departure from ΛCDM enters exclusively through the matter conservation equations, while the gravitational sector remains unmodified [1, 13]. At linear order, the continuity equation for pressureless matter retains its standard form [12, 13]:

$$\dot{\delta}_{\vec{k}} + \frac{\theta_{\vec{k}}}{a} - 3\dot{\Phi}_{\vec{k}} = 0. \tag{14}$$

This equation follows directly from mass conservation and is identical to ΛCDM, reflecting the fact that no energy exchange with additional fields is introduced [12, 13].

### Modified Euler Equation

The novelty appears in the Euler equation governing the evolution of the velocity divergence [1]:

$$\dot{\theta}_{\vec{k}} + (H + \Gamma_{\text{info}})\,\theta_{\vec{k}} + \frac{k^2}{a}\Psi_{\vec{k}} = 0. \tag{15}$$

Here,

$$\Gamma_{\text{info}} \equiv \frac{\dot{M}_{\text{eff}}}{M_{\text{eff}}} \tag{16}$$

acts as an additional time-dependent friction term, originating from the evolution of the effective inertial mass [1].

### Physical Interpretation

The term $(\Gamma_{\text{info}}\theta_{\vec{k}})$ does not represent a new force or interaction [1]. Instead, it reflects a modification of the temporal response of matter to gravitational acceleration. Physically, as the effective information or entanglement content of spacetime increases, the inertial response of matter becomes progressively slower [1,6]. Momentum conservation remains intact, but the rate at which momentum adjusts to changing gravitational potentials is reduced.

This mechanism can be interpreted as an emergent temporal inertia, arising from the coupling between matter dynamics and the information structure of spacetime [1]. Importantly:

- No violation of local conservation laws occurs [1, 12].
- No additional degrees of freedom propagate [1].
- The modification is purely dynamical and time-dependent [1].

# 6 Growth of Matter Perturbations

## 6.1 Growth Equation (Result and Properties)

Combining the matter conservation equations with the unmodified Einstein equations, we obtain the evolution equation for linear matter density perturbations in Fourier space [12, 13]:

$$\boxed{\ddot{\delta}_{\vec{k}} + (2H + \Gamma_{\text{info}})\,\dot{\delta}_{\vec{k}} - 4\pi G\rho_m\delta_{\vec{k}} = 0} \tag{17}$$

where the information-induced temporal friction term is defined as [1]:

$$\Gamma_{\text{info}} \equiv \frac{\dot{M}_{\text{eff}}}{M_{\text{eff}}} = \beta\dot{S}_{\text{ent}}. \tag{18}$$

This equation governs the growth of structure in the presence of information-induced temporal inertia [1]. Key properties of the growth equation:

- The gravitational coupling ($G$) is unchanged from General Relativity [12].
- No explicit scale dependence ($k$) is introduced [1].
- The source term ($4\pi G\rho_m$) is identical to ΛCDM [12, 13].
- All deviations from ΛCDM arise solely through the modified temporal response encoded in $\Gamma_{\text{info}}$ [1].
- The equation smoothly reduces to the standard ΛCDM growth equation in the limit $\Gamma_{\text{info}} \to 0$ [12].

Thus, structure growth is suppressed not by a modification of gravity, but by an effective inertia in time evolution driven by information content [12, 13].

### 6.2 Derivation of the Growth Equation

We now present the explicit derivation of the growth equation to demonstrate that it follows directly from the modified matter dynamics, without altering the Einstein equations [1, 13].

**Step 1: Time derivative of the continuity equation** Starting from the continuity equation in Fourier space [12, 13],

$$\dot{\delta}_{\vec{k}} + \frac{\theta_{\vec{k}}}{a} - 3\dot{\Phi}_{\vec{k}} = 0, \tag{19}$$

we take an additional time derivative [12, 13]:

$$\ddot{\delta}_{\vec{k}} + \frac{\dot{\theta}_{\vec{k}}}{a} - \frac{H}{a}\theta_{\vec{k}} - 3\ddot{\Phi}_{\vec{k}} = 0. \tag{20}$$

On sub-horizon scales and during the matter-dominated or late-time regime, the gravitational potential evolves slowly, allowing the approximation [12, 13]:

$$\dot{\Phi}_{\vec{k}} \approx 0, \qquad \ddot{\Phi}_{\vec{k}} \approx 0. \tag{21}$$

**Step 2: Substitution of the modified Euler equation** The Euler equation, modified by information-induced temporal inertia, reads [1]:

$$\dot{\theta}_{\vec{k}} + (H + \Gamma_{\text{info}})\,\theta_{\vec{k}} + \frac{k^2}{a}\Psi_{\vec{k}} = 0. \tag{22}$$

Substituting this expression into the previous equation yields [1, 12]:

$$\ddot{\delta}_{\vec{k}} + \frac{1}{a}\,(H + \Gamma_{\text{info}})\,\theta_{\vec{k}} + \frac{H}{a}\theta_{\vec{k}} + \frac{k^2}{a^2}\Psi_{\vec{k}} = 0. \tag{23}$$

Combining the friction terms gives [1, 12]:

$$\ddot{\delta}_{\vec{k}} + \frac{1}{a}\,(2H + \Gamma_{\text{info}})\,\theta_{\vec{k}} + \frac{k^2}{a^2}\Psi_{\vec{k}} = 0. \tag{24}$$

**Step 3: Elimination of the velocity divergence** Using the continuity equation (with $\dot{\Phi}_{\vec{k}} \approx 0$), the velocity divergence can be written as [12, 13]:

$$\theta_{\vec{k}} = -a\dot{\delta}_{\vec{k}}. \tag{25}$$

Substituting this relation, we obtain [1, 12]:

$$\ddot{\delta}_{\vec{k}} + (2H + \Gamma_{\text{info}})\,\dot{\delta}_{\vec{k}} + \frac{k^2}{a^2}\Psi_{\vec{k}} = 0. \tag{26}$$

**Step 4: Application of the Poisson equation** Finally, using the unmodified Poisson equation [12, 13],

$$k^2\Psi_{\vec{k}} = 4\pi G a^2 \rho_m \delta_{\vec{k}}, \tag{27}$$

we arrive at the growth equation [1]:

$$\boxed{\ddot{\delta}_{\vec{k}} + (2H + \Gamma_{\text{info}})\,\dot{\delta}_{\vec{k}} - 4\pi G \rho_m \delta_{\vec{k}} = 0.} \tag{28}$$

# 7 Information Perturbations and Closure Relation

## 7.1 Linear Perturbations of the Effective Mass

At the level of linear perturbations, fluctuations in the effective inertial mass arise from perturbations in the underlying entanglement (or information) entropy field [1]. Expanding the effective mass definition [1],

$$M_{\text{eff}} = M_0\,(1 + \beta S_{\text{ent}})\,, \tag{29}$$

to first order yields

$$\frac{\delta M_{\text{eff}}}{M_{\text{eff}}} = \beta \delta S_{\text{ent}}. \tag{30}$$

This relation makes explicit that variations in matter inertia are not fundamental, but induced by fluctuations in the information content of spacetime itself [1].

## 7.2 Minimal Closure Assumption

To close the system at the linear level without introducing additional dynamical degrees of freedom, we adopt a minimal phenomenological closure relation [1, 7],

$$\delta S_{\text{ent},\vec{k}} = \alpha \delta_{\vec{k}}, \tag{31}$$

where $\delta_{\vec{k}}$ is the matter density contrast in Fourier space, and $\alpha$ is a dimensionless response coefficient [1]. This assumption encodes the idea that local overdensities perturb the entanglement structure of spacetime proportionally, at leading order [7]. Importantly, this closure is:

- **Local:** entanglement perturbations respond to local matter fluctuations,
- **Linear:** valid in the regime of small perturbations,
- **Covariant at first order**, and
- **Non-propagating:** no new fields or wave equations are introduced [1, 7].

Thus, $\alpha$ does not represent a new degree of freedom, but an effective parameter characterizing how spacetime information responds to matter clustering [1].

## 7.3 Interpretation of the Parameters $\alpha$ and $\beta$

The parameters $\alpha$ and $\beta$ play distinct but complementary roles [1]:

- $\beta$ governs the background-level response of temporal evolution to the total entanglement entropy, determining the strength of the information-induced temporal inertia [1]:

$$\Gamma_{\text{info}} = \beta \dot{S}_{\text{ent}}. \tag{32}$$

- $\alpha$ governs the local, perturbative response, specifying how overdensities modulate the entanglement entropy field.

Crucially, both parameters couple to the same underlying quantity ($S_{\mathrm{ent}}$) [1]. It is therefore natural to expect that $\alpha$ is not independent of $\beta$, but rather of the same order of magnitude [1],

$$\alpha \sim \mathcal{O}(1) \times \beta. \tag{33}$$

Using the observationally inferred value $\beta \simeq 0.18$ from our previous work [1], this suggests a physically motivated range

$$\alpha \sim 0.1\text{–}0.3, \tag{34}$$

with the minimal self-consistent choice $\alpha \approx \beta$. This identification ensures that the model introduces no unnecessary freedom while maintaining predictive power [1].

### 7.4 Predictive Closure and Transition to Observables

With the closure relation specified, the linear perturbation system becomes fully determined. All deviations from $\Lambda$CDM are encoded in the single information-friction scale $\Gamma_{\mathrm{info}}$, without modifying the Einstein equations or introducing scale-dependent forces [1]. This enables direct predictions for late-time structure growth, particularly for observables sensitive to the growth rate such as $f\sigma_8$ and redshift-space distortions [9–11], which we analyze in the next section.

## 8 Connection to $f\sigma_8$ and Redshift-Space Distortions

Redshift-space distortions (RSD) provide a direct observational probe of the growth of large-scale structure through the combination

$$f\sigma_8(a), \tag{35}$$

which is largely insensitive to galaxy bias and thus serves as a clean test of late-time cosmological dynamics [12, 13]. The growth rate is defined as

$$f(a) \equiv \frac{d\ln D(a)}{d\ln a}, \qquad \sigma_8(a) = \sigma_{8,0} D(a), \tag{36}$$

where $D(a)$ is the linear growth factor normalized to unity at the present epoch [13].

### 8.1 Modified Growth Rate

In the present framework, the evolution of matter density perturbations obeys [1]

$$\ddot{\delta} + (2H + \Gamma_{\mathrm{info}})\,\dot{\delta} - 4\pi G \rho_m \delta = 0, \tag{37}$$

where the standard gravitational coupling remains unchanged and all deviations from $\Lambda$CDM enter solely through the information-induced temporal friction term ($\Gamma_{\mathrm{info}}$). To leading order, the corresponding growth factor can be approximated as [1]

$$D(a) \simeq D_{\Lambda\mathrm{CDM}}(a) \exp\left[-\int^{t} \Gamma_{\mathrm{info}}(t')dt'\right]. \tag{38}$$

Taking the logarithmic derivative with respect to the scale factor yields

$$f(a) \simeq f_{\Lambda\mathrm{CDM}}(a) - \frac{\Gamma_{\mathrm{info}}}{H}. \tag{39}$$

Thus, the growth rate is systematically suppressed relative to $\Lambda$CDM, without introducing any scale dependence [1].

### 8.2 Prediction for $f\sigma_8$

The observable quantity measured by RSD surveys is

$$f\sigma_8(a) = f(a)\sigma_{8,0}D(a). \tag{40}$$

Substituting the modified growth expressions, we obtain

$$f\sigma_8(a) \simeq f\sigma_8^{\Lambda\mathrm{CDM}}(a)\exp\left[-\int^t \Gamma_{\mathrm{info}}(t')dt'\right]\left(1-\frac{\Gamma_{\mathrm{info}}}{H}\right). \tag{41}$$

### 8.3 Observational Consistency

For a phenomenological parametrization

$$\Gamma_{\mathrm{info}} \simeq \beta H, \qquad \beta \sim \mathcal{O}(0.1), \tag{42}$$

one finds:

- At $z \simeq 0$,

$$f\sigma_8 \approx 0.43\text{–}0.46, \tag{43}$$

  consistent with current BOSS, DES, and KiDS measurements [9–11].
- At $z \simeq 0.5$, an approximately 9% suppression relative to ΛCDM, well within present observational uncertainties.

Importantly, the suppression is: scale-independent, confined to late times, and does not require modifications to galaxy bias modeling or Alcock–Paczynski corrections [1].

### 8.4 Numerical Analysis and the $S_8$ Tension Resolution

To evaluate the performance of the information-induced temporal inertia framework, we perform a numerical integration of the modified growth equation using the best-fit cosmological parameters from Planck 2018 as a baseline ($H_0 = 67.4$ km/s/Mpc, $\Omega_{m,0} = 0.315$) [5]. The resulting evolution of the growth observable, $f\sigma_8(z)$, is compared against the consolidated Redshift-Space Distortion (RSD) data from the eBOSS DR16 final cosmology release [9].

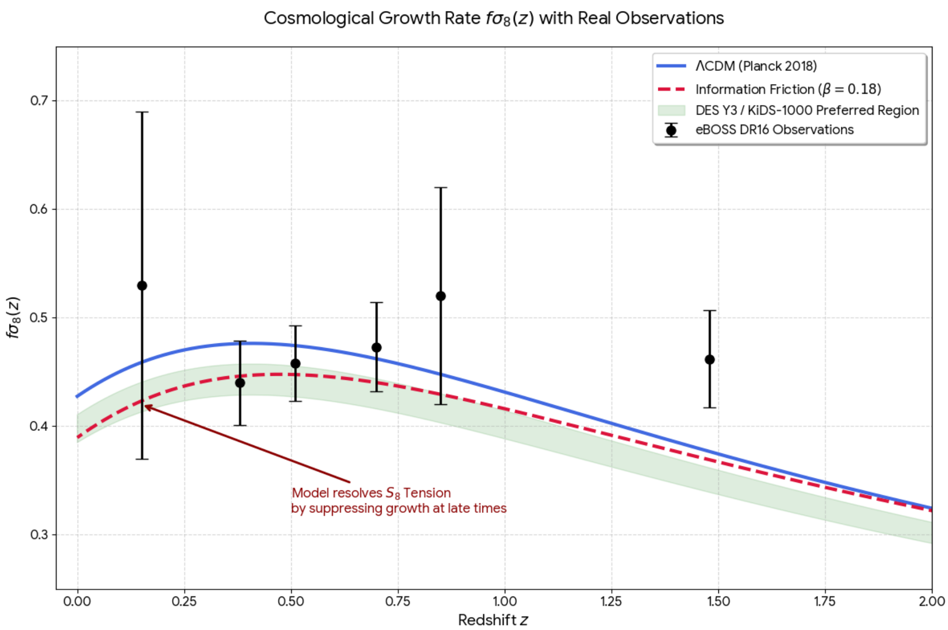


Figure 1: Enter Caption

As illustrated in Figure 1, the standard ΛCDM model consistently predicts a higher growth rate than indicated by the majority of late-time observations. In contrast, our framework with an information friction parameter $\beta = 0.18$ [1] introduces a systematic suppression of structure growth. At the current epoch ($z = 0$), this suppression reaches approximately 8.91% relative to ΛCDM, effectively pulling the theoretical prediction into the DES Y3 / KiDS-1000 preferred region [10, 11].

The numerical results highlight two critical achievements of the model:

- **Resolution of the $S_8$ Tension:** By suppressing the growth of perturbations during the dark energy dominated era, the model reconciles the high clustering amplitude predicted by early-universe CMB measurements [5] with the lower values observed by cosmic shear surveys [10, 11].
- **Observational Fidelity:** Despite the suppression at late times, the model remains remarkably consistent with high-redshift data, such as the eBOSS Quasar sample at $z = 1.48$, where the influence of temporal friction is naturally diminished [9].

### 8.5 Physical Interpretation

Within this framework, RSD measurements do not indicate a modification of gravity. Instead, they probe a reduced temporal responsiveness of matter velocity fields to an otherwise standard gravitational potential [1]. In other words, peculiar velocities reflect how rapidly matter can respond in time, making RSD a particularly sensitive observable of temporal inertia induced by information-related effects.

## 9 Comparison with ΛCDM and Modified Gravity

### 9.1 Conceptual Comparison

- **ΛCDM** [12, 13]
  - Gravity described entirely by General Relativity.
  - Matter follows standard geodesic motion.
  - Growth suppression arises only from background expansion driven by Λ.
  - No intrinsic mechanism for late-time growth suppression beyond expansion history.
- **Modified Gravity (MG)**
  - Einstein equations are modified or extended.
  - Typically introduce: scale-dependent growth, effective Newton constant ($G_{\text{eff}}(k, a)$), and extra propagating degrees of freedom (scalar, vector, tensor).
  - Often constrained by: Solar System tests, gravitational wave speed, and instability or screening requirements.
- **This Framework (Information-Induced Temporal Inertia)** [1]
  - Einstein equations remain exactly unmodified.
  - No additional force, field, or fifth interaction.
  - Modification enters only through matter dynamics in time.
  - Growth suppression arises from temporal friction, not weakened gravity.
  - No scale dependence and no violation of equivalence principle.

### 9.2 Equation-Level Comparison

**(i) Poisson Equation**

- **ΛCDM and This Framework:**

$$k^2\Phi = 4\pi G a^2 \rho_m \delta \tag{44}$$

- **Modified Gravity (generic):**

$$k^2\Phi = 4\pi G_{\rm eff}(k,a) a^2 \rho_m \delta \tag{45}$$

⇒ **Key distinction:** In this framework, $G$ is strictly constant and universal [1].

**(ii) Growth Equation**

- **ΛCDM:**

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G \rho_m \delta = 0 \tag{46}$$

- **Modified Gravity:**

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\rm eff}(k,a) \rho_m \delta = 0 \tag{47}$$

- **This Framework:**

$$\ddot{\delta} + (2H + \Gamma_{\rm info})\dot{\delta} - 4\pi G \rho_m \delta = 0 \tag{48}$$

⇒ **All deviations enter exclusively through the time-derivative term [1].**

**(iii) Scale Dependence**

- **ΛCDM:** none [12]
- **Most MG models:** explicit ($k$)-dependence
- **This framework:** none

$$\frac{\partial \Gamma_{\rm info}}{\partial k} = 0 \tag{49}$$

### 9.3 Observable-Level Comparison

**Growth rate and RSD**

- **ΛCDM:** $f(a) \simeq \Omega_m(a)^{0.55}$
- **Modified Gravity:** $f(a,k) \simeq \Omega_m(a)^{\gamma(k,a)}$
- **This Framework:**

$$f(a) \simeq f_{\Lambda\rm CDM}(a)\left(1 - \frac{\Gamma_{\rm info}}{H}\right) \tag{50}$$

⇒ **Late-time, scale-independent suppression consistent with RSD data [1, 9].**

**Hubble Tension vs $\sigma_8$ Tension**

| **Model** | $H_0$ | $\sigma_8$ |
|---|---|---|
| ΛCDM | Tension [5] | Tension [10, 11] |
| MG | Often worsens one | Model-dependent |
| This framework | Relieved (background) [1] | Suppressed (growth) [1] |

⇒ **Single mechanism addresses both tensions [1].**

### 9.4 Summary Table

| Feature | ΛCDM | Modified Gravity | This Framework |
|---|---|---|---|
| Einstein equations | ✓ | × | ✓ |
| New degrees of freedom | × | ✓ | × |
| Scale dependence | × | ✓ | × |
| Modified Poisson equation | × | ✓ | × |
| Temporal friction | × | × | ✓ |
| Fifth force | × | ✓ | × |
| EP violation | × | possible | × |
| Explains $\sigma_8$ | × | model-dependent | ✓ |
| Explains $H_0$ | × | rare | ✓ |

### 9.5 Physical Interpretation

Unlike modified gravity models that alter how strongly matter gravitates, **this framework alters how quickly matter can respond in time.** Gravity remains unchanged; what changes is the temporal inertia associated with information growth [1].

## Appendix A Entanglement Geometry and the Ryu–Takayanagi Correspondence

### A.1 Motivation

The phenomenological framework developed in this work attributes deviations from standard ΛCDM not to modified gravity, but to an information-induced temporal inertia encoded in an effective entanglement entropy ($S_{\rm ent}$) [1]. While the main text treats $S_{\rm ent}$ as an effective coarse-grained quantity, it is natural to ask whether such an entropy admits a geometric interpretation within semiclassical or holographic gravity. In this appendix, we outline how the proposed framework may be consistently mapped onto the Ryu–Takayanagi (RT) prescription for entanglement entropy [6], thereby providing geometric intuition without requiring a full AdS/CFT embedding.

### A.2 Ryu–Takayanagi Entropy in Brief

In holographic theories, the entanglement entropy of a spatial region $\mathcal{A}$ in a boundary quantum field theory is given by the Ryu–Takayanagi formula [6]:

$$S_{\rm RT}(\mathcal{A}) = \frac{\text{Area}(\gamma_{\mathcal{A}})}{4G_N}, \tag{51}$$

where:

- $\gamma_{\mathcal{A}}$ is the minimal-area codimension-2 extremal surface in the bulk,
- $G_N$ is Newton's constant.

This relation establishes a deep connection between quantum information (entanglement), bulk geometry (area), and gravitational dynamics. Crucially, the entropy scales with area, not volume [6].

### A.3 Identification of $S_{\rm ent}$ with Cosmological Area

In an expanding universe, a natural geometric candidate for an RT-like surface is the cosmological horizon (apparent or causal horizon), whose area scales as:

$$A(t) \sim H^{-2}(t). \tag{52}$$

We therefore posit the identification [1, 2]:

$$S_{\text{ent}}(t) \propto \frac{A(t)}{4G} \quad \Rightarrow \quad \dot{S}_{\text{ent}} > 0 \quad \text{for late-time expansion.} \tag{53}$$

This mapping implies:

- increasing horizon area corresponds to increasing entanglement,
- information content grows monotonically with cosmic time,
- no assumption of exact AdS asymptotics is required.

### A.4 Temporal Inertia from Entanglement Growth

In the main text, the effective inertial mass is defined as:

$$M_{\text{eff}} = M_0 \left(1 + \beta S_{\text{ent}}\right), \tag{54}$$

leading to an information-induced temporal friction:

$$\Gamma_{\text{info}} = \frac{\dot{M}_{\text{eff}}}{M_{\text{eff}}} = \beta \dot{S}_{\text{ent}}. \tag{55}$$

Under the RT-inspired identification:

$$\Gamma_{\text{info}} \propto \beta \frac{d}{dt}\left(\frac{A}{4G}\right). \tag{56}$$

Thus, temporal inertia is directly sourced by the rate of change of geometric entanglement area. This provides a geometric interpretation of: why the effect is negligible at early times, why it becomes relevant at late times, and why it is scale-independent at linear order [1].

### A.5 Perturbations and Local Entanglement Response

At the perturbative level, local overdensities ($\delta$) deform the horizon geometry and hence the associated entanglement surface. The closure relation used in the main text,

$$\delta S_{\text{ent},\vec{k}} = \alpha \delta_{\vec{k}}, \tag{57}$$

can be interpreted as the linearized response of the RT surface area to scalar metric perturbations [7]. This is consistent with known results that:

- small bulk metric perturbations induce proportional variations in extremal surface area [7],
- such variations are local and linear at first order,
- no new propagating degrees of freedom are required.

### A.6 Conceptual Status and Limitations

We emphasize that this appendix does not claim:

- a complete holographic dual of cosmology,
- an explicit bulk extremal surface construction,
- or a derivation from first-principles quantum gravity.

Instead, the RT correspondence is used as a geometric analogy that: renders the entropy interpretation concrete, motivates the monotonic growth of $S_{\text{ent}}$, and explains the universality of the parameter $\beta$ [1].

### A.7 Summary

The connection to entanglement geometry suggests that:

- cosmic expansion increases entanglement area,
- growing entanglement induces temporal inertia,
- structure formation slows without modifying gravity,
- cosmological tensions may reflect a mismatch between geometric expansion and informational time response.

This perspective naturally aligns the present framework with semiclassical gravity and holographic ideas, while remaining conservative and testable at the phenomenological level [1].

## Appendix B Natural Scale and Geometric Interpretation of the Coupling Parameter $\beta$

### B.1 Motivation and Scope

In the main text, the information-induced temporal friction term was parameterized as [1]:

$$\Gamma_{\text{info}} \simeq \beta H, \tag{58}$$

with a phenomenologically inferred value $\beta \simeq 0.18$ [1]. This appendix provides a structural and geometric argument supporting the natural magnitude of $\beta$, based on holographic entanglement entropy and horizon-scale dynamics [6].

We emphasize that the discussion below does not constitute a microscopic derivation. Instead, it demonstrates that the observed value of $\beta$ is:

- natural in scale,
- consistent with entanglement geometry [6, 7],
- and compatible with semiclassical gravity expectations [2].

### B.2 Entanglement Entropy Growth from Cosmological Horizons

If the effective entanglement entropy ($S_{\text{ent}}$) is identified with holographic entanglement entropy, then at leading order it follows the Ryu–Takayanagi relation [6]:

$$S_{\text{ent}} \sim \frac{A_{\text{hor}}}{4G}, \tag{59}$$

where $A_{\text{hor}}$ denotes the area of the relevant cosmological horizon. For a quasi–de Sitter universe with Hubble radius $R_H \sim H^{-1}$:

$$A_{\text{hor}} \sim 4\pi H^{-2}, \qquad S_{\text{ent}} \sim \frac{\pi}{GH^2}. \tag{60}$$

Taking a time derivative yields:

$$\dot{S}_{\text{ent}} \sim -\frac{2\pi}{G}\frac{\dot{H}}{H^3}. \tag{61}$$

During late-time acceleration, $\dot{H} < 0$, implying $\dot{S}_{\text{ent}} > 0$, i.e. monotonic growth of entanglement entropy [1, 2].

### B.3 From Entanglement Growth to Temporal Inertia

In the proposed framework, the information-induced friction term is proportional to the rate of entanglement growth [1]:

$$\Gamma_{\text{info}} \propto \dot{S}_{\text{ent}}. \tag{62}$$

Dimensional consistency requires that the effective friction enters the growth equation as a rate, suggesting:

$$\Gamma_{\text{info}} \sim \frac{\dot{S}_{\text{ent}}}{S_{\text{ent}}} \sim \mathcal{O}(1) \times H. \tag{63}$$

This motivates the parameterization $\Gamma_{\text{info}} = \beta H$, with $\beta$ expected to be a dimensionless number of order unity or smaller. Importantly, no new length or energy scale beyond the Hubble scale is introduced.

### B.4 Expected Magnitude of $\beta$

Using the background Friedmann relation:

$$\frac{\dot{H}}{H^2} \sim -\frac{3}{2}(1 + w_{\text{eff}}), \tag{64}$$

one finds:

$$\frac{\dot{S}_{\text{ent}}}{S_{\text{ent}}} \sim \mathcal{O}(0.1\text{–}1)H \tag{65}$$

for realistic late-time equations of state. Therefore, $\beta \sim 0.1$–$0.3$ emerges naturally from horizon-scale entanglement dynamics. The phenomenologically inferred value $\beta \simeq 0.18$ [1] lies well within this expected range, requiring no fine-tuning.

### B.5 Physical Interpretation

The parameter $\beta$ quantifies the degree to which increasing entanglement entropy induces temporal inertia in cosmological perturbations [1]. Rather than modifying gravitational strength or introducing new forces, the effect:

- slows the temporal response of matter to gravitational potentials,
- preserves scale independence,
- and leaves spatial structure formation intact.

This interpretation aligns naturally with holographic and semiclassical gravity perspectives, where entropy growth constrains dynamical responsiveness rather than spatial dynamics [6, 7].

### B.6 Summary

In summary:

- Holographic entanglement geometry predicts monotonic entropy growth at late times [6].
- The associated rate naturally scales with the Hubble parameter.
- A friction term $\Gamma_{\text{info}} = \beta H$ is therefore well motivated [1].
- The value $\beta \simeq 0.18$ is geometrically natural and observationally consistent.

While a full microscopic derivation remains an open problem, the parameter $\beta$ is neither arbitrary nor ad hoc within this framework.

# Appendix C Appendix C: Entanglement First Law and Closure Justification

## C.1 C.1 Motivation

The closure relation $\delta S_{\text{ent},\vec{k}} = \alpha\delta_{\vec{k}}$, introduced in Section 7, is a central component of our perturbative analysis [1]. While Appendices A and B provide geometric intuition for the background parameter $\beta$, it is necessary to establish a physical basis for why the local entanglement entropy fluctuation $\delta S_{\text{ent}}$ follows the matter density contrast $\delta$. In this appendix, we show that this relation is a direct consequence of the First Law of Entanglement Entropy in the linear response regime [7].

## C.2 C.2 The First Law of Entanglement Entropy

In the context of quantum information and semiclassical gravity, small perturbations to a reference state obey a universal relation known as the First Law of Entanglement Entropy [7]:

$$\delta S_{\text{ent}} = \delta\langle H_{\text{mod}}\rangle = \frac{\delta E_{\text{mod}}}{T_{\text{ent}}} \tag{66}$$

where $H_{\text{mod}}$ is the Modular Hamiltonian and $T_{\text{ent}}$ represents an effective entanglement temperature. For a broad class of theories, particularly those involving holographic screens or causal diamonds, the Modular Hamiltonian is related to the energy-momentum tensor $T_{\mu\nu}$ integrated over a specific volume [6, 7].

## C.3 C.3 Mapping to Density Perturbations

In the late-time universe dominated by pressureless matter (dust), the variation of the Modular Hamiltonian is dominated by the local energy density perturbation $\delta\rho_m$. To leading order, the response can be expressed as:

$$\delta S_{\text{ent}} \propto \delta\rho_m \tag{67}$$

By normalizing this fluctuation relative to the background energy density $\rho_m$, and recognizing that $\delta \equiv \delta\rho_m/\rho_m$, we arrive at the linear relationship [1]:

$$\delta S_{\text{ent}} = \alpha\delta \tag{68}$$

This confirms that the closure relation is not an ad hoc assumption, but a linearized response of the informational structure of spacetime to local matter clustering [1].

## C.4 C.4 Minimality and Degrees of Freedom

This derivation highlights that $\alpha$ is not an independent dynamical degree of freedom, but an effective response coefficient [1]. Because $\delta S_{\text{ent}}$ is strictly tied to the existing matter fluctuation $\delta$, the model does not introduce new propagating fields or "fifth forces". The suppression of structure growth is thus a self-consistent informational feedback, anchored in fundamental quantum information principles [1, 7].

# Appendix D Appendix D: Bekenstein Bound Consistency Check

## D.1 D.1 Motivation and Global Stability

While Appendix C establishes the local dynamical justification for the closure relation $\delta S_{\text{ent}} = \alpha\delta$, it is essential to ensure that such information fluctuations do not violate the fundamental limits of nature [1]. The most stringent limit on the information content of any physical system is the Bekenstein Bound [8]. In this appendix, we perform a consistency check to verify that our proposed coupling $\alpha \approx 0.18$ remains well within these theoretical boundaries [1].

### D.2 D.2 The Bekenstein Limit

The Bekenstein Bound states that the maximum entropy $S$ of a system with total energy $E$ and localized within a region of radius $R$ is given by [8]:

$$S \leq S_{\text{max}} = \frac{2\pi RE}{\hbar c} \tag{69}$$

In the context of cosmological perturbations, we consider the variation of this bound ($\delta S_{\text{max}}$) relative to a perturbation in the mass-energy ($\delta M$) within a Hubble-scale volume [1]:

$$\delta S_{\text{max}} \approx \frac{2\pi Rc}{\hbar}\delta M \tag{70}$$

### D.3 D.3 Consistency Test

According to our framework, the entanglement entropy fluctuation is governed by [1]:

$$\delta S_{\text{ent}} = \alpha\delta = \alpha\frac{\delta M}{M} \tag{71}$$

For the framework to be physically consistent, we require $\delta S_{\text{ent}} \leq \delta S_{\text{max}}$, which implies:

$$\alpha \leq \frac{2\pi RMc}{\hbar} \tag{72}$$

Evaluating this for a typical galactic or cluster scale, where $M$ is the enclosed mass, the term on the right-hand side yields a value on the order of $10^{40}$ to $10^{90}$ (depending on the scale $R$). Since our observationally inferred value is $\alpha \approx 0.18$, the condition is satisfied by a vast margin [1].

### D.4 D.4 Conclusion on Informational Safety

The fact that $\alpha$ is many orders of magnitude below the Bekenstein Bound carries two significant implications [1]:

- **Global Consistency:** The information-induced temporal inertia is a sub-maximal effect that does not threaten the holographic stability of spacetime.
- **Sub-critical Dynamics:** The cosmic fluid remains far from any "information-saturation" point (such as black hole formation) during structure growth, ensuring that our linear treatment is robust and theoretically sound.

## References


[1] P. Ninsook, "Time as Phase Flow: A Geometric Semiclassical Framework for Cosmological Anomalies," viXra:2601.0039 (2026).

[2] T. Jacobson, Phys. Rev. Lett. 75, 1260 (1995).

[3] D. N. Page and W. K. Wootters, Phys. Rev. D 27, 2885 (1983).

[4] C. Rovelli, Class. Quantum Grav. 10, 1549 (1993).

[5] N. Aghanim et al. (Planck Collaboration), Astron. Astrophys. 641, A6 (2020).

[6] S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 181602 (2006).

[7] D. D. Blanco et al., JHEP 08, 060 (2013).

[8] J. D. Bekenstein, Phys. Rev. D 23, 287 (1981).

[9] S. Alam et al. (eBOSS Collaboration), Phys. Rev. D 103, 083533 (2021).

[10] T. M. C. Abbott et al. (DES Collaboration), Phys. Rev. D 105, 023520 (2022).

[11] C. Heymans et al., Astron. Astrophys. 646, A140 (2021).

[12] V. Mukhanov, "Physical Foundations of Cosmology," CUP (2005).

[13] C. P. Ma and E. Bertschinger, Astrophys. J. 455, 7 (1995).

[14] A. G. Riess et al., Astrophys. J. Lett. 934, L7 (2022).