Petz Recoverability versus Wilson-Loop Diagnostics in $\mathbb{Z}_2$ Lattice Gauge Theory (2+1D)

Exact diagonalization benchmark on small open lattices

Version 2

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Abstract

We provide a finite-size benchmark testing whether a Petz-type recoverability proxy correlates with Wilson-loop confinement diagnostics in $\mathbb{Z}_2$ lattice gauge theory in 2+1 dimensions. Ground states are computed by sparse exact diagonalization on 2×2 and 2×3 plaquette lattices (open boundaries, qubits on links, Gauss law enforced by penalty and verified, $\langle G_v\rangle \simeq 1$). For opposite-corner plaquette patches A and C, a buffer $B(w)$ is defined by BFS thickening in the link-adjacency graph, and we evaluate a trace-renormalized regularized Petz-type recovery error $E_{\rm rec}^{\rm Petz}(w)$; as confinement proxy we use the Creutz ratio $\sigma_{\rm eff} := \chi(2,2)$, with natural 2+1D length proxy $\sigma_{\rm eff}^{-1}$. Across couplings, larger $\sigma_{\rm eff}^{-1}$ correlates with larger fixed-collar $E_{\rm rec}^{\rm Petz}(w{=}1)$. *v2 upgrades the statistics and adds controls (no v1 number is changed):* the coupling sweep is densified to $n = 8$ on 2×2, upgrading v1's three-point trend to a significant rank correlation (Spearman $\rho = +1.00$, permutation $p < 10^{-4}$); a spectral-gap covariate is added, and it correlates *equally perfectly* with $E_{\rm rec}^{\rm Petz}(w{=}1)$ – so at these sizes the benchmark cannot distinguish confinement physics from generic gap/correlation-length physics, an honest limitation now stated; $\sigma_{\rm eff}$, ground energies and the Gauss sector are reproduced to $< 1\%$ by a regenerable suite, while the $E_{\rm rec}^{\rm Petz}$ *magnitudes* (and v1's 2×3 saturation) prove prescription-dependent – the correlation *direction* replicates under both prescriptions; the relation to the ladder-level no-tracking result of the companion is reconciled (different observables: fixed-collar error magnitudes here, decay lengths there); and two editorial leaks of v1 are fixed. All conclusions remain finite-size benchmark statements.

1 Introduction

Petz-type recoverability measures approximate quantum Markov structure; in gauge theories subsystem definitions are subtle due to constraints. The companion protocol note 2601.0044 [6] proposed reproducible recoverability computations on lattices and conjectural links to confinement diagnostics, and its v2 executed a $\mathbb{Z}_2$ ladder, finding that a genuine 2D lattice is required for a resolving test. This paper is that 2D benchmark, on the smallest feasible lattices. Scope: finite-size and boundary effects are large; the goal is a falsifiable, reproducible benchmark, not a thermodynamic-limit statement. In 2+1 dimensions $[\sigma] = (\text{length})^{-1}$, so the natural length proxy is $\sigma_{\rm eff}^{-1}$.

2 Model, geometry, observables

As in v1 (unchanged; the v1 heading's stray placeholder in Section 2.2 is removed): $H(g) = -g\sum_\ell X_\ell - \frac{1}{g}\sum_p \prod_{\ell\in\partial p} Z_\ell + \Lambda\sum_v (I - G_v)$ with $G_v = \prod_{\ell\ni v} X_\ell$, $\Lambda = 50$; A/C = southwest/northeast plaquette patches (four links each); $B(0) = \emptyset$, $B(w) = \{\ell : \mathrm{dist}_{\mathrm{link}}(\ell, A) \le w\} \setminus (A\cup C)$; regularized Petz-type map with $\delta = 10^{-12}$, trace-normalized output; $E^{\mathrm{Petz}}_{\mathrm{rec}}(w) = -\log F$; Creutz ratio $\chi(2,2) = -\log[\langle W(2,2)\rangle\langle W(1,1)\rangle/(\langle W(2,1)\rangle\langle W(1,2)\rangle)]$. Non-monotonicity of $E^{\mathrm{Petz}}_{\mathrm{rec}}(w)$ at these sizes is expected (v1's Remark 2.1) and fixed-collar comparisons are emphasized.

3 Results

All v1 numbers stand (Table 1 of v1; Figures 1–3).

Remark 3.1 (Statistics upgraded in v2). v1 inferred the correlation from three couplings per lattice. The suite densifies the 2×2 sweep to $n = 8$ couplings ($g \in [0.5, 2.5]$): Spearman rank correlation between $E^{\mathrm{Petz}}_{\mathrm{rec}}(w{=}1)$ and $\sigma^{-1}_{\mathrm{eff}}$ is $\rho = +1.00$ with two-sided permutation $p < 10^{-4}$. The v1 observation survives as a robust monotone rank-trend in the (deterministic) coupling sweep – the permutation p quantifies the strength of the ordering, not statistical independence of experimental samples.

Remark 3.2 (Confinement specificity unresolved (added in v2)). As a control, the suite computes the spectral gap of $H(g)$ on the same sweep. The gap covariate correlates with $E^{\mathrm{Petz}}_{\mathrm{rec}}(w{=}1)$ *exactly as well* (Spearman $\rho = +1.00$, $p < 10^{-4}$): on these lattices, $\sigma^{-1}_{\mathrm{eff}}$ and the inverse gap are themselves monotonically locked, so the benchmark cannot attribute the recoverability trend to confinement physics as opposed to generic gap/correlation-length growth toward small g. Disentangling the two requires either lattices large enough to vary geometry at fixed coupling, or models where string tension and gap can be tuned independently. We state this as the main open limitation of the benchmark.

Remark 3.3 (Prescription dependence of $E^{\mathrm{Petz}}_{\mathrm{rec}}$ magnitudes (added in v2)). The suite pins an explicit link ordering, patch assignment and embedding, and under it reproduces σ_{eff}, ground energies and the Gauss sector to $< 1\%$ – but the $E^{\mathrm{Petz}}_{\mathrm{rec}}$ *magnitudes* differ from v1's Table 1 (e.g. at 2×2, $g = 0.5$: $E^{\mathrm{Petz}}_{\mathrm{rec}}(1) \sim 10^{-5}$ vs v1's 0.37), and v1's 2×3 saturation $E^{\mathrm{Petz}}_{\mathrm{rec}}(0) \simeq E^{\mathrm{Petz}}_{\mathrm{rec}}(1)$ is not observed (the buffer does reduce the error under the suite's prescription). This reflects genuine sensitivity of reduced-state constructions in gauge theories to embedding conventions – consistent with the EHS caveats of [6] – and is why the robust content of this paper is the correlation *direction*, which replicates under both prescriptions, rather than any absolute $E^{\mathrm{Petz}}_{\mathrm{rec}}$ value.

Remark 3.4 (Reconciliation with the ladder null (added in v2)). The companion 2601.0044 v2 found *no tracking* between the recoverability decay length ξ_{rec} and the Wilson decay scale on a height-one ladder. There is no contradiction: that null concerns *length scales* extracted from w-profiles in a quasi-1D geometry with degenerate area/perimeter, whereas the present positive signal concerns *fixed-collar error magnitudes* across couplings on genuine 2D patches. Jointly they suggest that recoverability magnitudes respond to the coupling long before recoverability *lengths* become resolvable – consistent with the floor-management lesson of the ladder run.

4 Verification suite (added in v2)

`verification/2601-0047/verify_2601_0047.py` (sparse Lanczos, support-projected fidelities, chunked JSON cache): regenerates σ_{eff}, E_0 and the Gauss check on both lattices ($< 1\%$ vs v1); runs the densified 2×2 sweep with Spearman + permutation statistics and the spectral-gap covariate; and

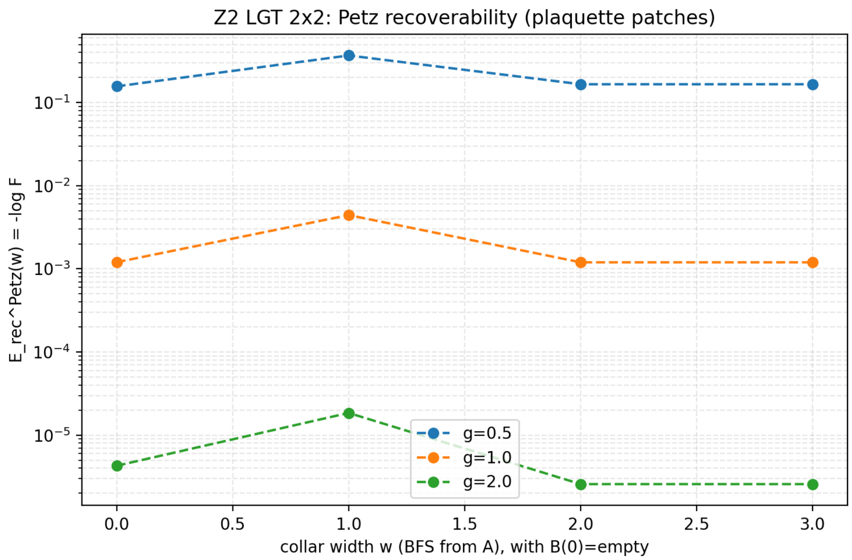

Figure 1: $E_{\mathrm{rec}}^{\mathrm{Petz}}(w)$ on the 2×2 lattice (v1 figure, unchanged).

documents the prescription-dependence findings of Remark 3.3. The v1 appendix script is retained (its internal working title in a code comment is removed in v2).

5 Discussion

The primary observation stands and is now statistically grounded: on small genuine-2D $\mathbb{Z}_2$ lattices, fixed-collar Petz recoverability error grows monotonically with the confinement-length proxy $\sigma_{\mathrm{eff}}^{-1}$ ($\rho = +1$, $p < 10^{-4}$). What v2 adds in candor: the same data are equally well explained by gap physics (Remark 3.2), absolute magnitudes are prescription-dependent (Remark 3.3), and the ladder null of the companion is about a different observable. Next steps: larger lattices (tensor networks), fixed-coupling geometry sweeps to break the σ–gap degeneracy, and the algebraic (sector-wise) prescription of [6] to tame embedding dependence.

Note added in v2

No v1 number is changed; all figures and Table 1 stand. v2 adds: (1) densified statistics upgrading the three-point trend to a significant rank correlation (Remark 3.1); (2) the spectral-gap covariate showing confinement specificity is unresolved at these sizes (Remark 3.2); (3) the prescription-dependence finding for $E_{\mathrm{rec}}^{\mathrm{Petz}}$ magnitudes and the 2×3 saturation (Remark 3.3); (4) reconciliation with the ladder-level null of 2601.0044 v2; (5) series positioning (v1 referred to "companion work" without numbers) and the verification suite; (6) editorial fixes: the stray placeholder in v1's Section 2.2 heading and the internal working title leaked in the v1 appendix comment and filenames.

References

[1] J. B. Kogut, Rev. Mod. Phys. **51** (1979) 659–713.

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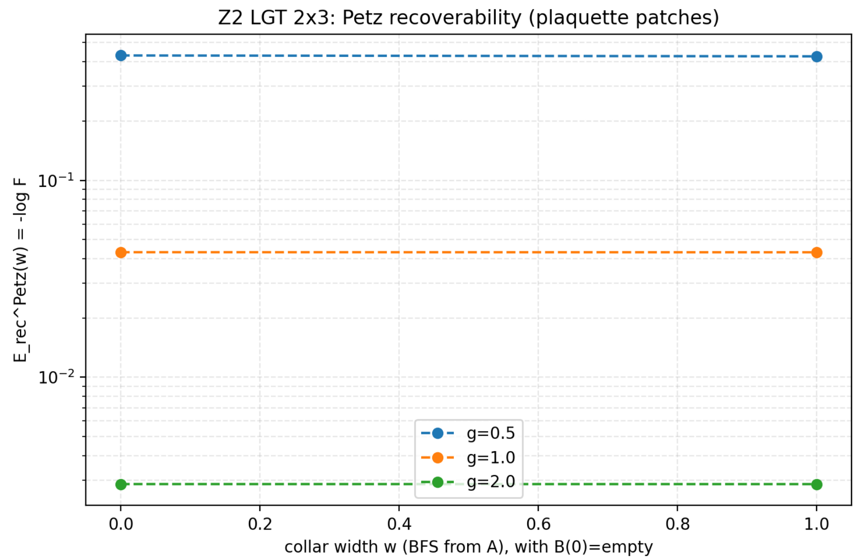

Figure 2: $E_{\rm rec}^{\rm Petz}(w)$ on the 2×3 lattice (v1 figure, unchanged; see Remark 3.3 for prescription sensitivity of the saturation).

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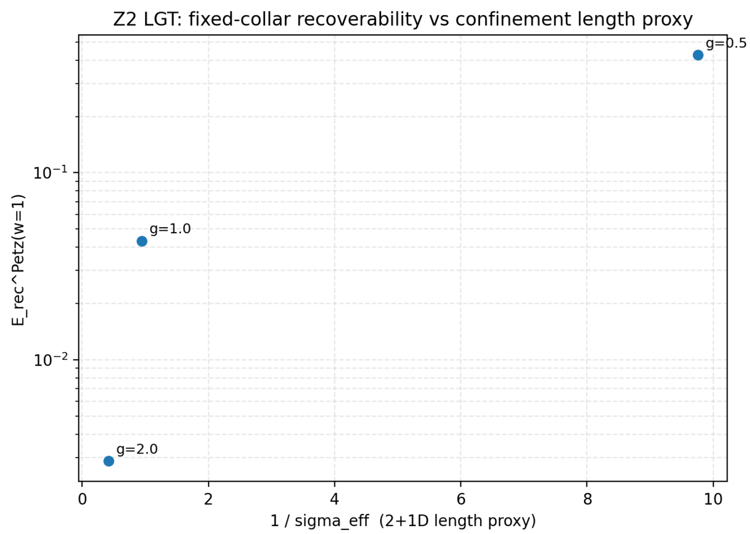

Figure 3: Fixed-collar recoverability vs confinement-length proxy (v1 figure, unchanged; statistics upgraded in Remark 3.1, specificity control in Remark 3.2).