Recoverability Length Scales and Wilson Loops in Lattice Gauge Theories

Protocol, definitions, and conjectural links to confinement diagnostics

Version 2

Lluis Eriksson
Independent Researcher
lluiseriksson@gmail.com

January 2026 (v1); July 2026 (v2)

Abstract

We propose a numerical protocol and falsifiable conjectures relating quantum-information recoverability measures to confinement diagnostics in lattice gauge theories. For a tripartition A–B–C and collar width w, we define a Petz-type recovery error $E_{\mathrm{rec}}^{\mathrm{Petz}}(w)$ and extract a recoverability length from threshold and fit criteria. Since gauge constraints obstruct naive factorization, the protocol is formulated in an extended-Hilbert-space (EHS) prescription by default, with an algebraic (gauge-invariant) variant outlined together with its subtleties (centers, sectors). We conjecture that $E_{\mathrm{rec}}^{\mathrm{Petz}}(w)$ decays exponentially in gapped phases and that its scale tracks confinement scales set by Wilson loops. *v2 executes the testbed that v1 only specified:* on a $\mathbb{Z}_2$ ladder of four plaquettes (14 links, Gauss law enforced exactly, $\langle G_v \rangle = 1$ to 10^{-6}), the suite computes $E_{\mathrm{rec}}^{\mathrm{Petz}}(w)$ and Wilson decay on the same ground states across five couplings. Findings: the pipeline runs end to end; ξ_{rec} is identifiable at three of five couplings and nearly coupling-independent (0.21–0.31, spread $\times 1.4$), while the Wilson decay length varies strongly (0.38–6.9, spread $\times 8.6$): *no tracking at ladder level.* Because height-one ladders degenerate area and perimeter, the Wilson scale there is not a pure confinement scale, so this first dataset is inconclusive-but-cautionary for the tracking conjecture rather than a falsification – and it sharpens the requirement: a genuine 2D lattice is needed. v2 further flags the TFIM control row shared with the companion framework note as unstable under regeneration, and adds series positioning. No v1 definition is changed.

1 Introduction, preliminaries, prescriptions, definitions

As in v1 (unchanged): squared Uhlmann fidelity and $E_{\mathrm{rec}} = -\log F$; EHS prescription as default (reduced states are ordinary density matrices; edge modes on cut links) with the algebraic prescription outlined (centers, sector-wise fidelity/recovery) [5, 6]; regularized Petz-type map (CP, trace-normalized output; used as an operational diagnostic), global $\delta = 10^{-12}$ with stability window. For self-containedness, the two central formulas: with $\rho_{B,\delta} = \rho_B + \delta \mathbf{1}$,

$$\mathcal{R}_{B\to BC}^{(\delta)}(X_B) = \rho_{BC}^{1/2}\big(\rho_{B,\delta}^{-1/2} X_B \rho_{B,\delta}^{-1/2} \otimes \mathbf{1}_C\big)\rho_{BC}^{1/2}, \qquad E_{\mathrm{rec}}^{\mathrm{Petz}}(w) = -\log F\big(\rho_{ABC},\, \hat{\rho}_{ABC}(w)\big),$$

where $\hat{\rho}_{ABC}(w)$ is the trace-normalized output of $(\mathrm{id}_A \otimes \mathcal{R}_{B\to BC}^{(\delta)})(\rho_{AB})$ and F is the squared Uhlmann fidelity. Recoverability profile, threshold length $d_{\mathrm{rec}}(\varepsilon)$ and rate ξ_{rec} as in v1; Wilson loops, effective string tension and Creutz ratios; the $\mathbb{Z}_2$ Kogut–Susskind testbed [4]; the conjectures (exponential recoverability in gapped phases; $\xi_{\rm rec}$ tracks confinement scales, $f(\sigma) \propto \sigma^{-1/2}$ in 3+1 by dimensional analysis); and the numerical protocol steps.

Series positioning (added in v2). This note is the gauge-theory extension of the recoverability-geometry framework 2601.0043 [7] (whose collaring-rule separation lesson and fixed-target design apply here), built on the $d_{\rm eff}$ mini-block 2601.0038/0040/ 0042 [8, 9, 10] and the pipeline of 2601.0035 [11]. The AQFT companion cited as [1] in v1 remains forthcoming.

2 Control experiment: TFIM validation (flag added in v2)

The TFIM control (Table 1 of v1; $L = 14$, $|A| = 3$, $|C| = 4$, complement traced, fits on $w \in \{2, 3, 4\}$) is *shared* with the companion 2601.0043, with slightly different $g = 0.5$ entries across the two papers ($\xi_{\rm rec} = 1.424$ here vs 1.433 there; $R^2 = 0.6712$ vs 0.6501) – v1 already attributed this to solver-level variability. The regeneration performed for 2601.0043 v2 ($\xi_{\rm rec} = 0.986$, $R^2 = 0.83$) confirms that the $g = 0.5$ row is unstable under platform/detail changes and, by the fit-window policy both papers share, it should not be quoted as a scale; the $g = 1.0$ and $g = 2.0$ rows are stable. The v1 appendix script (`tfim_reproduce.py`) is retained in the repository.

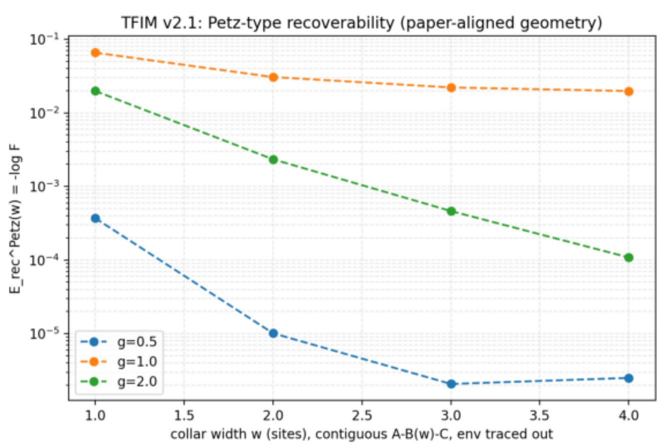

Figure 1: TFIM control (v1 figure, unchanged).

3 First execution of the $\mathbb{Z}_2$ testbed (added in v2)

v1 specified the gauge testbed but reported no gauge data. v2 executes a minimal instance: a ladder of $P = 4$ plaquettes (2×5 vertices, 14 links, qubits on links), $H(g) = -g \sum_\ell \sigma^x_\ell - \frac{1}{g} \sum_p \prod_{\ell \in \partial p} \sigma^z_\ell$, Gauss law enforced by a commuting penalty ($\lambda = 10$); ground states by sparse Lanczos; EHS reduced states by partial trace over links. *Recoverability:* A = leftmost rung (fixed), $B(w)$ = all links of the first w columns (admissible separation per the 2601.0043 v2 lesson), C = the remainder ($|C|$ varies – declared, as in the original $d_{\rm eff}$ design); $E^{\rm Petz}_{\rm rec}(w)$, $w \in \{1, \ldots, 4\}$, by exact pure-state low-rank evaluation. *Wilson decay:* $\langle W(1 \times R) \rangle$, $R \in \{1, \ldots, 4\}$; on a height-one ladder, area (R) and perimeter ($2R+2$) are degenerate, so the extracted ξ_W is the *combined* Wilson decay length, declared as the ladder proxy.

g	$\langle G_v \rangle_{\min}$	ξ_{rec} (n pre-floor)	ξ_W	comment
0.45	1.000000	0.215 (4)	6.851	
0.60	1.000000	0.231 (4)	2.437	
0.80	1.000000	— (2)	1.179	floor-limited
1.00	1.000000	0.308 (4)	0.798	
2.00	1.000000	— (2)	0.383	floor-limited (deep gap)

Remark 3.1 (What the first dataset shows). (i) The full pipeline – Gauss sector, EHS reduced states, regularized Petz, Wilson loops – runs end to end with exact gauge invariance. (ii) $E^{\text{Petz}}_{\text{rec}}(w)$ collapses to the numerical floor within one to two columns at all sampled couplings: the ladder recoverability length is very short ($\xi_{\text{rec}} \approx 0.2$–$0.3$ lattice units) and nearly coupling-independent (spread $\times 1.4$), while ξ_W varies by $\times 8.6$ over the same window. *At ladder level there is no tracking between the two scales.* (iii) Because of the area/perimeter degeneracy, ξ_W on the ladder is not a pure confinement scale (especially at $g < 1$), so this dataset is *inconclusive-but-cautionary* for the tracking conjecture, not a falsification. (iv) The concrete lesson for the program: a resolving test requires a genuine 2D lattice (e.g. $2 \times N$ or $3 \times N$ plaquette arrays), where Creutz ratios disentangle σ from perimeter terms and wider buffers postpone the numerical floor.

4 Verification suite (added in v2)

`verification/2601-0044/verify_2601_0044.py` (sparse Lanczos; chunked JSON cache; low-rank pure-state fidelities): Gauss-sector check, $E^{\text{Petz}}_{\text{rec}}(w)$ profiles, Wilson decays, ξ_{rec}/ξ_W extraction with the $n \geq 3$ identifiability policy, and the no-tracking summary of Remark 3.1. The v1 TFIM appendix script is kept alongside.

5 Discussion and outlook

As in v1, updated: the open directions (fully algebraic sector-wise implementation; comparison with string-tension proxies) now have a first empirical anchor – the executed ladder shows the protocol is sound and cheap, and locates precisely where it must grow (2D geometry, floor management, fixed-target variants per 2601.0043) before the tracking conjecture can be meaningfully tested.

Note added in v2

No v1 definition, conjecture or protocol step is changed. v2 adds: (1) the first execution of the $\mathbb{Z}_2$ testbed (Section 3), with exact Gauss invariance and the no-tracking-at-ladder-level finding, framed as inconclusive for the tracking conjecture due to the declared area/perimeter degeneracy; (2) the shared-control acknowledgment with 2601.0043 and the instability flag on the $g = 0.5$ TFIM row (confirmed by regeneration in the companion's v2); (3) series positioning (v1 cited only the forthcoming AQFT companion); (4) the verification suite.

References

[1] L. Eriksson, *Petz recoverability in AQFT via conditional expectations*, preprint (2026).

[2] D. Petz, Commun. Math. Phys. **105** (1986) 123–131.

[3] O. Fawzi and R. Renner, Commun. Math. Phys. **340** (2015) 575–611.

[4] J. B. Kogut, Rev. Mod. Phys. **51** (1979) 659.

[5] W. Donnelly, Phys. Rev. D **85** (2012) 085004; Class. Quantum Grav. **31** (2014) 214003.

[6] H. Casini, M. Huerta, and J. A. Rosabal, Phys. Rev. D **89** (2014) 085012.

[7] L. Eriksson, ai.viXra:2601.0043 (2026; v2 2026).

[8] L. Eriksson, ai.viXra:2601.0038 (2026; v2 2026).

[9] L. Eriksson, ai.viXra:2601.0040 (2026; v2 2026).

[10] L. Eriksson, ai.viXra:2601.0042 (2026; v2 2026).

[11] L. Eriksson, ai.viXra:2601.0035 (2026; v2 2026).