Emergent Information Distance from Petz Recovery

Temperature and perturbation dependence in TFIM exact diagonalization

Version 2

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Abstract

We define an operational notion of effective distance from approximate quantum state recovery. Given a tripartition A–B–C with B a collar of width w separating A from C, we compute a Petz recovery reconstruction error $E_{\mathrm{Petz}}(w) = -\log F(\rho_{ABC}, \tilde{\rho}^{\mathrm{Petz}}_{ABC}(w))$ (squared Uhlmann fidelity) and define an emergent distance $d_{\mathrm{eff}}(\varepsilon)$ as the minimal collar width such that the best-achieved error up to w falls below a threshold ε. Using exact diagonalization for the transverse-field Ising chain at $N = 11$, $h_x = 1.05$, $|A| = 2$, we find that $d_{\mathrm{eff}}(10^{-3})$ grows strongly with inverse temperature β in the unperturbed case ($h_z = 0$), from 1.00 at $\beta = 0.5$ to 3.57 at $\beta = 5.0$, while remaining near-minimal in the longitudinally perturbed case ($h_z = 0.5$), close to 1.0 across the same range. We also introduce a discrete curvature diagnostic based on second differences of $\log E_{\mathrm{Petz}}(w)$ on a pre-floor window, reported only when identifiable. *v2 (no v1 number is changed):* the garbled reproducibility paragraph of v1 is replaced by a real, regenerable verification suite, which reproduces the β-sweep to two decimals already at $N = 9$ ($d_{\mathrm{eff}} = 1.00, 1.00, 1.51, 2.11, 2.82, 3.55$ vs 3.57 at $N = 11$; μ_{prefloor} endpoints $8.44 \to 1.35$ vs 1.33; PSD-projection sensitivity 3.3×10^{-8} vs $\sim 3 \times 10^{-8}$) – independently confirming the finite-size robustness of the appendix; the mild $|C|$-shrinkage caveat is stated with cross-references to its quantified analysis in the companions; and series positioning is added.

1 Introduction

Motivated by the theme that locality and entanglement structure constrain information flow, we extract a geometry-like diagnostic directly from approximate recoverability: recovery performance is the primitive observable, and an effective distance is the minimal separation needed to reach a fixed recovery-error threshold. Scope: we do not attempt to derive gravitational field equations; the goal is a minimal, falsifiable construction of an emergent geometric *diagnostic* – an operational quantity depending on the threshold, temperature, protocol and geometry, not a universal length.

Series positioning (added in v2). This note completes the d_{eff} mini-block of the recovery program: 2601.0038 [5] introduced d_{eff} and its criticality signature in h_x; 2601.0040 [6] studied its finite-size scaling (whose $|C|$/baseline and functional-form caveats apply mutatis mutandis here); the present note fixes $h_x = 1.05$ and varies (β, h_z). The ED/Petz pipeline and censored-reporting discipline are those of 2601.0035 [7], with the non-Gaussian bridge upstream in 2512.0101 [8].

2 Setup and definitions

As in the companions: $E_{\mathrm{Petz}}(w) := -\log F(\rho_{ABC}, \tilde{\rho}^{\mathrm{Petz}}_{ABC}(w))$, $F(\rho,\sigma) = (\mathrm{Tr}\sqrt{\sqrt{\rho}\,\sigma\sqrt{\rho}})^2$; $\mathcal{R}^{\mathrm{Petz}}_{B\to BC}(X_B) = \rho_{BC}^{1/2}\rho_B^{-1/2}X_B\rho_B^{-1/2}\rho_{BC}^{1/2}$ with spectral floor $\lambda_{\min} = 10^{-12}$ applied only in $\rho_B^{-1/2}$; hermitization, PSD projection and trace renormalization applied to the output (direct check: this post-processing changes E_{Petz} by at most $\sim 3\times10^{-8}$; the suite reproduces 3.3×10^{-8}); $E_{\mathrm{best}}(w) := \min_{w'\le w} E_{\mathrm{Petz}}(w')$; $d_{\mathrm{eff}}(\varepsilon) := \min\{w : E_{\mathrm{best}}(w) \le \varepsilon\}$ with log-linear interpolation. Pre-floor window $E_{\mathrm{Petz}}(w) \ge 10\, y_{\min}$ with $y_{\min} = 10^{-12}$ (distinct from $\lambda_{\min}$); discrete curvature $\kappa(w)$ by second differences of $\log E_{\mathrm{Petz}}$, summarized by $\bar{\kappa}$ only when at least three consecutive widths are pre-floor.

Remark 2.1 ($|C|$ caveat (added in v2)). Here $w \le 5$ at $N = 11$, so $|C| \ge 4$ and the $|C|$-shrinkage confound quantified in 2601.0038/0040 v2 is mild; nevertheless, absolute d_{eff} values inherit the tripartition geometry, and all claims below are regime *comparisons* at fixed geometry ($h_z = 0$ vs 0.5 at the same $(N, |A|, w)$), which are unaffected.

3 Data and methodology

As in v1: $H = -J\sum Z_iZ_{i+1} - h_x\sum X_i - h_z\sum Z_i$, open boundaries, $J = 1$, $h_x = 1.05$, $h_z \in \{0, 0.5\}$; Gibbs states by ED; $A = \{1,2\}$, $B = \{3,\dots,2+w\}$, C the rest; $w \in \{1,\dots,5\}$ at $N = 11$; pre-floor fits $\log E_{\mathrm{Petz}}(w) \approx a - \mu_{\mathrm{prefloor}} w$ with identifiability reported via $(n_{\mathrm{prefloor}}, n_\kappa)$.

4 Results

All v1 numbers stand. **Effective distance vs temperature:** at $\varepsilon = 10^{-3}$, $d_{\mathrm{eff}} = 1.00, 1.00, 1.51, 2.11, 2.82, 3.57$ for $h_z = 0$ and $1.00, 1.00, 1.00, 1.00, 1.02, 1.04$ for $h_z = 0.5$ ($\beta = 0.5, \dots, 5$); Figures 1–3. **Threshold sensitivity:** the regime ordering $d_{\mathrm{eff}}|_{h_z=0} \ge d_{\mathrm{eff}}|_{h_z=0.5}$ holds for $\varepsilon \in \{10^{-2}, 10^{-3}, 10^{-4}\}$ across all sampled β. **Decay proxy and curvature:** μ_{prefloor} falls from ≈ 8.44 ($\beta = 0.5$) to ≈ 1.33 ($\beta = 5$) at $h_z = 0$ while staying in $\approx [4.16, 8.56]$ at $h_z = 0.5$; $\bar{\kappa}$ is identifiable only at $h_z = 0$, $\beta \ge 1$ ($(n_{\mathrm{prefloor}}, n_\kappa) \in \{(3,1),(5,3)\}$; e.g. $\bar{\kappa} \approx 1.97$ at $\beta = 1$, 0.44 at $\beta = 3$), and undefined at $h_z = 0.5$ ($n_{\mathrm{prefloor}} = 2$) – by policy, reported as "—".

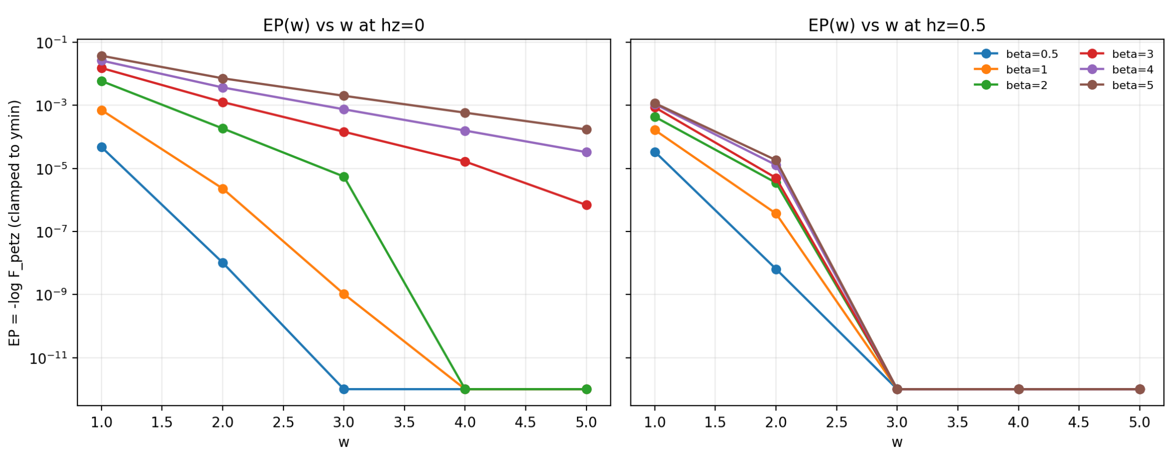

Figure 1: $E_{\mathrm{Petz}}(w)$ vs w (log scale, clamped at $y_{\min} = 10^{-12}$). Left: $h_z = 0$; right: $h_z = 0.5$ (v1 figure, unchanged).

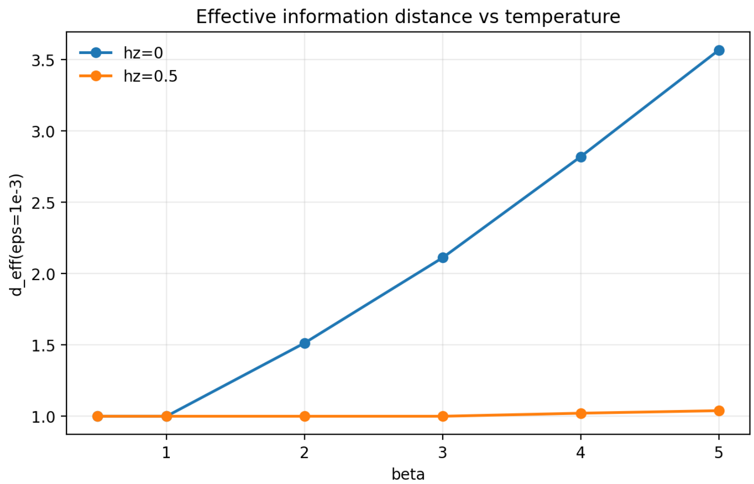

Figure 2: $d_{\text{eff}}(10^{-3})$ vs β for both regimes (v1 figure, unchanged).

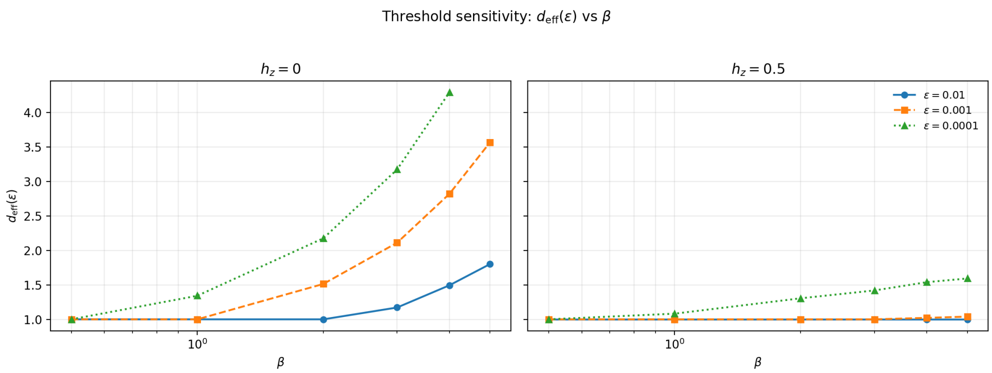

Figure 3: Threshold sensitivity: $d_{\text{eff}}(\varepsilon)$ vs β, $\varepsilon \in \{10^{-2}, 10^{-3}, 10^{-4}\}$ (v1 figure, unchanged).

5 Verification suite (rewritten in v2)

v1's reproducibility paragraph was corrupted in production (a garbled reference to a patched driver `run_edp_sdproj.py`) and referred vaguely to CSV artifacts "possibly included" in an Overleaf bundle. v2 replaces it: `verification/2601-0042/verify_2601_0042.py` (regenerable ED, chunked JSON cache, default $N = 9$, `-N11` for the paper size) recomputes the entire pipeline from scratch and checks: the β-growth of d_{eff} at $h_z = 0$ and near-minimality at $h_z = 0.5$ (reproducing the v1 values to two decimals already at $N = 9$); the regime ordering across all three thresholds; the decrease of μ_{prefloor} with β; the $\bar{\kappa}$ identifiability policy; and the PSD-projection sensitivity bound. The close $N = 9$ vs $N = 11$ agreement independently confirms the finite-size robustness check of the appendix ($N = 11$ vs 12).

6 Limitations and outlook

As in v1, sharpened: d_{eff} is an operational diagnostic – threshold-, temperature-, protocol- and geometry-dependent – not a universal correlation length; the connection of its β-growth at $h_x = 1.05$ to the near-critical ground state (and its suppression by a symmetry-breaking h_z) is physically natural and consistent with the h_x-signature of 2601.0038, but establishing scaling requires the fixed-$|C|$ protocols and larger sizes discussed in 2601.0040 v2. Comparing to CMI-based diagnostics (cf. 2601.0038 v2) is a natural next step.

A Finite-size robustness (v1 figure, unchanged)

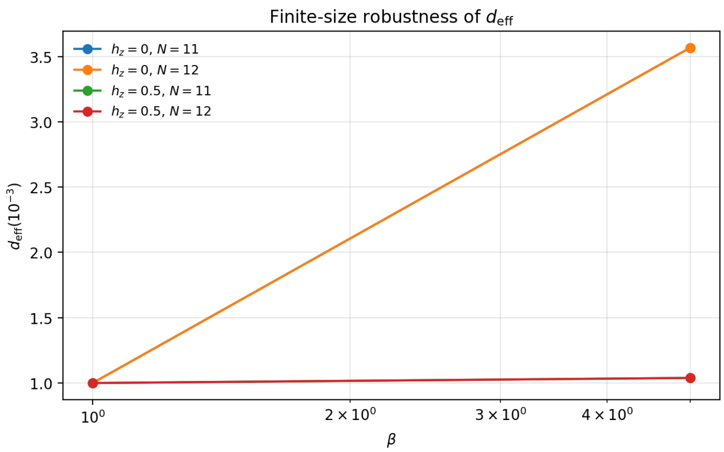

Figure 4: $N = 11$ vs $N = 12$ robustness check from v1; independently corroborated by the $N = 9$ reproduction of the suite.

Note added in v2

No v1 number is changed; all figures, tables and the appendix stand, and the suite reproduces the headline numbers to two decimals at $N = 9$. Changes: (1) the corrupted reproducibility paragraph of v1 is replaced by a real verification suite; (2) the mild $|C|$ caveat is stated with cross-references to its quantified analysis in 2601.0038/0040 v2; (3) series positioning added (v1 cited no companion papers despite sharing definitions verbatim with 2601.0038); (4) the "emergent distance" reading is explicitly framed as an operational diagnostic.

References

[1] O. Fawzi and R. Renner, Commun. Math. Phys. **340** (2015) 575–611.

[2] M. Junge et al., Ann. Henri Poincaré **19** (2018) 2955–2978.

[3] D. Petz, Commun. Math. Phys. **105** (1986) 123–131.

[4] S. Sachdev, *Quantum Phase Transitions*, 2nd ed., Cambridge University Press (2011).

[5] L. Eriksson, ai.viXra:2601.0038 (2026; v2 2026).

[6] L. Eriksson, ai.viXra:2601.0040 (2026; v2 2026).

[7] L. Eriksson, ai.viXra:2601.0035 (2026; v2 2026).

[8] L. Eriksson, ai.viXra:2512.0101 (2025; v2 2026).