Operational Signatures of Criticality from Petz Recovery

Collar-length requirements in TFIM exact diagonalization

Version 2

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Abstract

We study whether recovery-based operational distances exhibit a distinctive finite-size signature near quantum criticality. For a tripartition A–B–C of a 1D chain with a collar B of width w separating A from C, we compute a Petz-based reconstructed state and the recovery error $E_{\mathrm{Petz}}(w) = -\log F$ (squared Uhlmann fidelity), and define an effective recovery distance $d_{\mathrm{eff}}(\varepsilon)$ as the minimal collar width achieving error below ε, stabilized by $E_{\mathrm{best}}(w) = \min_{w' \le w} E_{\mathrm{Petz}}(w')$ and reported with explicit censoring. Using exact diagonalization of the transverse-field Ising chain at $N = 11$ with $|A| = 2$, we sweep h_x across the critical region at $h_z = 0$ and compare to a longitudinally perturbed control $h_z = 0.5$: pronounced growth and extensive censoring of $d_{\mathrm{eff}}(\varepsilon)$ appear in the critical region at low temperature, while the control remains comparatively featureless; an extended-collar spot-check yields $d_{\mathrm{eff}}(10^{-3}) \approx 7.6$–$7.7$ at $\beta = 12$ near $h_x \in \{0.96, 1.00\}$. *v2 adds (no v1 result is changed):* an explicit $|C|$-shrinkage caveat – at fixed N, growing w also shrinks C, so the *absolute* scale of d_{eff} near $w_{\max}$ conflates buffer growth with a shrinking reconstruction target, while fixed-geometry comparisons across h_x (the criticality signature) are unaffected; a window-relativity remark (ε, β, N jointly set what is resolvable: at smaller N the $\beta = 12$, $\varepsilon = 10^{-3}$ window censors even off-critical points, consistent with v1's own zoom); delivery of v1's "future work" item: a CMI-based distance computed on the same sweep shows the *same* criticality signature at its own threshold (CMI decays about half as fast as the Petz error, cf. 2601.0035); series positioning; and a fully regenerable verification suite.

1 Introduction

Approximate quantum state recovery provides an operational way to quantify how much information about a subsystem can be reconstructed from a surrounding collar. We test whether recovery performance exhibits a characteristic finite-size signature when a lattice model is tuned through a quantum critical region. We do not claim a thermodynamic divergence at finite size: the object is a finite-N, finite-β diagnostic – whether the collar length required for a fixed recovery threshold grows sharply near the expected critical point and differs from an explicitly perturbed control.

Series positioning (added in v2). This note extends the non-Gaussian recovery block of the series to criticality: the ED pipeline, model family and censored-fit discipline are those of 2601.0035 [3] (whose corrected FR scale also calibrates how to read $-\log F$ against CMI); the crossover-width diagnostic w^* of 2512.0101 v2 [4] is the cousin of d_{eff}; the upstream program is 2512.0060 [5] with Type III capstone 2601.0034 [6].

2 Setup and definitions

As in v1: $E_{\text{Petz}}(w) := -\log F(\rho_{ABC}, \tilde{\rho}^{\text{Petz}}_{ABC}(w))$ with $F(\rho,\sigma) = (\operatorname{Tr}\sqrt{\sqrt{\rho}\,\sigma\sqrt{\rho}})^2$; $E_{\text{best}}(w) := \min_{w' \leq w} E_{\text{Petz}}(w')$; $d_{\text{eff}}(\varepsilon) := \min\{w : E_{\text{best}}(w) \leq \varepsilon\}$ with log-linear interpolation; censoring $d_{\text{eff}}(\varepsilon) > w_{\max}$ reported explicitly, always against the *effective* $w_{\max}$ actually evaluated.

Remark 2.1 ($|C|$-shrinkage caveat (added in v2)). At fixed N and $|A|$, increasing w necessarily shrinks $|C| = N - |A| - w$: at the extended-collar spot-check values $w = 7, 8$ with $N = 11$, the reconstruction target C has only $2, 1$ sites. Reconstructing a smaller C is intrinsically easier, so the *absolute* reading "operational recovery length ≈ 8 sites" conflates genuine buffer-length requirements with a shrinking target and should be treated as an upper-bound-flavored diagnostic at this system size. The *signature* claims are unaffected: all critical vs off-critical vs control comparisons are made at fixed $(N, |A|, w)$ geometry, where $|C|$ is identical across the compared points. A fixed-$|C|$ protocol (growing N with w) is the natural sequel and is listed in the outlook.

3 Model and numerics

As in v1: $H = -J\sum Z_i Z_{i+1} - h_x \sum X_i - h_z \sum Z_i$, open boundaries, $J = 1$, $h_z \in \{0, 0.5\}$, Gibbs states by ED, $|A| = 2$ at the left edge, coarse sweep $h_x \in [0.7, 1.3]$ and zoom $[0.95, 1.05]$, $\beta \in \{4, 8, 12\}$.

4 Results

All v1 results stand unchanged: Figure 1 (coarse sweep: growth and censoring near $h_x = 1$ at $h_z = 0$, featureless control at $h_z = 0.5$); the zoom behavior (at $N = 11$: $\beta = 12$, $\varepsilon = 10^{-3}$ fully censored within $w_{\max} = 7$ across the zoom window; $\beta = 8$ uncensored; $\varepsilon = 10^{-4}$ censored for both); and the extended-collar spot-check $d_{\text{eff}}(10^{-3}) \approx 7.710$ at $h_x = 0.96$, ≈ 7.627 at $h_x = 1.00$ ($\beta = 12$, effective range $w \leq 8$; see Remark 2.1 for how to read the absolute value).

Remark 4.1 (Window relativity (added in v2)). What is resolvable depends jointly on (ε, β, N). The regenerated benchmark at $N = 9$ makes this concrete: at $\beta = 12$, $\varepsilon = 10^{-3}$, *every* point of the coarse sweep censors, including far off-critical ones (the off-critical correlation length already exceeds the accessible window at that threshold and size) – consistent with v1's own fully-censored zoom at $\beta = 12$. The critical/off-critical contrast reappears at $\varepsilon = 10^{-2}$: $d_{\text{eff}} = 3, 5, 5, 5, 4, 3$ across $h_x = 0.7, \ldots, 1.3$ at $h_z = 0$, with the $h_z = 0.5$ control flat at $d_{\text{eff}} = 1$, and is attenuated at $\beta = 4$ (flat $d_{\text{eff}} = 2$). Signature claims are therefore always window-relative statements, as v1's finite-N/finite-β framing intended.

5 CMI companion diagnostic (added in v2)

v1 listed comparing d_{eff} to CMI-based diagnostics as future work; the suite delivers it on the same sweep. Define $d^{\text{CMI}}_{\text{eff}}(\varepsilon_I)$ by replacing E_{best} with $I_\rho(A : C|B)$ (which is monotone decreasing in w up to numerical noise). Because CMI decays about half as fast as the Petz error ($\mu_{\text{Petz}} \approx 2\mu_{\text{CMI}}$ in this model family, cf. 2601.0035 [3]), it needs its own threshold; choosing the first ε_I that resolves both off-critical endpoints ($\varepsilon_I = 3 \times 10^{-2}$ at $N = 9$, $\beta = 12$) gives $d^{\text{CMI}}_{\text{eff}} = 3, {>}5, {>}5, {>}5, 4, 3$ across $h_x = 0.7, \ldots, 1.3$ at $h_z = 0$ – the same signature as the Petz distance, if anything sharper – with the $h_z = 0.5$ control flat at 1. The criticality signature is thus not an artifact of the Petz construction: it is visible directly in the information-theoretic quantity that upper-bounds optimal recovery.

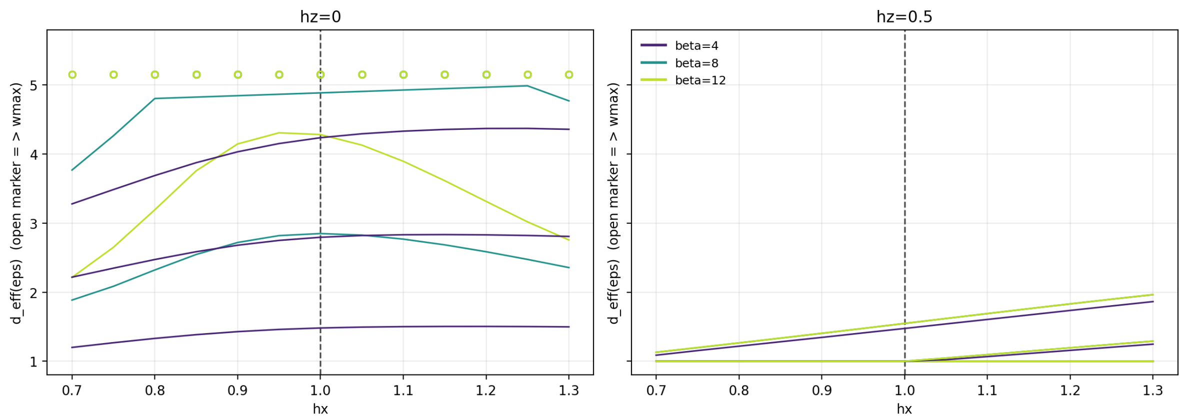

Figure 1: Coarse sweep of $d_{\text{eff}}(\varepsilon)$ vs h_x for $h_z = 0$ and $h_z = 0.5$ at $N = 11$, $|A| = 2$ (v1 figure, unchanged). Open markers: censored ($d_{\text{eff}} > w_{\max} = 5$). Dashed line: $h_x = 1$.

6 Verification suite (added in v2)

`verification/2601-0038/verify_2601_0038.py` regenerates the pipeline from scratch (ED; default $N = 9$, `-N11` for the paper size; chunked with a JSON cache for constrained environments): the Petz criticality signature at its resolvable window (Remark 4.1), the flat control, the CMI companion signature (Section 5), the $\beta = 4$ attenuation, the all-censored $\beta = 12$, $\varepsilon = 10^{-3}$ window, $|C|$ per w reported explicitly, and monotonicity of E_{best}.

7 Discussion and outlook

At finite size and temperature the TFIM critical point manifests as a broadened crossover; the recovery distance d_{eff} grows and censors there while the perturbed control stays featureless, and the same signature appears in the CMI diagnostic. Future work: fixed-$|C|$ protocols (growing N with w, addressing Remark 2.1), finite-size scaling in N, tensor-network extension of the collar window.

Note added in v2

No v1 result is changed: all figures, tables and numbers stand. v2 adds what a careful reader needs to weigh them: (1) the $|C|$-shrinkage caveat (Remark 2.1) qualifying the absolute "≈ 8 sites" reading while leaving the signature intact; (2) the window-relativity remark (Remark 4.1), grounded in the regenerated $N = 9$ benchmark; (3) the CMI companion diagnostic (Section 5), delivering v1's future-work item and showing the signature is not Petz-specific; (4) series positioning (v1 cited no companion papers) and a fully regenerable verification suite.

References

[1] D. Petz, Commun. Math. Phys. **105** (1986) 123–131.

[2] S. Sachdev, *Quantum Phase Transitions*, 2nd ed., Cambridge University Press (2011).

[3] L. Eriksson, ai.viXra:2601.0035 (2026; v2 2026).

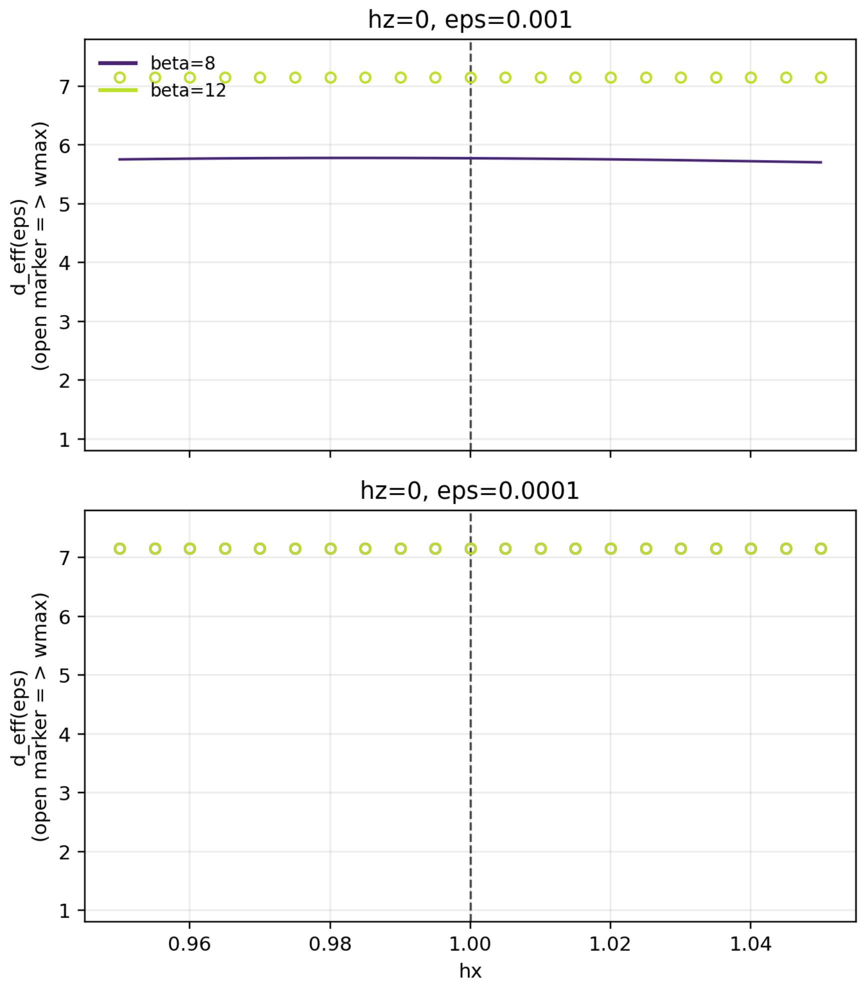

Figure 2: Zoom sweep at $h_z = 0$, $N = 11$, $\beta \in \{8, 12\}$, $\varepsilon \in \{10^{-3}, 10^{-4}\}$, $w_{\max} = 7$ (v1 figure, unchanged).

[4] L. Eriksson, ai.viXra:2512.0101 (2025; v2 2026).

[5] L. Eriksson, ai.viXra:2512.0060 (2025; v2 2026).

[6] L. Eriksson, ai.viXra:2601.0034 (2026; v2 2026).