Operational signatures of criticality from Petz recovery
Collar-length requirements in TFIM exact diagonalization

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Abstract

We study whether recovery-based operational distances exhibit a distinctive finite-size signature near quantum criticality. For a tripartition A–B–C of a 1D chain with a collar B of width w separating A from C, we compute a Petz-based reconstructed state $\widetilde{\rho}_{ABC}^{\mathrm{Petz}}(w)$ and the corresponding recovery error $E_{\mathrm{Petz}}(w) = -\log F(\rho_{ABC}, \widetilde{\rho}_{ABC}^{\mathrm{Petz}}(w))$ (squared Uhlmann fidelity). We define an effective recovery distance $d_{\mathrm{eff}}(\varepsilon)$ as the minimal collar width needed to achieve error below a target threshold ε, using $E_{\mathrm{best}}(w) = \min_{w' \leq w} E_{\mathrm{Petz}}(w')$ to stabilize non-monotonicity. Using exact diagonalization of the transverse-field Ising chain at $N = 11$ with $|A| = 2$, we sweep the transverse field h_x across the critical region at $h_z = 0$ and compare to a longitudinally perturbed control $h_z = 0.5$. We find a pronounced growth and extensive censoring of $d_{\mathrm{eff}}(\varepsilon)$ in the critical region at low temperature, while the perturbed control remains comparatively featureless. An extended-collar spot-check partially resolves the censoring and yields $d_{\mathrm{eff}}(10^{-3}) \approx 7.6$–$7.7$ at $\beta = 12$ near $h_x \in \{0.96, 1.00\}$, indicating an operational recovery length scale of order eight lattice sites at this system size.

Contents

1   Introduction

Approximate quantum state recovery provides an operational way to quantify how much information about a subsystem can be reconstructed from a surrounding collar. Motivated by the general link between locality, correlation length, and recoverability, we test whether recovery performance exhibits a characteristic finite-size signature when a lattice model is tuned through a quantum critical region.

   We do not claim a thermodynamic divergence at finite size. Instead, we focus on a finite-N, finite-β diagnostic: whether the collar length required to achieve a fixed recovery threshold grows sharply near the expected critical point and differs from an explicitly perturbed control.

2   Setup and definitions

We consider a 1D chain and a tripartition A–B–C, where B is a collar of width w separating A from C. For each w we form a Petz-based reconstructed state $\widetilde{\rho}_{ABC}^{\text{Petz}}(w)$ by applying the Petz map [1] to the input marginal ρ_{AB}, using ρ_{BC} (and hence ρ_B) extracted from the same target state ρ_{ABC}. We define

$$E_{\text{Petz}}(w) := -\log F\big(\rho_{ABC}, \widetilde{\rho}_{ABC}^{\text{Petz}}(w)\big), \qquad F(\rho, \sigma) := \left(\text{Tr}\sqrt{\sqrt{\rho}\,\sigma\,\sqrt{\rho}}\right)^2.$$

   To stabilize non-monotonic finite-size behavior, define

$$E_{\text{best}}(w) := \min_{w' \leq w} E_{\text{Petz}}(w').$$

For a target threshold $\varepsilon > 0$, define the effective distance

$$d_{\text{eff}}(\varepsilon) := \min\{w : \ E_{\text{best}}(w) \leq \varepsilon\},$$

with log-linear interpolation between adjacent w values when needed.

Censoring.   All runs have a finite accessible collar window. When $E_{\text{best}}(w_{\max}) > \varepsilon$, we report the distance as censored:

$$d_{\text{eff}}(\varepsilon) > w_{\max}.$$

Effective $w_{\max}$ in practice.   The numerical pipeline accepts a requested maximum collar width, but the actually evaluated maximum depends on the tripartition geometry and system size. We therefore report the effective maximum collar width actually reached in each run (for example, in an extended-collar spot-check at $N = 11$ we obtain data up to $w = 8$). When quoting censoring thresholds we always use the effective $w_{\max}$ actually evaluated in that run.

3   Model and numerics

We analyze the transverse-field Ising chain with open boundary conditions [2]:

$$H = -J \sum_{i=1}^{N-1} Z_i Z_{i+1} - h_x \sum_{i=1}^{N} X_i - h_z \sum_{i=1}^{N} Z_i,$$

with $J = 1$ and $h_z \in \{0, 0.5\}$. We fix $|A| = 2$ at the left edge and vary the collar width $w \in \{1, \ldots, w_{\max}\}$.

   We consider Gibbs states $\rho \propto e^{-\beta H}$ obtained by exact diagonalization. For each parameter setting we evaluate $E_{\text{Petz}}(w)$ for all accessible w values.

Parameter sweeps. We sweep h_x over a coarse range $[0.7, 1.3]$ and a zoom range $[0.95, 1.05]$ at $\beta \in \{4, 8, 12\}$. The critical point in the thermodynamic limit at $h_z = 0$ is at $h_x/J = 1$, but at finite size and finite temperature we expect a broadened crossover region and possible pseudocritical shifts.

4 Results

4.1 Coarse sweep: $h_z = 0$ versus $h_z = 0.5$

Figure 1 shows $d_{\text{eff}}(\varepsilon)$ across a coarse sweep in h_x for an integrable case ($h_z = 0$) and a longitudinally perturbed control ($h_z = 0.5$). We mark censored points ($d_{\text{eff}}(\varepsilon) > w_{\text{max}}$) explicitly.
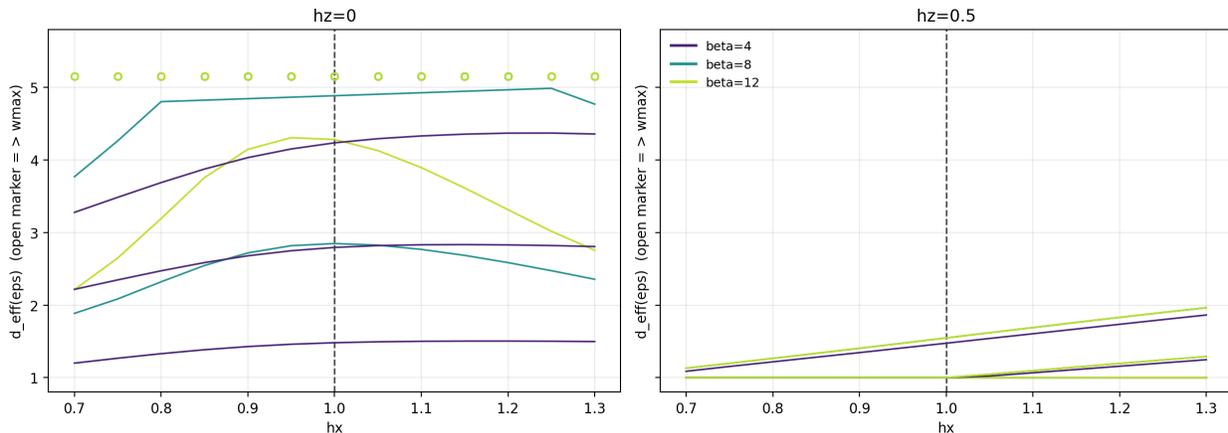

Figure 1: Coarse sweep of the recovery distance $d_{\text{eff}}(\varepsilon)$ versus transverse field h_x for $h_z = 0$ and $h_z = 0.5$ at $N = 11$ and $|A| = 2$. Open markers indicate censoring, i.e. $d_{\text{eff}}(\varepsilon) > w_{\text{max}}$ (here $w_{\text{max}} = 5$). The dashed line marks $h_x = 1$.

4.2 Zoom: resolving censoring by extending the collar window

At low temperature the critical-region distance can exceed the accessible collar window. For $N = 11$ and $\varepsilon = 10^{-3}$, we find that at $\beta = 12$ the distance remains fully censored within $w_{\text{max}} = 7$ throughout $0.95 \leq h_x \leq 1.05$, i.e. $d_{\text{eff}}(10^{-3}) > 7$ across the full zoom window. At $\beta = 8$, the same threshold $\varepsilon = 10^{-3}$ is uncensored throughout the zoom window, i.e. $d_{\text{eff}}(10^{-3}) \leq 7$ for all sampled $h_x \in [0.95, 1.05]$, while the stricter threshold $\varepsilon = 10^{-4}$ remains fully censored for both $\beta = 8$ and $\beta = 12$ ($d_{\text{eff}}(10^{-4}) > 7$) throughout $0.95 \leq h_x \leq 1.05$.

4.3 Extended-collar spot-check

To confirm that the censoring reflects a genuine increase in the operational recovery length scale (rather than a plotting artifact), we perform an extended-collar spot-check at $\beta = 12$ near the critical region. Since $d_{\text{eff}}(\varepsilon)$ is defined from $E_{\text{best}}(w)$, extending the accessible collar window can only decrease censoring. In this run the effective collar range reaches $w \leq 8$, and the threshold $\varepsilon = 10^{-3}$ becomes resolvable:

$$d_{\text{eff}}(10^{-3}) \approx 7.710 \text{ at } h_x = 0.96, \qquad d_{\text{eff}}(10^{-3}) \approx 7.627 \text{ at } h_x = 1.00.$$
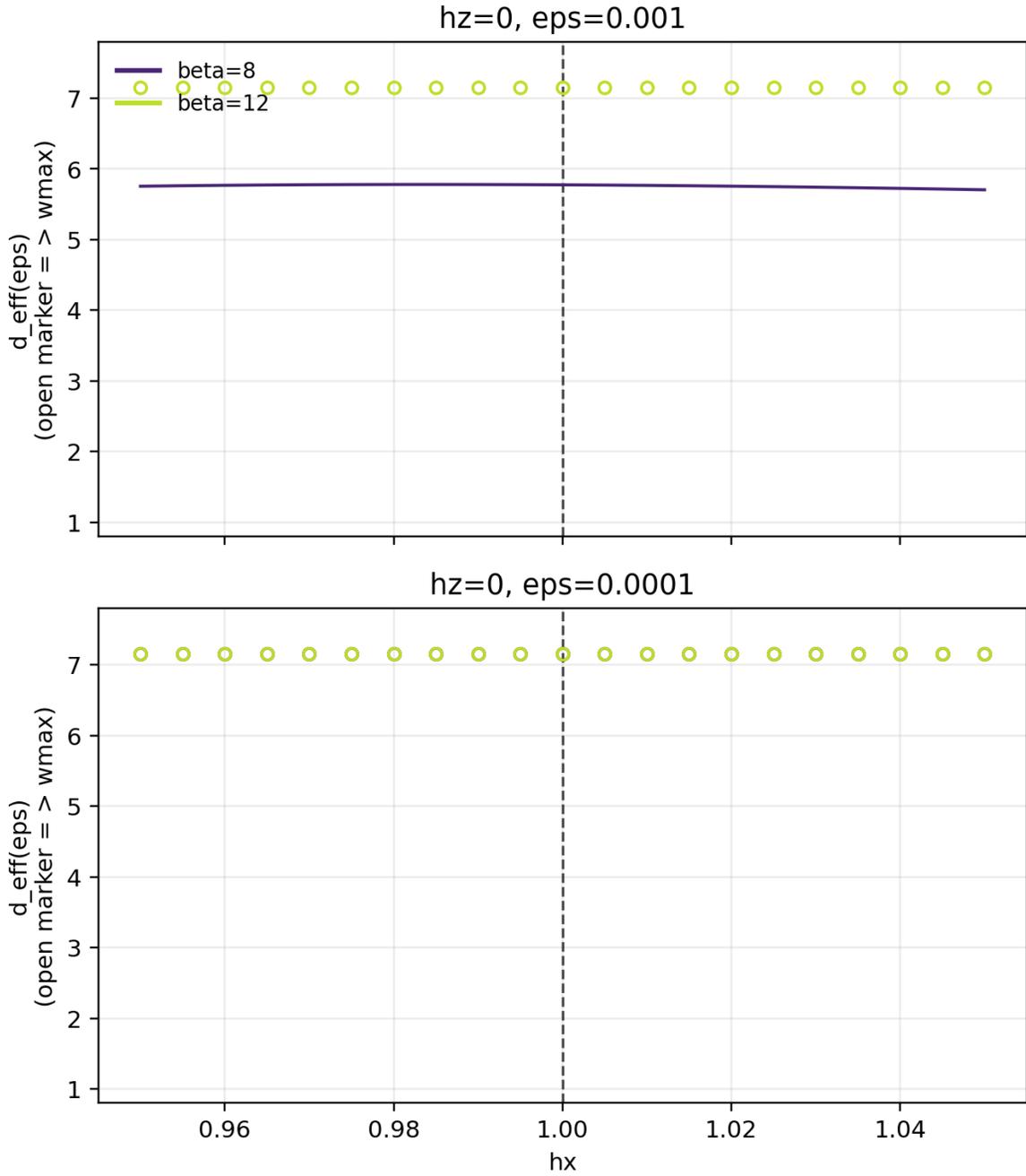
Figure 2: Zoom sweep in the integrable case ($h_z = 0$) at $N = 11$, showing $d_{\text{eff}}(\varepsilon)$ versus h_x for $\beta \in \{8, 12\}$ and thresholds $\varepsilon \in \{10^{-3}, 10^{-4}\}$ with an extended collar window up to $w_{\max} = 7$. Open markers indicate censoring ($d_{\text{eff}}(\varepsilon) > 7$).

N	h_z	β	ε	h_x	$d_{\text{eff}}(\varepsilon)$
11	0.0	12	10^{-3}	0.96	7.710
11	0.0	12	10^{-3}	1.00	7.627

Table 1: Extended-collar spot-check in the critical region. In this run the effective collar range reached $w \leq 8$ (see text).

5   Discussion and outlook

At finite size and finite temperature, the TFIM critical point manifests as a broadened crossover region. We find that the recovery-based distance $d_{\text{eff}}(\varepsilon)$ exhibits strong growth and censoring near the expected critical region in the integrable case ($h_z = 0$), while a longitudinally perturbed control ($h_z = 0.5$) remains comparatively featureless over the same sweep range. Extending the accessible collar window partially resolves censoring and yields an operational recovery length scale of order eight lattice sites at $N = 11$ and $\beta = 12$ for $\varepsilon = 10^{-3}$.

Future work includes finite-size scaling in N and extending the collar window using tensor-network methods, as well as comparing Petz-based recovery distances to diagnostics based on conditional mutual information.

References

[1] Dénes Petz. Sufficient subalgebras and the relative entropy of states of a von Neumann algebra. *Communications in Mathematical Physics*, 105(1):123–131, 1986.

[2] Subir Sachdev. *Quantum Phase Transitions*. Cambridge University Press, Cambridge, 2 edition, 2011.