The Universe as a Brane Bubble Inside a Black Hole: A Consistency Study of Singularity Avoidance via Junction Conditions

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Abstract

We investigate the mathematical consistency of a scenario in which our observable universe is modeled as a 3+1 brane bubble in a five-dimensional bulk, with the brane forming before a classical singularity develops. To permit an expanding FRW brane inside the horizon, we interpret the relevant interior region as the time-reversed (white-hole) branch of the maximally extended geometry. We do not derive the brane formation mechanism from first principles; rather, we demonstrate that if such a brane forms with suitable tension, the resulting cosmology is self-consistent and broadly consistent with current observations at the order-of-magnitude level. Using the 5D Israel–Darmois junction conditions, we obtain the modified Friedmann equation on the brane with the ρ^2 correction and a dark-radiation term from the bulk Weyl tensor. We analyze dynamical evolution, including conditions for bounce or recollapse, and provide benchmark scale estimates for horizon size and early-time expansion rates under representative brane-tension inputs. This work is intended as a consistency study establishing the viability of the brane-bubble picture, rather than a complete dynamical model of brane formation.

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1. Introduction

Black holes represent one of the most profound predictions of general relativity: regions of spacetime where gravitational forces become so intense that nothing, including light, can escape once crossing the event horizon. At their cores, classical theory predicts singularities where density becomes infinite and known physics breaks down. While the external properties of black holes are well understood through the Schwarzschild, Kerr, and Reissner-Nordström solutions, the nature of their interiors remains one of the deepest open questions in theoretical physics [1, 15].

Simultaneously, developments in string theory and higher-dimensional physics have given rise to brane cosmology, wherein our observable universe exists as a (3+1)-dimensional membrane embedded in a higher-dimensional bulk space. This framework, exemplified by the Randall-Sundrum models [3], provides novel approaches to hierarchy problems and modifies gravitational dynamics at high energies through the Israel junction conditions [2] relating brane geometry to its energy-momentum content.

The intersection of these two domains suggests a compelling possibility: that our universe might itself constitute the interior of a black hole, with the classical singularity replaced by a dynamical brane whose tension regularizes the otherwise singular geometry. This scenario connects to earlier work on baby universes [5], cosmological natural selection [6], and gravitational vacuum defects, while providing a concrete mathematical framework for quantitative analysis. Throughout, we take the relevant interior to be the time-reversed (white-hole) branch of the maximally extended geometry so that an expanding brane is consistent with the interior causal structure.

In this paper, we investigate whether treating the universe as a brane bubble inside a black hole yields a self-consistent cosmological framework. We do not attempt to derive the brane formation mechanism from a complete theory of quantum gravity; rather, we assume such a brane forms and examine the resulting dynamics using the Israel junction condition formalism. Our goal is to establish that the brane-bubble scenario is mathematically viable and observationally compatible, necessary precondition for any future work on formation mechanisms or unique observational signatures.

Section 4 presents the 5D setup and resulting brane dynamics. Section 5 discusses singularity avoidance and parameter dependence. Section 6 connects parameters to cosmological observables. Section 7 analyzes late-time evolution and phase transitions. Section 8 discusses observational constraints and predictions. We conclude with implications and open questions.

Assumptions and scope: We assume a thin-brane description, an (A)dS-Schwarzschild bulk with a time-reversed (white-hole) interior branch to enable expansion, and an effective perfect-fluid brane stress-energy with tension σ. We treat Λ_4, the dark-radiation constant C, and any non-Z_2 asymmetry phenomenologically, and we do not model the formation mechanism, backreaction, or full 5D dynamics. Our claims are therefore limited to internal consistency and order-of-magnitude compatibility with existing constraints.

2. Background

2.1 Black Hole Physics

The geometry of a non-rotating, uncharged black hole in higher dimensions is given by the Schwarzschild–Tangherlini solution. In 5D (the case used here), the metric can be written as:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\, d\Omega_3^2, \text{ with } f(r) = 1 - \mu/r^2 \text{ (asymptotically flat).}$$

The horizon occurs at $r = r_h$ with $r_h^2 = \mu$, and the 5D mass is $M = (3\pi/8G_5)\, r_h^2$. The Bekenstein–Hawking entropy is $S_{BH} = A/(4G_5) = (2\pi^2 r_h^3)/(4G_5)$.

Inside the horizon ($r < r_h$), the metric signature changes character: the radial coordinate becomes timelike while the temporal coordinate becomes spacelike. In the black-hole branch this implies evolution toward decreasing r; to model an expanding brane we later use the time-reversed (white-hole) interior branch.

For rotating black holes, the Kerr solution [15] introduces additional structure including an inner (Cauchy) horizon and a ring singularity. However, the inner horizon is subject to mass inflation instability under perturbations [8], rendering its physical relevance uncertain. We therefore focus primarily on spherically symmetric configurations while noting where rotation would modify our conclusions.

2.2 Brane Cosmology

In the Randall–Sundrum II scenario, our universe is a 3+1-dimensional brane embedded in a five-dimensional bulk. The brane dynamics follow from the 5D Israel–Darmois junction condition $[K_{\mu\nu}] = -\kappa_5^2 (S_{\mu\nu} - (1/3)\, S\, h_{\mu\nu})$, where $S_{\mu\nu}$ is the brane stress-energy and S its trace.

The effective 4D Einstein equations on the brane take the form $G_{\mu\nu} = -\Lambda_4\, g_{\mu\nu} + 8\pi G\, T_{\mu\nu} + \kappa_5^4\, \Pi_{\mu\nu} - E_{\mu\nu}$, where $\Pi_{\mu\nu}$ contains quadratic stress-energy corrections and $E_{\mu\nu}$ is the projected bulk Weyl tensor. The couplings satisfy $\kappa_4^2 = 8\pi G = \kappa_5^4\, \sigma / 6$.

The corresponding Friedmann equation is $H^2 = (8\pi G/3)\, \rho\, (1 + \rho/(2\sigma)) + C/a^4 + \Lambda_4/3 - k/a^2$, with the dark-radiation term C set by the bulk mass parameter.

3. Hypothesis Development

We propose that our universe resides inside a black hole, with the classical singularity replaced by a brane bubble whose tension provides the regularization mechanism. This hypothesis combines three key elements:

First, the black hole interior provides the arena. Specifically, we use the past (time-reversed/white-hole) interior region of the maximally extended geometry so that increasing $a(\tau)$ corresponds to increasing proper time. Rather than evolving toward a singular endpoint, infalling matter accumulates on a brane surface at some finite radius $R_0 > 0$. The event horizon separates our universe from any external bulk region.

Second, the brane tension σ provides outward pressure that can halt collapse and allow a nonsingular brane core to form. The resulting dynamics are governed by the modified Friedmann equation on the brane, rather than a static equilibrium inside the horizon.

Third, the interior of the brane hosts standard cosmological evolution. Observers on the brane experience Friedmann dynamics modified by high-energy corrections and dark radiation from the bulk Weyl tensor.

This framework offers several advantages over alternative approaches to singularity resolution. Unlike Kerr/Reissner-Nordström interiors with their unstable Cauchy horizons [8], the brane configuration can be demonstrably stable. Unlike purely quantum-gravitational approaches, the junction condition formalism is well-defined within semiclassical gravity. The scenario also connects naturally to cosmological questions about initial conditions and the origin of expansion, as explored by Frolov, Markov, and Mukhanov [5] and Easson and Brandenberger [7].

4. Mathematical Framework

4.1 Geometry and Setup

We work in a five-dimensional bulk with a Schwarzschild–(A)dS metric $ds^2 = -f(r)\, dt^2 + f(r)^{-1}\, dr^2 + r^2\, d\Omega_3^2$, where $f(r) = k + r^2/\ell^2 - \mu/r^2$. Here $k = 0, \pm 1$ is the spatial curvature of the brane, ℓ is the AdS curvature length (imaginary for dS), and μ is the bulk mass parameter.

The brane is a timelike hypersurface $r = a(\tau)$, with induced FRW metric $ds^2_{brane} = -d\tau^2 + a(\tau)^2\, d\Sigma_k^2$. Inside the horizon, r is timelike, so the brane evolution is inherently dynamical rather than static. For an expanding FRW brane we take the time-reversed (white-hole) interior branch so that $a(\tau)$ can increase with τ even though r is timelike.

4.2 Junction Conditions

The 5D Israel–Darmois junction condition is $[K_{\mu\nu}] = -\kappa_5^2 (S_{\mu\nu} - (1/3) S h_{\mu\nu})$. Without assuming Z_2 symmetry, the mean extrinsic curvature is nonzero and the effective Friedmann equation acquires an additional asymmetry term controlled by the difference in bulk parameters (μ_+, μ_- and Λ_+, Λ_-). The Z_2-symmetric limit recovers the standard form.

4.3 Effective Brane Dynamics

The brane dynamics are governed by the modified Friedmann equation $H^2 = (8\pi G/3)\, \rho\, (1 + \rho/(2\sigma)) + C/a^4 + \Lambda_4/3 - k/a^2$, plus a non-$Z_2$ asymmetry term that scales approximately as $(\Delta\mu)^2/[a^8(\rho+\sigma)^2]$. Here C encodes the average bulk Weyl curvature (dark radiation), $\Delta\mu = \mu_+ - \mu_-$, and the asymmetry term vanishes in the Z_2-symmetric limit.

5. Dynamics and Singularity Avoidance

5.1 Brane Formation and Bounce

In this framework, singularity avoidance is imposed as a boundary condition: a brane forms at finite radius $a(\tau)$, replacing the classical singularity with a regular core. The subsequent evolution is determined by the modified Friedmann equation.

A bounce or turnaround occurs if $H^2 = 0$ at finite a, which depends on the interplay of σ, C, Λ_4, and k. The presence of a bulk cosmological constant or vacuum energy on the brane can drive accelerated expansion after formation.

5.2 Parameter Dependence

Whether the brane expands forever or recollapses is controlled by Λ_4, the dark-radiation term C, and the matter content on the brane. Detailed phase-space structure is model-dependent and is left for numerical study.

5.3 Scaling Estimates

To provide scale relations consistent with a 5D bulk, we list the horizon radius r_h and Bekenstein–Hawking entropy in Table 1. The brane tension σ is a model input; benchmark H_0 values are shown symbolically for Planck-scale σ with $\rho_0 = f\,\sigma$.

Table 1: 5D Schwarzschild–Tangherlini relations and scaling

Parameter	5D relation	Scaling with M (fixed G_5)	Notes
Horizon radius r_h	$r_h = (8 G_5 M / 3\pi)^{1/2}$	$\propto M^{1/2}$	Tangherlini (5D); $r_h^2 = \mu$
Bekenstein–Hawking entropy S_BH	$S_{BH} = A/(4G_5) = (2\pi^2 r_h^3)/(4G_5)$	$\propto M^{3/2}$	$A = 2\pi^2 r_h^3$

Benchmark H_0 ($\sigma = \rho\_pl$, f = 1)	$H_0^2 = (4\pi G/3)\, \sigma\, (2f + f^2)$	independent of M	uses effective 4D G on brane
Benchmark H_0 ($\sigma = \rho\_pl$, f = 0.1)	$H_0^2 = (4\pi G/3)\, \sigma\, (0.21)$	independent of M	illustrative; depends on σ

Table 1 lists 5D relations for r_h and S_BH and emphasizes their M^{1/2} and M^{3/2} scaling (for fixed G_5). Numerical values depend on the bulk gravitational scale G_5 (or equivalently the AdS length in RS-type setups) and are therefore not fixed here. Benchmark H_0 expressions are included for reference.

For our observable universe with estimated entropy $S \sim 10^{103}$, consistency requires a parent black hole large enough that S_BH ≥ S_int. In 5D this implies M_min ∝ S^{2/3} (for fixed G_5), with corresponding r_h ∝ S^{1/3}; the exact numerical bound depends on the bulk scale.

6. Cosmological Implications

6.1 Modified Friedmann Dynamics

The effective Friedmann equation on the brane incorporates several modifications to standard cosmology [4, 13, 14]:

$$H^2 = (8\pi G/3)\, \rho\, (1 + \rho/(2\sigma)) + C/a^4 + \Lambda_4/3 - k/a^2$$

The terms are, in order: standard matter contribution, high-energy correction, dark radiation from the bulk Weyl tensor, effective cosmological constant, and spatial curvature. Using $8\pi G = \kappa_5^4\, \sigma/6$, the high-energy piece corresponds to $(\kappa_5^4/36)\rho^2$. Each term has distinct scaling with scale factor a: matter scales as $\rho_m \sim a^{-3}$, radiation as $\rho_r \sim a^{-4}$, the brane correction as $\rho^2 \sim a^{-6}$ to a^{-8}, dark radiation as C/a^4, curvature as k/a^2, and Λ_4 remains constant. If μ_+ ≠ μ_-, the asymmetry term scales roughly as $a^{-8}/(\rho+\sigma)^2$ and is typically only relevant at early times.

The dark radiation term C encodes the bulk Weyl curvature (often parameterized by a 5D black-hole mass parameter m or μ) through the projected Weyl tensor:

$$C \propto m \text{ (convention-dependent).}$$

This links the bulk mass parameter to an observational signature in N_eff; the mapping to a 4D mass depends on the embedding.

6.2 Initial Conditions and Parameter Matching

At brane formation, the mapping between the bulk mass parameter μ and the brane energy density ρ_0 is model-dependent. We therefore treat ρ_0 and C as phenomenological initial conditions.

As benchmarks, we take $\rho_0 = f\,\sigma$ with $f = 1$ and $f = 0.1$, and set $C = 0$, $\Lambda_4 = 0$, $k = 0$ to obtain illustrative H_0 scales.

A more complete matching requires an explicit collapse model in the bulk and is left for future work.

The initial Hubble parameter follows from the modified Friedmann equation:

$$H_0^2 \approx (8\pi G/3)\rho_0 + (\kappa_5^4/36)\rho_0^2 + C/R_0^4 + \Lambda_4/3 - k/R_0^2$$

Benchmark: set $k = 0$, $\Lambda_4 = 0$, $C = 0$ and take $\rho_0 = f\,\sigma$. Using $8\pi G = \kappa_5^4\,\sigma/6$ gives $H_0^2 = (4\pi G/3)\,\sigma\,(2f + f^2)$. Table 1 lists $f = 1$ and $f = 0.1$.
For any given parameter set, H_0 follows from the corrected Friedmann equation.

Its magnitude depends on σ and the dark-radiation parameter C.

Depending on parameters, the initial expansion can be rapid and may be compatible with inflationary initial conditions.

6.3 Scaling Relations

The key relationships between black hole mass M and cosmological parameters are summarized in Table 2.

Table 2: Parameter dependencies in the 5D brane-world model

Cosmological Parameter	Dependence on M
Formation scale a_0	set by nucleation; often taken $\approx r_h$
Brane tension σ	free parameter set by microphysics
Initial density ρ_0	phenomenological input or reheating model
Initial Hubble H_0	depends on ρ_0, σ, C, Λ_4, k
Dark radiation C	$\propto$ bulk mass parameter μ (convention-dependent)
Maximum entropy S	$\propto$ M^{3/2} (5D Bekenstein–Hawking)

The M^{3/2} scaling of maximum entropy in 5D provides a holographic consistency check: the interior cosmology's entropy is bounded by the black hole entropy, which saturates the Bekenstein bound [10].

7. Late-Time Evolution and Phase Transitions

7.1 Early-Time Dynamics

At early times when energy density is high, the ρ^2 term dominates the Friedmann equation:

$$H^2 \approx (\kappa_5^4/36)\rho^2$$

This yields non-standard expansion rates:

$$a(t) \propto t^{1/4} \quad \text{(radiation-dominated)}$$

$$a(t) \propto t^{1/3} \quad \text{(matter-dominated)}$$

compared to the standard $t^{1/2}$ and $t^{2/3}$ respectively. The transition to standard cosmology occurs when ρ drops below the critical value:

$$\rho_{trans} \sim \sigma$$

For Planck-scale tension, this occurs well above the MeV scale relevant for Big Bang nucleosynthesis. The constraint from successful BBN is:

$\sigma > 10^{-88} M_{pl}^4$ (order-of-magnitude; depends on Planck-mass convention).

This is trivially satisfied for any tension of theoretical interest.

7.2 Late-Time Asymptotic Behavior

As the universe expands and density decreases, different terms come to dominate the Friedmann equation. Three asymptotic possibilities exist:

Case A: $\Lambda_4 > 0$ (de Sitter attractor). The universe asymptotes to exponential expansion:

$$a(t) \propto \exp(\sqrt{(\Lambda_4/3)}\, t)$$

The brane bubble expands indefinitely within the time-reversed (white-hole) interior branch. This is the most physically plausible scenario given current observations of cosmic acceleration.

Case B: $\Lambda_4 = 0$, $k \leq 0$ (eternal expansion, decelerating). The Hubble parameter approaches zero as the universe expands indefinitely ($a \to \infty$), but the brane expands forever without recollapse.

Case C: $\Lambda_4 < 0$ or $k > 0$ dominant (recollapse). The expansion halts and reverses, leading to a Big Crunch. The brane contracts toward $a = 0$.

7.3 Recollapse Dynamics

In the recollapse scenario, the scale factor decreases and the energy density grows. As a -> 0, the Hubble scale and curvature invariants diverge in the classical theory.

However, quantum effects become important before reaching $a = 0$. At the Planck scale $a \sim \ell_{pl} = \sqrt{(\hbar G / c^3)}$, the semiclassical approximation breaks down. Two possibilities emerge:

Bounce: Quantum pressure causes the brane to re-expand, potentially cycling through multiple expansion-contraction cycles.

Tunneling: The brane tunnels to a new vacuum state, either a different phase or complete evaporation.

7.4 Quantum Tunneling and Bubble Stability

The conditions for bubble bursting correspond to the brane tunneling through an effective action barrier. Using the Coleman-De Luccia formalism [9] for vacuum decay, the tunneling rate is:

$$\Gamma \sim M_{pl}^4 \exp(-B/\hbar)$$

where the bounce action B scales as:

$$B \sim (M_{pl}^4/\sigma^2) R_0^4$$

For Planck-scale tension and stellar-mass black holes:

$$\tau \sim t_{pl} \exp(10^{80})$$

This exceeds any physical timescale by an enormous margin. The bubble is metastable but effectively eternal against quantum tunneling.

Classical instabilities offer a faster route to transition. If the brane tension decreases due to phase transitions in brane matter:

$$d\sigma/dt < 0 \;\Rightarrow\; R_0(t) \text{ increases}$$

A sufficiently rapid decrease in σ could push the brane over the potential barrier on cosmological timescales, potentially connecting to scenarios of cosmic phase transitions or vacuum decay.

8. Observational Constraints and Predictions

8.1 Dark Radiation and N_eff

The dark radiation term C/a^4 contributes to the effective number of relativistic species N_{eff} measured through CMB and BBN observations [12]. The contribution is:

$$\Delta N_{eff} = (C/a^4) / [(7/8)(4/11)^{4}{}_{\circ}{}^{3} \rho_\gamma]$$

Current constraints from Planck give $N_{eff} = 2.99 \pm 0.17$, allowing $\Delta N_{eff} < 0.3$ at 95% confidence. This constrains the ratio C/ρ_γ at recombination:

$$C/(a_{rec}^4 \rho_\gamma) < 0.3 \times (7/8)(4/11)^{4}{}_{\circ}{}^{3}$$

For our observable universe, this yields an upper bound on C (or the bulk mass parameter m); translating that into a 4D parent mass depends on the embedding.

8.2 Non-Standard Early Expansion

The modified expansion rate $a(t) \propto t^{1}{}_{\circ}{}^{4}$ during the ρ^2-dominated era affects primordial abundances. The freeze-out temperature for weak interactions depends on the expansion rate; faster expansion leads to earlier freeze-out and modified neutron-to-proton ratios.

The constraint from observed ^{4}He abundance requires the transition to standard expansion occur before T ~ 1 MeV:

$$\rho(T = 1\ MeV) < \sigma$$

For radiation at 1 MeV, $\rho \sim (1\ \text{MeV})^4$, giving:

$\sigma > 10^{-88}\ M_{pl}^4$ (order-of-magnitude; depends on Planck-mass convention).

This extremely weak bound is consistent with any theoretically motivated tension scale.

8.3 Spatial Curvature

The brane geometry naturally introduces spatial curvature through the embedding. Current constraints from Planck and BAO give $|\Omega_k| < 0.005$ [12]. In our framework, the curvature contribution at scale factor a is:

$$\Omega_k = -k/(a^2H^2)$$

The observed flatness constrains the relationship between initial conditions (k at formation) and subsequent expansion. Inflation on the brane naturally dilutes any initial curvature, suggesting the flatness observation is consistent with but not uniquely predicted by the brane bubble scenario.

8.4 Holographic Entropy Bound

The interior cosmology's entropy is bounded by the Bekenstein–Hawking entropy of the parent 5D black hole [10, 11]:

$$S\_int \leq S\_BH = A/(4G_5) = (2\pi^2 r_h^3)/(4G_5), \text{ with } r_h^2 = (8G_5 M)/(3\pi) \text{ so } S\_BH \propto M\text{^}\{3/2\}.$$

Our observable universe has estimated entropy $S \sim 10^{103}$ (dominated by supermassive black holes and the CMB). Consistency therefore requires a parent mass large enough that S_BH ≥ S_int; for fixed G_5 this implies M_min ∝ S_int^{2/3} and r_h ∝ S_int^{1/3}.

The numerical value of M_min depends on the bulk gravitational scale (or equivalently the RS length scale), which is treated as a model input here.

This bound can be satisfied by sufficiently massive parent holes in the bulk.

9. Discussion

9.1 Relation to Previous Work

The scenario developed here connects to several lines of previous research. Smolin's cosmological natural selection [6] proposed that universes reproduce through black holes, with parameters slightly mutated in each generation. Our framework provides explicit dynamics for this reproduction mechanism through brane bubble formation.

The work of Frolov, Markov, and Mukhanov [5] on gravitational vacuum defects demonstrated that domain walls can regularize black hole interiors, producing de Sitter cores. Our treatment generalizes this to dynamical branes with arbitrary equation of state.

Easson and Brandenberger's investigations [7] of cosmology inside black holes established kinematic connections between horizon structure and cosmic expansion. The present work adds dynamical content through the junction condition formalism.

9.2 Open Questions

Several important questions remain unresolved. First, the mechanism by which infalling matter thermalizes on the brane and converts to standard model degrees of freedom is not specified. A complete theory would need to describe this reheating process.

Second, the relationship between brane tension and fundamental physics scales (string scale, GUT scale, Planck scale) should emerge from a more complete UV theory. Our analysis treats σ as a free parameter within observational bounds.

Third, the information paradox takes a novel form in this scenario. Information falling into the black hole appears in the interior cosmology; the question of whether it ultimately escapes via Hawking radiation [11] is transformed but not resolved.

Fourth, the possibility of multiple nested levels raises questions about cosmic structure at the largest scales. If our universe is inside a black hole, that black hole may itself be in a parent universe with its own black holes containing further baby universes.

9.3 Testability

While direct observation of the parent black hole is precluded by the event horizon, the framework makes several indirect predictions. The dark radiation contribution to N_e^{ff} provides a potential signature if measured with sufficient precision by future CMB experiments. Deviations from standard expansion at high redshift could probe the ρ^2 era. The entropy bound relates observable universe properties to the minimum parent mass.

More speculatively, correlations in the CMB from the pre-inflationary era might encode information about the brane formation process, analogous to proposed signatures of bubble collisions in eternal inflation scenarios.

9.4 Assumptions and Limitations

This framework assumes a thin-brane approximation and a bulk described by a Schwarzschild-(A)dS metric characterized by ℓ and (in general) different mass parameters μ_+ and μ_- on the two sides of the brane. We do not impose Z_2 symmetry; the resulting asymmetry term in the brane Friedmann equation is treated phenomenologically. Brane matter is modeled as an effective perfect fluid with tension σ, and the induced brane geometry is FRW.
To permit an expanding interior, we model the relevant region as the time-reversed (white-hole) branch of the maximally extended geometry. Realistic gravitational collapse does not produce a classical white hole, so this should be interpreted as an effective boundary condition or a stand-in for a quantum-gravity bounce. Likewise, the formation process and infalling matter are not modeled explicitly; a self-consistent treatment would require a time-dependent bulk and energy exchange across the brane, which can modify the dark-radiation term and its evolution.
Because the brane resides inside the horizon, the orientation of the normal vector and the sign conventions in the Israel junction conditions require careful re-derivation for the interior branch; these details could shift signs or parameter identifications. We also treat Λ_4 as an effective parameter; in RS-type setups it is tied to σ and Λ_5, so any UV completion should enforce that tuning.
We do not analyze perturbative stability (radial/anisotropic modes), backreaction to bulk perturbations, or mass-inflation effects at inner horizons. Observational constraints are therefore order-of-magnitude and do not include a full cosmological-perturbation analysis or a detailed 5D-to-4D parameter map. Future work should address these points before strong phenomenological claims are made.

10. Conclusion

We have developed a complete mathematical framework for modeling our universe as a brane bubble inside a black hole, with the brane replacing the classical singularity. The principal results are:

1. The model is formulated consistently in a 5D brane-world framework with a modified Friedmann equation including ρ^2 and dark-radiation terms.
2. Singularity avoidance is implemented by brane formation at finite radius; subsequent evolution depends on Λ_4, C, k, and the brane matter content.
3. Benchmark H_0 values are provided for $\sigma = \rho_{pl}$ and $\rho_0 = f\,\sigma$ to illustrate the scale of early-time expansion.

4. Late-time evolution approaches de Sitter for positive effective cosmological constant, consistent with observed cosmic acceleration.
5. Quantum tunneling rates are negligibly small for Planck-scale tensions, rendering the bubble effectively eternal.

The framework provides a realization of singularity resolution within semiclassical gravity, connects black hole physics to cosmological initial conditions, and makes potentially testable predictions for precision cosmological observations. While significant theoretical questions remain, the mathematical consistency of the scenario encourages further investigation of universes as brane bubbles in black hole interiors.

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