From Static Recoverability to Maintenance Power: A Typed Pipeline with $\omega = 0$ Obstructions

Version 2

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Abstract

We study when geometric separation in gapped quantum systems yields a genuine reduction in thermodynamic resources required to maintain coherence against uncontrolled open-system dynamics. Our analysis separates three layers. First, in a regularized Gaussian split regime motivated by algebraic QFT, we state an explicit static reconstruction bound: collar suppression of vacuum cross-correlations enables approximate state recovery via a conditional-reattachment covariance rule with fidelity error controlled by a cross-block recovery norm. Second, we show why static recoverability does not automatically imply suppression of dynamical decay rates: fixed-point structure and Bohr-zero ($\omega = 0$) channels can generate obstructions invisible to static clustering alone. We formalize this using an exact $\omega = 0$ Dirichlet identity and implement finite-size commutator-witness diagnostics in the transverse-field Ising chain, finding no evidence of a size-independent $\omega = 0$ floor in that benchmark regime for tested sizes. Third, we give an autocontained finite-dimensional core linking coherence loss to incremental maintenance power under an explicit battery-assisted thermal-operations model with paired strategies, and we state a typed rate-inheritance hypothesis identifying precisely what additional dynamical input is required to propagate collar suppression into power suppression. We conclude with a Type III blueprint. *v2 corrects two points and adds verification:* (i) the recovery rule of the static layer is conditional reattachment – *not* the Petz map for correlated Gaussian references (aligned with 2601.0007 v2 and 2512.0060 v2), with an explicit admissibility hypothesis; (ii) the v1 work-cost bookkeeping (its Eq. (25)) carried a sign error – the work cost is the battery free-energy *decrease* – which made v1's one-step work lemma false as printed (random energy-conserving unitaries violate it in 57/100 draws) while its own proof chain and everything downstream hold verbatim with the corrected sign; this is the same error class repaired in 2512.0061 v2. v2 further adds a GNS/KMS convention bridge to the companion witness papers, series positioning, an updated status of the rate-inheritance hypothesis, and a verification suite.

1 Introduction

Geometric separation in gapped many-body systems suppresses static correlations, motivating the expectation that thick buffers reduce cross-region influence. A distinct operational question is dynamical: even if a global state is recoverable from local data, what resources are required to keep a target state stable under uncontrolled open-system dynamics? This paper isolates a frequent logical pitfall: static locality controls an *amplitude*, whereas maintenance cost is governed by *rates*. Our contribution is a typed pipeline separating static recoverability from dynamic rate control, identifying the unique missing hinge (rate inheritance on a family), and clarifying the role of $\omega = 0$/fixed-point obstructions.

Series positioning (added in v2). This note is the integrative closure of the 2601 block: the static layer instantiates 2601.0007 [12] (finite-mode core of 2512.0060 [6]); the $\omega = 0$ obstruction and witnesses develop 2601.0022/0023 [14, 15]; the maintenance bound is the paired-strategy theorem of 2512.0061 v2 [7], reproved here self-contained; the typed rate-inheritance hypothesis is the family-level version of the hinge formulated in 2512.0072 [10], stress-tested in 2512.0070 [9], proved in the quasi-free class in 2512.0064 [8], and reduced to a Dirichlet comparison in 2601.0020 [13]; the Type III blueprint points at 2512.0101 [11].

2 Setup: conditional expectations, coherence functionals, and operational families

Let $\mathcal{H}$ be finite-dimensional, H_S with spectral projectors $\{\Pi_n\}$, $\beta := 1/(k_B T)$, $\sigma := e^{-\beta H_S} / \operatorname{Tr}(e^{-\beta H_S})$, and $T_t = e^{t\mathcal{L}_*}$ an uncontrolled GKLS semigroup with $\mathcal{L}_*[\sigma] = 0$.

Definition 2.1 (Coherence relative to a conditional expectation)**.** *For a σ-preserving conditional expectation E with predual E_*, $C_E(\rho) := S(\rho \,\|\, E_*[\rho])$. Instance 1 (law-grade): energy pinching $\Delta[A] = \sum_n \Pi_n A \Pi_n$. Instance 2 (static, geometric): a vacuum-preserving split expectation E_{split} in a split configuration $\mathcal{O}_1 \Subset \mathcal{O}_2$ of collar width r.*

Loss rate: $\dot{C}_{E,\text{loss}}(\rho) := -\frac{d}{dt} C_E(T_t\rho)|_{t=0}$; local rate $r_E(\rho) := \dot{C}_{E,\text{loss}}(\rho)/C_E(\rho)$ for $C_E(\rho) > 0$; typed envelopes $r_E^{\downarrow}(r) := \inf_{\rho\in\mathcal{F}_r} r_E(\rho)$ and $r_E^{\uparrow}(r) := \sup_{\rho\in\mathcal{F}_r} r_E(\rho)$ on an operational family $\mathcal{F}_r$ of locally prepared states ($\rho = \Lambda_A(\sigma)$, Λ_A with Stinespring dilation localized in A) with energy budget, optional coherence budget, and fixed-point exclusion $\|\rho - E_{\text{fix},*}[\rho]\|_1 \geq \delta_{\text{fix}}$, where $E_{\text{fix},*}$ is the ergodic mean of T_t.

3 Static layer: quantitative recoverability from collar suppression (Gaussian split regime; terminology corrected in v2)

With centered quasi-free covariances Γ, Γ_0 in block form, vacuum correlation factor $\eta_{\text{vac}} := \|A_0^{-1/2} X_0 B_0^{-1/2}\|_{\text{op}}$ and perturbation $\delta_X := \|A_0^{-1/2}(X - X_0)B_0^{-1/2}\|_{\text{op}}$, the *conditional-reattachment* covariance rule is

$$\tilde{A} = A, \qquad \tilde{X} = AA_0^{-1}X_0, \qquad \tilde{B} = B_0 + X_0^T A_0^{-1}(A - A_0)A_0^{-1}X_0, \qquad \Delta^{(12)} := X - \tilde{X}.$$

Remark 3.1 (Terminology corrected in v2). v1 called this rule “Petz recovery in block form.” Per 2601.0007 v2 [12] (aligned with the repaired Proposition 2.14 of 2512.0060 v2 [6]), the rule is conditional reattachment: it agrees with the Petz output only for factorized references ($X_0 = 0$); for correlated references the Petz map does not preserve the input marginal. All uses below are of the explicit covariance rule on its admissible domain ($\tilde{\Gamma} + \frac{i}{2}\Omega \succeq 0$).

Theorem 3.2 (Gaussian clustering–recovery bridge (static; imported))**.** *Under quasi-free regularity, coercivity of A relative to A_0, cross-correlation control, $\|K\|_{\text{op}} \leq 1/2$, and admissibility of $\tilde{\Gamma}$, the reattached state $\tilde{\omega}$ satisfies $1 - F(\omega, \tilde{\omega}) \leq C(\epsilon, \eta_{\text{vac}}, \delta)\, \|\Delta^{(12)}\|_{\text{HS}}^2$, with prefactor controlled by ϵ and the correlation denominator $1 - \epsilon^{-1}(\eta_{\text{vac}} + \delta)^2$. See [12] for the proved quantitative version (quadratic+quartic, with certified constants).*

Remark 3.3 (Static only). Theorem 3.2 is a static reconstruction guarantee; it implies nothing about dynamical rates.

4 The $\omega = 0$ obstruction and witness diagnostics

With $S(0) := \sum_n \Pi_n S \Pi_n$ and an $\omega = 0$ Davies channel $\mathcal{L}_0(O) = \gamma(0)(S(0)OS(0) - \frac{1}{2}\{S(0)^2, O\})$, we use the σ-weighted (GNS) inner product $\langle A, B\rangle_\sigma := \mathrm{Tr}(A^\dagger \sigma B)$, matching the numerical diagnostics (for Pauli strings $\|O\|_{2,\sigma}^2 = 1$).

Lemma 4.1 (Exact $\omega = 0$ Dirichlet identity (GNS))**.** *If* $[S(0), \sigma] = 0$*, then* $\mathcal{E}_\sigma^{(0)}(O) = \frac{\gamma(0)}{2}\|[S(0), O]\|_{2,\sigma}^2$ *for all* O*.*

Remark 4.2 (Convention bridge (added in v2)). The companions 2601.0022/0023 [14, 15] state the same identity in the *KMS* inner product $\mathrm{Tr}(\sigma^{1/2}A^\dagger\sigma^{1/2}B)$; the proof only uses $[S(0), \sigma] = 0$ and holds in both. The optimized witness values differ numerically between conventions (suite benchmark, TFIM $N = 8$: ratio GNS/KMS ≈ 1.15 at $\epsilon = 1$, 1.24 at $\epsilon = 2$), so cross-paper comparisons of R_{opt} must fix the convention. This paper's numbers are GNS throughout.

Witness families: $R_{\mathrm{opt}}^{(k)}(\epsilon; N) := \max_O \|[S(0), O]\|_{2,\sigma}^2 / \|O\|_{2,\sigma}^2$ over one-site and two-site Pauli families at distance ϵ, certifying $\kappa_{\mathrm{wit}}^{(k)} = \frac{\gamma(0)}{2} R_{\mathrm{opt}}^{(k)}$ within the family.

Remark 4.3 (Consistency with the companions (added in v2)). Figure 1 shows R_{opt} *decreasing with* N at fixed ϵ: no evidence of a size-independent $\omega = 0$ floor in this benchmark. This refines – and does not contradict – 2601.0022/0023, which certify a strictly positive floor *mechanism at fixed* N: the mechanism is real (and switches off for couplings with vanishing $\omega = 0$ component, per the 0022 v2 null case), while its N-dependence in this benchmark suggests it need not survive the thermodynamic limit for this coupling family. The suite reproduces the decreasing trend ($N \in \{6, 8, 10\}$, $\epsilon \in \{1, 2\}$).

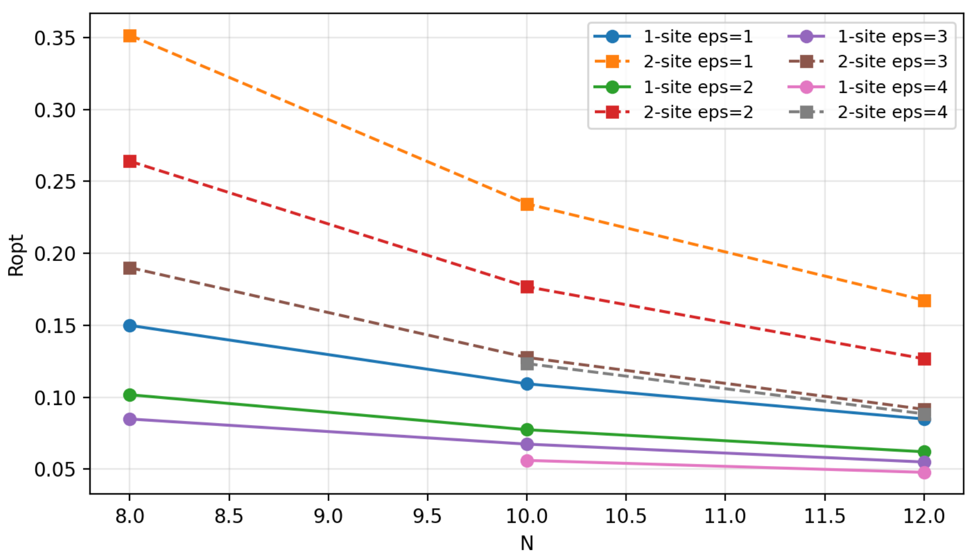

Figure 1: Optimized $\omega = 0$ commutator witnesses in the TFIM ($J = 1$, $h = 1.5$, $\beta = 1$). Solid: one-site family; dashed: adjacent two-site family. Both decrease with N at fixed ϵ for $N \leq 12$.

5 Thermodynamic control model and the power bound (sign corrected in v2)

Battery-assisted thermal operations: $\Phi_{SW}(\cdot) = \mathrm{Tr}_B[U(\cdot \otimes \gamma_B)U^\dagger]$ with $[U, H_S + H_B + H_W] = 0$; battery free energy $F_\beta(\omega_W) := k_B T\, S(\omega_W \| \gamma_W)$.

Definition 5.1 (Work cost; sign corrected in v2)**.** *For a battery process $\omega_W \mapsto \omega'_W$, the work* expenditure *is*

$$W := F_\beta(\omega_W) - F_\beta(\omega'_W),$$

the battery free-energy decrease*. (v1's Eq. (25) defined W with the opposite sign, which renders its one-step lemma false as printed: the suite exhibits violations in 57/100 random energy-conserving unitaries, while the corrected sign yields 0/100. This is the same error class as the work-sign bug repaired in 2512.0061 v2 [7].)*

Instantaneous maintenance and *paired* incremental power $P_{\mathrm{extra}}(\rho)$ are defined as in v1 (paired strategies maintaining ρ and $\Delta[\rho]$ with the same bath model and initial battery state; one never subtracts unrelated infima).

Lemma 5.2 (One-step work lower bound, corrected sign)**.** *For Φ_{SW} as above mapping $\rho \otimes \omega_W$ to a state with marginals ρ', ω'_W: $W \geq F^S_\beta(\rho') - F^S_\beta(\rho)$, with $F^S_\beta(\rho) := k_B T\, S(\rho\|\sigma) + \mathrm{const}$.*

Proof. Energy conservation gives $U\Gamma U^\dagger = \Gamma$ for $\Gamma = \sigma \otimes \gamma_B \otimes \gamma_W$. Monotonicity of relative entropy under $\mathrm{Tr}_B\, U(\cdot)U^\dagger$ plus the chain rule with $I(S{:}W) \geq 0$ yield $S(\rho\|\sigma) + S(\omega_W\|\gamma_W) \geq S(\rho'\|\sigma) + S(\omega'_W\|\gamma_W)$, i.e. total nonequilibrium free energy does not increase. Rearranged: $F_\beta(\omega_W) - F_\beta(\omega'_W) \geq F^S_\beta(\rho') - F^S_\beta(\rho)$, which is the claim with the corrected sign convention. (v1 derived exactly this display – its Eq. (36) – and then transcribed it with the inverted work sign.) ☐

Assumption 5.3 (Energy covariance of the drift)**.** $\Delta[T_t(\rho)] = T_t(\Delta[\rho])$ *(holds for Davies/secular generators; verified in the suite).*

Theorem 5.4 (Incremental maintenance power lower bound (paired strategies))**.** *Under the maintenance model, Definition 5.1, and Assumption 5.3, if paired strategies exist for ρ and $\Delta[\rho]$, then $P_{\mathrm{extra}}(\rho) \geq k_B T\, \dot{C}_{\Delta,\mathrm{loss}}(\rho)$.*

Proof. Verbatim as in v1 with W of Definition 5.1: apply Lemma 5.2 to both maintenance steps, subtract the paired inequalities, use the pinching Pythagoras identity $S(\rho\|\sigma) = S(\rho\|\Delta[\rho]) + S(\Delta[\rho]\|\sigma)$ and Assumption 5.3, divide by δt and pass to the limit. ☐

6 Main results: finite-dimensional core and Type III blueprint

Corollary 6.1 (Typed family lower bound)**.** *For $\rho \in \mathcal{F}_r$ with $C_\Delta(\rho) > 0$, with P_{extra} the* paired *incremental power of Theorem 5.4: $P_{\mathrm{extra}}(\rho) \geq k_B T\, r_\Delta(\rho)\, C_\Delta(\rho) \geq k_B T\, r^\downarrow_\Delta(r)\, C_\Delta(\rho)$.*

Hypothesis 6.2 (Typed rate inheritance on $\mathcal{F}_r$; status updated in v2)**.** *On $\mathcal{F}_r$, $r^\downarrow_\Delta(r) > 0$ with a collar-dependent envelope (e.g. $r^\downarrow_\Delta(r) \gtrsim e^{-\alpha r}$ up to polynomial prefactors), and fixed-point/$\omega = 0$ mechanisms do not induce a collar-independent floor in the relevant regime.* Status in the series: *proved exactly in the quasi-free local-sink class [8]; failure scenario realized within the secular Davies class [9] (with the nonlocality caveat there); reduced to an explicit bulk–collar Dirichlet comparison in [13]; the $\omega = 0$ no-floor input is supported in the TFIM benchmark by Remark 4.3 at tested sizes.*

Corollary 6.3 (Collar-dependent power suppression (conditional))**.** *Assuming Hypothesis 6.2 and $C_\Delta \geq C_0 > 0$ on $\mathcal{F}_r$, the paired incremental power satisfies $\inf_{\rho\in\mathcal{F}_r} P_{\mathrm{extra}}(\rho) \gtrsim k_B T\, r_\Delta^\downarrow(r)\, C_0$.*

Conjecture 6.4 (Typed pipeline in AQFT (blueprint))**.** *As in v1: with a split inclusion of collar width r, static residue $C_{E_{\mathrm{split}}}(\omega) := S_{\mathrm{Araki}}(\omega\|\omega\circ E_{\mathrm{split}})$, a physically specified dissipative dynamics and a well-posed maintenance notion, a Type III analogue of Hypothesis 6.2 yields, for the paired incremental power, $P_{\mathrm{extra}}(\omega) \gtrsim k_B T\, r_\Delta^\downarrow(r)\, C_{E_{\mathrm{split}}}(\omega)$ on a typed fast-sector family. The CMI route to the static side is 2512.0101 [11].*

7 Verification suite (added in v2)

`verification/2601-0031/verify_2601_0031.py`: (1) the $\omega = 0$ identity in the GNS convention (machine precision); (2) the GNS/KMS bridge of Remark 4.2; (3) the decreasing-with-N witness trend ($N \in \{6, 8, 10\}$, both ϵ); (4) the pinching Pythagoras identity (exact); (5) the work-sign refutation/repair of Definition 5.1 (57/100 violations with v1's sign; 0/100 corrected) on random energy-conserving unitaries over $S \otimes B \otimes W$; (6) Assumption 5.3 for a secular Davies generator (exact). The Figure 1 script of v1's appendix is relocated to the same directory.

8 Conclusion

Geometry buys recoverability (Theorem 3.2); it does not buy efficiency freely: maintenance is governed by rate envelopes vulnerable to fixed-point and $\omega = 0$ obstructions that static clustering cannot rule out. The missing dynamical hinge is explicit (Hypothesis 6.2, with its series status), and the witness diagnostics delineate when "isolation implies stability" becomes thermodynamically valid.

Note added in v2

Two corrections, both verified in the suite, plus positioning. (1) *Terminology/admissibility (static layer):* "Petz" $\to$ conditional reattachment per 2601.0007 v2 and 2512.0060 v2 (Remark 3.1), with the admissibility hypothesis stated. (2) *Work-sign error:* v1's Eq. (25) inverted the work-cost sign, making its one-step lemma false as printed; the corrected bookkeeping (Definition 5.1) restores it, and Theorem 5.4 holds verbatim – v1's own proof chain (its Eq. (36)) already established the corrected display. Same error class as 2512.0061 v1 (repaired in its v2). (3) Additions: GNS/KMS convention bridge (Remark 4.2); consistency remark with the fixed-N floor mechanism of 2601.0022/0023 (Remark 4.3); series citations (v1 cited no companion); updated status of Hypothesis 6.2; verification suite; appendix script relocated.

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