Zero as Non-Number: The Origin Criterion of Existence Value with 0(n) Notation

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Abstract

Numbers are defined as symbols denoting the value of existent being. However, zero does not denote an existent value; instead, it functions as the absolute origin criterion before existence emerges. Through the operational contradiction of $0 \times 9 = 0$ and the $0(n)$ notation, we prove the complete non-numerical nature of zero. This is a philosophical proposal that fundamentally reconsiders the ontological foundations of mathematical operations.

1 Introduction

1.1 Problem Statement

In traditional mathematics, zero is regarded as an element of the integer set $\mathbb{Z}$ and the identity element of an additive group. However, this definition overlooks zero's essential role. While all other numbers denote "existent quantities," zero uniquely represents "the absence of quantity." This paper argues that zero is not a number in the conventional sense, but rather a meta-mathematical reference point.

1.2 Core Premise

Numbers always presuppose "existent values." This research redefines zero as a meta-mathematical reference point, not a number itself. The foundation of this argument rests on distinguishing between entities that represent existing states and those that represent the threshold of existence.

2 The Essence of Numbers: Indicators of Existent Values

2.1 Philosophical Foundation of Existence Value

We propose the following definition:

Definition 2.1: *Existence Value* = a numerical quantity representing the state of specific entities.

Every conventional number carries an implicit ontological commitment—it presupposes the existence of something. Consider the following taxonomy:

- **1 represents:** one existent entity (minimal unit of being)
- **5 represents:** five existent entities (cardinality of set)
- **9 represents:** nine existent entities (quantitative expression)
- -3 **represents:** a directional state with reverse polarity (relative magnitude)

Each of these numbers assumes a fundamental precondition: there exists something to count, measure, or quantify.

2.2 Peano Axioms and the Structure of Numbers

In Peano Axioms, numbers are defined recursively as $S(n) = n + 1$, where S is the successor function. This definition ensures that all numbers exhibit finite growth and operate within states of existence. The axioms do not, however, clarify the ontological status of the starting point zero.

Core Observation: The Peano system begins with an arbitrary point (traditionally 0) but never justifies why that starting point should itself be called a “number.” All subsequent elements are generated by successive application of the successor function to entities that presuppose being. Zero, by contrast, precedes this entire structure.

3 Proof of Zero’s Non-Numerical Nature

3.1 Zero as Origin Criterion

We propose a revised definition:

Definition 3.1: *Zero* = the absolute origin criterion of being’s emergence—the threshold at which existence transitions from potential to actual.

The distinction can be visualized as:

Continuum of Existence:

$$0 \text{ (Reference Point/Pre-existence)} \longleftrightarrow 1, -3, 9, \ldots \text{ (Existence Values)}$$

This framework aligns with observations across multiple disciplines:

- **Physics:** In cosmology, $t = 0$ marks the Big Bang singularity—the universe begins with zero total energy-matter before expansion begins.
- **Mathematics:** The Cartesian origin $(0, 0)$ serves as the reference point of spatial measurement, not as a quantity itself.
- **Ontology:** In philosophical creation doctrines (ex nihilo), being emerges from nothingness—zero represents this threshold.

3.2 The Operational Contradiction: $0 \times 9 = 0$

Traditional mathematics asserts that $0 \times 9 = 0$ via the "absorption property of zero." However, this is a circular argument that presupposes zero is a number.

Consider the logical structure:

Operation	**Left Presupposition**	**Right Presupposition**	**Logical Possibility**	**Explanation**
1×9	Existent value $\times$ existent value	9 value exists	Possible	Existence $\times$ Existence
-3×9	Negated value $\times$ existent value	-27 value exists	Possible	Sign reversal
0×9	Reference criterion $\times$ existent value	Reference point?	Impossible	Category Error

Problem: If 0×9 were legitimate, the result should be "0(9)" (9 instances of reference point). Yet mathematicians simply assert this equals 0.

3.3 Logical Proof Process

Step 1: Define 9 as "the value representing 9 existent entities"

Step 2: Define 0 as "the criterion marking pre-existence" (not itself an existent value)

Step 3: Consider the operation "criterion $\times$ existent value"

- The criterion (0) is not an operand but a precondition of the operational system itself
- Applying a pre-existential operator to a post-existential value commits a category error
- The result would be ontologically incoherent

Conclusion: The equation $0 \times 9 = 0$ assumes zero is a number. This is circular reasoning.

3.4 The $0(n)$ Notation: An Alternative Expression

To resolve this contradiction, we introduce the $0(n)$ **notation**:

$$0 \times 9 = \text{"state of 9 reference points"} \to 0(9)$$

Definition 3.2: $0(n) =$ set of n reference points $\equiv$ single reference point 0

Logical Justification:

The reference point (0) is defined as the **unique and singular criterion of pre-existence**. It is not quantifiable in the way that numbers are. Therefore:

- The reference point cannot be pluralized without losing its essential property
- "9 reference points" is a logically impossible expression
- When we force the operation 0×9, the result remains a single reference point 0
- Therefore: $0(n) \equiv 0$ for all positive integers n

This notation clarifies what traditional mathematics leaves ambiguous:

Operation	Traditional	0(n) Notation	Interpretation
0×1	0	0(1)	Reference (non-quantifiable; singularity cannot plur
0×5	0	0(5)	Reference (non-quantifiable; "5 origins" is incoher
0×9	0	0(9)	Reference (non-quantifiable; uniqueness prevents enum

The $0(n)$ notation makes explicit what multiplication by zero means: absorption of quantitative content into the pre-existential reference state.

4 Philosophical and Scientific Implications

4.1 Interdisciplinary Reinterpretation

Our framework reveals deep parallels across domains:

Field	**Traditional View**	**This Research**
Mathematics	$0 \in \mathbb{Z}$ (identity element)	0 = meta-symbol (existence reference criterion)
Physics	$t = 0$ (Big Bang singularity)	$t = 0$ marks transition from non-being to being
Philosophy	0 = nothingness	0 = absolute ground enabling being to emerge

This reinterpretation does not reject mathematics' computational use of zero but rather clarifies its ontological status.

4.2 Reconsideration of Operations Involving Zero

When zero's non-numerical nature is recognized, other mathematical enigmas become clarified:

- $0 \div 0$ **(indeterminate form):** No longer a computational puzzle but a clear category error.
- 0^0 **(conventional assignment):** The arbitrary assignment becomes justified because zero is not a number subject to normal exponentiation rules.
- **Limit processes:** Calculus's use of zero finds firmer conceptual ground when understood as approach toward the reference criterion, not toward an actual value.

5 Discussion and Broader Implications

5.1 Preservation of Mathematical Utility

This redefinition does not invalidate mathematics. Zero continues to function perfectly as a computational tool in algebraic structures, calculus, and physics. What changes is our conceptual foundation: by abandoning the fictitious assumption that zero is a number and instead recognizing **zero as a meta-mathematical reference point**, we can explain the structural basis of mathematics more clearly and consistently. Zero's true nature is not hidden; rather, it strengthens the logical robustness of mathematics.

5.2 Complementarity with Infinity

Interestingly, infinity (∞) and zero occupy dual ontological positions:

- **Zero:** Pre-existential reference point (absolute minimum)
- **Infinity:** Post-existential unboundedness (absolute maximum)

Just as zero is not a number, infinity is not a number. Both are meta-mathematical concepts describing limits of the numerical domain rather than elements within it.

5.3 Implications for Set Theory and Logic

In set theory, the empty set ($\emptyset$) and the number zero (0) are often conflated, yet they represent distinct concepts:

- **$\emptyset$:** the absence of members (existential emptiness)
- **0:** the reference point before quantity itself emerges (pre-existential criterion)

Our framework suggests these require clearer distinction in foundational mathematics.

6 Conclusion

Zero is not a number. This conclusion follows rigorously from recognizing that numbers denote existent values while zero denotes the threshold before existence. The operational contradiction inherent in $0 \times 9 = 0$ dissolves when we cease treating zero as an operand and recognize it instead as the origin criterion of the numerical system itself.

The $0(n)$ notation makes this distinction explicit and tractable. This is not merely a philosophical exercise—it points toward deeper truths about the relationship between ontology (being) and mathematics (quantitative representation of being). Recognizing zero's true status opens new pathways for understanding the foundations of logic, set theory, and physics.

Future research should explore:

1. Formal axiomatization of mathematical systems where zero is explicitly excluded from numerical set $\mathbb{Z}$
2. Reconstruction of calculus and real analysis on this revised foundation

3. Implications for quantum mechanics and vacuum states

4. Connection between this framework and recent work on the ontology of nothingness

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