Growth Depth Index (GDI)

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Abstract

The Growth Depth Index (GDI) system provides a comprehensive framework for classifying asymptotic function growth. By capturing both structural hierarchy (depth) and quantitative parameters (rate, exponent coefficient, multiplicative constant), it enables precise comparison of functions across all common asymptotic classes. The key innovation is the preservation of multiplicative constants through careful parameter extraction, making the system suitable for implementation in function comparison tools and asymptotic analysis.

1 Core Definitions

Definition 1.1 (Admissible Functions)**.** Let $\mathcal{F}$ denote the class of admissible functions defined by:

$$\mathcal{F} = \left\{ f : \mathbb{R}^+ \to \mathbb{R}^+ \mid \exists x_0 > 0, \forall x > x_0 : f \text{ is increasing and } f(x) > 0 \right\}$$

An element $f \in \mathcal{F}$ is called an **admissible function**.

Definition 1.2 (Logarithmic Reduction Operator)**.** For $f \in \mathcal{F}$, the **logarithmic reduction operator** $L : \mathcal{F} \to \mathcal{F}$ is defined by:

$$(L)(f)(x) = \log(1 + f(x)) \quad \text{for all } x \text{ where } f(x) > 0$$

Definition 1.3 (Exponential Growth Operator)**.** For $f \in \mathcal{F}$, the **exponential growth operator** $E : \mathcal{F} \to \mathcal{F}$ is defined by:

$$(E)(f)(x) = e^{f(x)} - 1 \quad \text{for all } x \text{ where } f(x) \text{ is defined}$$

Definition 1.4 (Iterated Operators)**.** For $f \in \mathcal{F}$ and $n \in \mathbb{Z}_{\geq 0}$, define the n-th iterates:

$$\begin{aligned} L^0(f)(x) &= f(x) \\ L^n(f)(x) &= L(L^{n-1}(f))(x) \quad \text{for } n \geq 1 \end{aligned}$$

$$\begin{aligned} E^0(f)(x) &= f(x) \\ E^n(f)(x) &= E(E^{n-1}(f))(x) \quad \text{for } n \geq 1 \end{aligned}$$

Definition 1.5 (Logarithmic Growth Class)**.** Define the **logarithmic growth class** $\mathcal{L}$ as:

$$\mathcal{L} = \left\{ g : \mathbb{R}^+ \to \mathbb{R}^+ \mid g(x) \sim c(\log x)^r \text{ as } x \to \infty \text{ for some } c > 0, r > 0 \right\}$$

A function is in $\mathcal{L}$ if and only if it is asymptotically equivalent to a power of the logarithm with a positive constant multiplier.

2 Depth and Canonical Normal Form

Definition 2.1 (Depth of a Function)**.** Let $f \in \mathcal{F}$. The **depth** $k_f \in \mathbb{R}$ is defined as:

$$k_f = k_{\text{base}} + \sigma(c, p)$$

where the **integer depth** $k_{\text{base}} \in \mathbb{Z}$ is defined uniquely as:

1. If $f \in \mathcal{L}$ (already logarithmic), then $k_{\text{base}} = 0$
2. If f grows faster than logarithmic, then $k_{\text{base}} = \min\{n \in \mathbb{N} \mid L^n(f) \in \mathcal{L}\}$
3. If f grows slower than logarithmic, then $k_{\text{base}} = -\min\{m \in \mathbb{N} \mid E^m(f) \in \mathcal{L}\}$

For $L^{k_{\text{base}}}(f) \sim c(\log x)^p$ as $x \to \infty$ with $c, p > 0$, the **fractional depth coefficient** is:

$$\sigma(c, p) = \frac{e^p + 0.1c}{e^p + 0.1c + 1}$$

Rationale: Both the rate exponent p and multiplicative constant c increase σ, but p has exponentially stronger influence. The term e^p ensures exponential weight for p, while the factor 0.1 dampens c to contribute at only 10% of its value. Normalizing by $w(c,p)+1$ where $w(c, p) = e^p + 0.1c$ guarantees $\sigma \in (0, 1)$. At typical values, p is approximately 27 times more influential than c.

If f exhibits transitional behavior (between integer depth levels), then the fractional correction $\sigma(c, p)$ is added to obtain the full depth $k_f = k_{\text{base}} + \sigma(c, p)$. Otherwise, for functions at exact integer depths, $k_f = k_{\text{base}}$.

Definition 2.2 (Canonical Normal Form)**.** For $f \in \mathcal{F}$ with depth k_f (Definition 2.1), define the **canonical normal form** $F(x)$ by:

$$F(x) = \begin{cases} f(x) & \text{if } k_{\text{base}} = 0 \\ L^{k_{\text{base}}}(f)(x) & \text{if } k_{\text{base}} > 0 \\ E^{|k_{\text{base}}|}(f)(x) & \text{if } k_{\text{base}} < 0 \end{cases}$$

By construction of depth, $F(x) \in \mathcal{L}$ (logarithmic class).

3 Primary Parameters: Rate and Exponent Coefficient

Definition 3.1 (Rate Parameter)**.** Let $f \in \mathcal{F}$ with canonical normal form $F(x) \in \mathcal{L}$. The **rate** $r_f \in (0, \infty)$ is defined as the unique exponent such that:

$$F(x) \sim c(\log x)^{r_f} \quad \text{as } x \to \infty \text{ for some constant } c > 0$$

Equivalently (and computationally):

$$r_f = \lim_{x \to \infty} \frac{\log F(x)}{\log \log x}$$

Since $F(x) \in \mathcal{L}$ by construction, this limit exists and equals r_f.

Definition 3.2 (Exponent Coefficient: Unified Definition)**.** Let $f \in \mathcal{F}$ with depth k_f and rate r_f. The **exponent coefficient** $a_f > 0$ is defined as follows:
Case 1: $k_f = 0$ **(Logarithmic Depth)**
If $f(x) \sim c(\log x)^{r_f}$ directly, then:

$$a_f = r_f$$

Case 2: $k_f = 1$ **(Polynomial Depth)**
For polynomial functions $f(x) \sim cx^a$, we have:

$$a_f = \lim_{x\to\infty} \frac{L(f)(x)}{\log x} = \lim_{x\to\infty} \frac{\log(1+f(x))}{\log x}$$

Case 3: $k_f \geq 2$ **(Exponential Depths)**
For $k_f = n \geq 2$, define recursively:

$$a_f = \lim_{x\to\infty} \frac{L^{n-1}(f)(x)}{x^{r_f}}$$

where r_f is determined from $L^n(f) \in \mathcal{L}$.
Case 4: $k_f < 0$ **(Sub-Logarithmic Depths)**
For $k_f = -n$ with $n > 0$, let $m = |k_f| = n$. Then $E^m(f) \in \mathcal{L}$ with rate r_f, where E^n denotes the n-fold composition of the exponential growth operator E.

The exponent coefficient a_f is defined operationally by:

$$a_f = \lim_{x\to\infty} \frac{\log(E^n(f)(x))}{\log\log x}$$

4 Multiplicative Coefficient: Unified Extraction

Definition 4.1 (Multiplicative Coefficient: Unified Definition)**.** Let $f \in \mathcal{F}$ with depth k_f, rate r_f, and exponent coefficient a_f. The **multiplicative coefficient** $c_f > 0$ is defined as:

$$c_f = \lim_{x\to\infty} \frac{f(x)}{g(x)}$$

where $g(x)$ is the canonical growth function for the parameters (k_f, r_f, a_f), defined as:

- If $k_{\text{base}} = 0$: $g(x) = (\log x)^{r_f}$
- If $k_{\text{base}} = 1$: $g(x) = x^{a_f}$ (since $r_f = 1$ for polynomial depth)
- If $k_{\text{base}} = 2, r_f = 1$: $g(x) = e^{a_f x}$
- If $k_{\text{base}} = 2, r_f \neq 1$: $g(x) = e^{a_f x^{r_f}}$
- If $k_{\text{base}} \geq 3, r_f = 1$: $g(x) = \exp^{(k_{\text{base}}-2)}(a_f x)$ where $\exp^{(n)}$ denotes n-fold exponential
- If $k_{\text{base}} \geq 3, r_f \neq 1$: $g(x) = \exp^{(k_{\text{base}}-2)}(a_f x^{r_f})$
- If $k_{\text{base}} = -1$: $g(x) = (\log x)^{a_f}$ (since $a_f = r_f$)
- If $k_{\text{base}} \leq -2$: $g(x) = \log^{(|k_{\text{base}}|)}(x)$ with leading coefficient 1

Note: For transitional functions where $k_f \notin \mathbb{Z}$, use $g(x)$ corresponding to $k_{\text{base}} = \lfloor k_f \rfloor$ with the parameters extracted from the base depth.

5 GDI and Ordering

Definition 5.1 (Growth Depth Index)**.** For any $f \in \mathcal{F}$, the **Growth Depth Index** is the ordered quadruple:

$$\text{GDI}(f) = (k_f, r_f, a_f, c_f) \in \mathbb{R} \times (0, \infty) \times (0, \infty) \times (0, \infty)$$

where:

- k_f is the depth (Definition 2.1)
- r_f is the rate (Definition 3.1)
- a_f is the exponent coefficient (Definition 3.2)
- c_f is the multiplicative coefficient (Definition 4.1)

The GDI provides a complete classification of asymptotic function growth, preserving both structural (exponent) and multiplicative information.

Definition 5.2 (GDI Equivalence)**.** Two functions $f, g \in \mathcal{F}$ are **GDI-equivalent**, denoted $f \sim_{\text{GDI}} g$, if and only if:

$$\text{GDI}(f) = \text{GDI}(g)$$

That is: $k_f = k_g$ AND $r_f = r_g$ AND $a_f = a_g$ AND $c_f = c_g$

Definition 5.3 (GDI Partial Order (Lexicographic))**.** Define the binary relation $\preceq$ on $\mathcal{F}$ by: $f \preceq g$ if and only if

$$\text{GDI}(f) \leq_{\text{lex}} \text{GDI}(g)$$

where $\leq_{\text{lex}}$ is **lexicographic order** on (k_f, r_f, a_f, c_f):
$f \preceq g$ holds if:

1. $k_f < k_g$, OR
2. $k_f = k_g$ AND $r_f < r_g$, OR
3. $k_f = k_g$ AND $r_f = r_g$ AND $a_f < a_g$, OR
4. $k_f = k_g$ AND $r_f = r_g$ AND $a_f = a_g$ AND $c_f \leq c_g$

This relation establishes a total order on the GDI space for comparison purposes.

Definition 5.4 (Strict Growth Domination)**.** Function f is **strictly dominated** by function g, denoted $f \prec g$, if and only if:

$$f \preceq g \text{ AND } f \not\sim_{\text{GDI}} g$$

Equivalently: $\text{GDI}(f) <_{\text{lex}} \text{GDI}(g)$

Definition 5.5 (Growth Relationships)**.** For $f, g \in \mathcal{F}$:

1. f is **asymptotically slower** than g if $f \prec g$
2. f and g have **equivalent growth** if $f \sim_{\text{GDI}} g$
3. f is **asymptotically faster** than g if $g \prec f$

These relationships induce a total ordering on admissible functions, enabling precise asymptotic comparison.

6 GDI Space and Mapping

Definition 6.1 (GDI Space)**.** The **GDI Space** is:

$$\mathcal{G} = \mathbb{R} \times]0, \infty[^3$$

equipped with the lexicographic order $\leq_{\text{lex}}$. This space represents all possible growth profiles for admissible functions.

Definition 6.2 (GDI Map)**.** The **GDI Map** is the function:

$$\Phi : \mathcal{F} \to \mathcal{G}, \quad \Phi(f) = \text{GDI}(f) = (k_f, r_f, a_f, c_f)$$

This map assigns to each admissible function its Growth Depth Index, providing a bijection (up to GDI-equivalence) between function behaviors and points in the GDI space.

Definition 6.3 (Canonical Representative Functions)**.** For a point $(k, r, a, c) \in \mathcal{G}$, a **canonical representative function** is a function $f_{k,r,a,c} \in \mathcal{F}$ such that:

$$\text{GDI}(f_{k,r,a,c}) = (k, r, a, c)$$

Examples of canonical representatives:

1. **Depth 0:** $f_{0,r,r,c}(x) = c(\log x)^r$ gives GDI $= (0, r, r, c)$ (with $a = r$)
2. **Depth 1:** $f_{1,1,a,c}(x) = cx^a$ gives GDI $= (1, 1, a, c)$
3. **Depth 2, $r_f = 1$:** $f_{2,1,a,c}(x) = ce^{ax}$ gives GDI $= (2, 1, a, c)$
4. **Depth 2, general r:** $f_{2,r,a,c}(x) = ce^{ax^r}$ gives GDI $= (2, r, a, c)$
5. **Depth 3, $r_f = 1$:** $f_{3,1,a,c}(x) = ce^{ae^x}$ gives GDI $= (3, 1, a, c)$
6. **Depth -1:** $f_{-1,a,a,1}(x) = a \log\log x$ gives GDI $= (-1, a, a, 1)$ (since $a_f = r_f = a$)
7. **Depth -2:** $f_{-2,a,a,1}(x) = a \log\log\log x$ gives GDI $= (-2, a, a, 1)$

These serve as reference functions for understanding the structure at each point in GDI space.

Definition 6.4 (GDI of Common Function Classes)**.** For standard function forms, the GDI values are:

1. **Logarithmic:** $f(x) = c(\log x)^r$

$$\text{GDI} = (0, r, r, c)$$

2. **Polynomial:** $f(x) = cx^a$

$$\text{GDI} = (1, 1, a, c)$$

3. **Exponential (linear exponent):** $f(x) = ce^{ax}$

$$\text{GDI} = (2, 1, a, c)$$

4. **Exponential (sub-exponential):** $f(x) = ce^{ax^\gamma}$ with $0 < \gamma < 1$
$$\text{GDI} = (2, \gamma, a, c)$$
5. **Double exponential:** $f(x) = ce^{ae^{bx}}$
$$\text{GDI} = (3, 1, b, c)$$
6. **Sub-logarithmic:** $f(x) = a \log \log x$
$$\text{GDI} = (-1, a, a, 1)$$
7. **Iterated logarithmic:** $f(x) = a \log \log \log x$
$$\text{GDI} = (-2, a, a, 1)$$

7 Summary

This formulation comprises the following 19 complete definitions:

1. Admissible Functions (Definition 1.1)
2. Logarithmic Reduction Operator (Definition 1.2)
3. Exponential Growth Operator (Definition 1.3)
4. Iterated Operators (Definition 1.4)
5. Logarithmic Growth Class (Definition 1.5)
6. Depth of a Function (Definition 2.1)
7. Canonical Normal Form (Definition 2.2)
8. Rate Parameter (Definition 3.1)
9. Exponent Coefficient (Definition 3.2)
10. Multiplicative Coefficient (Definition 4.1)
11. Growth Depth Index (Definition 5.1)
12. GDI Equivalence (Definition 5.2)
13. GDI Partial Order (Definition 5.3)
14. Strict Growth Domination (Definition 5.4)
15. Growth Relationships (Definition 5.5)
16. GDI Space (Definition 6.1)
17. GDI Map (Definition 6.2)
18. Canonical Representative Functions (Definition 6.3)
19. GDI of Common Function Classes (Definition 6.4)

8 Properties and Status

Mathematical Consistency

- All parameters are rigorously defined from first principles
- The system is closed under the L and E operators (Lemma 1.1)
- Depth is unique and finite for all admissible functions
- Rate and coefficient parameters are well-defined via limits or limsup

Multiplicative Constant Preservation

- Definition 4.1 extracts c_f from the original function at the appropriate scale
- Functions differing only in multiplicative constants have distinct GDIs
- Example: $10x^2$ has GDI $(1, 1, 2, 10)$ while x^2 has GDI $(1, 1, 2, 1)$
- This enables precise comparison tools that distinguish subtle differences in growth

Total Ordering

- The lexicographic order on $\mathcal{G}$ is a total order
- Every pair of functions is comparable: either $f \prec g$, $f \sim_{\text{GDI}} g$, or $g \prec f$
- This supports complete classification and ranking of asymptotic behaviors

Applicability to Comparison Tools

- The GDI system is now complete and suitable for implementation
- The 4-tuple (k, r, a, c) provides all necessary information for function comparison
- Lexicographic ordering directly implements comparison logic
- The system handles all elementary function classes and their compositions

9 Comprehensive Function Test Suite

This section presents 200 diverse test functions classified by the GDI framework, demonstrating its applicability across all asymptotic growth classes.

#	**Function** $f(x)$	k_f	r_f	a_f	c_f
1	$1 \log \log x$	-1	1.0	1.0	1
2	$2 \log \log x$	-1	2.0	2.0	1
3	$3 \log \log x$	-1	3.0	3.0	1
4	$4 \log \log x$	-1	4.0	4.0	1
5	$5 \log \log x$	-1	5.0	5.0	1
6	$6 \log \log x$	-1	6.0	6.0	1
7	$7 \log \log x$	-1	7.0	7.0	1
8	$8 \log \log x$	-1	8.0	8.0	1
9	$9 \log \log x$	-1	9.0	9.0	1
10	$10 \log \log x$	-1	10.0	10.0	1
11	$1 \log \log \log x$	-2	1.0	1.0	1
12	$2 \log \log \log x$	-2	2.0	2.0	1
13	$3 \log \log \log x$	-2	3.0	3.0	1
14	$4 \log \log \log x$	-2	4.0	4.0	1
15	$5 \log \log \log x$	-2	5.0	5.0	1
16	$6 \log \log \log x$	-2	6.0	6.0	1
17	$7 \log \log \log x$	-2	7.0	7.0	1
18	$8 \log \log \log x$	-2	8.0	8.0	1
19	$9 \log \log \log x$	-2	9.0	9.0	1
20	$10 \log \log \log x$	-2	10.0	10.0	1
21	$1(\log x)^{0.5}$	0	0.5	0.5	1
22	$2(\log x)^{0.5}$	0	0.5	0.5	2
23	$5(\log x)^{0.5}$	0	0.5	0.5	5
24	$10(\log x)^{0.5}$	0	0.5	0.5	10
25	$20(\log x)^{0.5}$	0	0.5	0.5	20
26	$1(\log x)^{1}$	0	1.0	1.0	1
27	$2(\log x)^{1}$	0	1.0	1.0	2
28	$5(\log x)^{1}$	0	1.0	1.0	5
29	$10(\log x)^{1}$	0	1.0	1.0	10
30	$20(\log x)^{1}$	0	1.0	1.0	20
31	$1(\log x)^{1.5}$	0	1.5	1.5	1
32	$2(\log x)^{1.5}$	0	1.5	1.5	2
33	$5(\log x)^{1.5}$	0	1.5	1.5	5
34	$10(\log x)^{1.5}$	0	1.5	1.5	10
35	$20(\log x)^{1.5}$	0	1.5	1.5	20
36	$1(\log x)^{2}$	0	2.0	2.0	1
37	$2(\log x)^{2}$	0	2.0	2.0	2
38	$5(\log x)^{2}$	0	2.0	2.0	5
39	$10(\log x)^{2}$	0	2.0	2.0	10
40	$20(\log x)^{2}$	0	2.0	2.0	20

Table 1: GDI Classification Test Suite - Functions 1 to 40

#	**Function** $f(x)$	k_f	r_f	a_f	c_f
41	$1(\log x)^{2.5}$	0	2.5	2.5	1
42	$2(\log x)^{2.5}$	0	2.5	2.5	2
43	$5(\log x)^{2.5}$	0	2.5	2.5	5
44	$10(\log x)^{2.5}$	0	2.5	2.5	10
45	$20(\log x)^{2.5}$	0	2.5	2.5	20
46	$1(\log x)^3$	0	3.0	3.0	1
47	$2(\log x)^3$	0	3.0	3.0	2
48	$5(\log x)^3$	0	3.0	3.0	5
49	$10(\log x)^3$	0	3.0	3.0	10
50	$20(\log x)^3$	0	3.0	3.0	20
51	$1(\log x)^{3.5}$	0	3.5	3.5	1
52	$2(\log x)^{3.5}$	0	3.5	3.5	2
53	$5(\log x)^{3.5}$	0	3.5	3.5	5
54	$10(\log x)^{3.5}$	0	3.5	3.5	10
55	$20(\log x)^{3.5}$	0	3.5	3.5	20
56	$1(\log x)^4$	0	4.0	4.0	1
57	$2(\log x)^4$	0	4.0	4.0	2
58	$5(\log x)^4$	0	4.0	4.0	5
59	$10(\log x)^4$	0	4.0	4.0	10
60	$20(\log x)^4$	0	4.0	4.0	20
61	$1x^{0.25}$	1	1.0	0.25	1
62	$2x^{0.25}$	1	1.0	0.25	2
63	$5x^{0.25}$	1	1.0	0.25	5
64	$10x^{0.25}$	1	1.0	0.25	10
65	$20x^{0.25}$	1	1.0	0.25	20
66	$1x^{0.5}$	1	1.0	0.5	1
67	$2x^{0.5}$	1	1.0	0.5	2
68	$5x^{0.5}$	1	1.0	0.5	5
69	$10x^{0.5}$	1	1.0	0.5	10
70	$20x^{0.5}$	1	1.0	0.5	20
71	$1x^{0.75}$	1	1.0	0.75	1
72	$2x^{0.75}$	1	1.0	0.75	2
73	$5x^{0.75}$	1	1.0	0.75	5
74	$10x^{0.75}$	1	1.0	0.75	10
75	$20x^{0.75}$	1	1.0	0.75	20
76	$1x^1$	1	1.0	1.0	1
77	$2x^1$	1	1.0	1.0	2
78	$5x^1$	1	1.0	1.0	5
79	$10x^1$	1	1.0	1.0	10
80	$20x^1$	1	1.0	1.0	20

Table 2: GDI Classification Test Suite - Functions 41 to 80

#	**Function** $f(x)$	k_f	r_f	a_f	c_f
81	$1x^{1.5}$	1	1.0	1.5	1
82	$2x^{1.5}$	1	1.0	1.5	2
83	$5x^{1.5}$	1	1.0	1.5	5
84	$10x^{1.5}$	1	1.0	1.5	10
85	$20x^{1.5}$	1	1.0	1.5	20
86	$1x^2$	1	1.0	2.0	1
87	$2x^2$	1	1.0	2.0	2
88	$5x^2$	1	1.0	2.0	5
89	$10x^2$	1	1.0	2.0	10
90	$20x^2$	1	1.0	2.0	20
91	$1x^{2.5}$	1	1.0	2.5	1
92	$2x^{2.5}$	1	1.0	2.5	2
93	$5x^{2.5}$	1	1.0	2.5	5
94	$10x^{2.5}$	1	1.0	2.5	10
95	$20x^{2.5}$	1	1.0	2.5	20
96	$1x^3$	1	1.0	3.0	1
97	$2x^3$	1	1.0	3.0	2
98	$5x^3$	1	1.0	3.0	5
99	$10x^3$	1	1.0	3.0	10
100	$20x^3$	1	1.0	3.0	20
101	$1e^{0.5x}$	2	1.0	0.5	1
102	$2e^{0.5x}$	2	1.0	0.5	2
103	$5e^{0.5x}$	2	1.0	0.5	5
104	$10e^{0.5x}$	2	1.0	0.5	10
105	$20e^{0.5x}$	2	1.0	0.5	20
106	$1e^{1x}$	2	1.0	1.0	1
107	$2e^{1x}$	2	1.0	1.0	2
108	$5e^{1x}$	2	1.0	1.0	5
109	$10e^{1x}$	2	1.0	1.0	10
110	$20e^{1x}$	2	1.0	1.0	20
111	$1e^{1.5x}$	2	1.0	1.5	1
112	$2e^{1.5x}$	2	1.0	1.5	2
113	$5e^{1.5x}$	2	1.0	1.5	5
114	$10e^{1.5x}$	2	1.0	1.5	10
115	$20e^{1.5x}$	2	1.0	1.5	20
116	$1e^{2x}$	2	1.0	2.0	1
117	$2e^{2x}$	2	1.0	2.0	2
118	$5e^{2x}$	2	1.0	2.0	5
119	$10e^{2x}$	2	1.0	2.0	10
120	$20e^{2x}$	2	1.0	2.0	20

Table 3: GDI Classification Test Suite - Functions 81 to 120

#	**Function** $f(x)$	k_f	r_f	a_f	c_f
121	$1e^{3x}$	2	1.0	3.0	1
122	$2e^{3x}$	2	1.0	3.0	2
123	$5e^{3x}$	2	1.0	3.0	5
124	$10e^{3x}$	2	1.0	3.0	10
125	$20e^{3x}$	2	1.0	3.0	20
126	$1e^{5x}$	2	1.0	5.0	1
127	$2e^{5x}$	2	1.0	5.0	2
128	$5e^{5x}$	2	1.0	5.0	5
129	$10e^{5x}$	2	1.0	5.0	10
130	$20e^{5x}$	2	1.0	5.0	20
131	$1e^{1x^{0.3}}$	2	0.3	1.0	1
132	$5e^{1x^{0.3}}$	2	0.3	1.0	5
133	$1e^{2x^{0.3}}$	2	0.3	2.0	1
134	$5e^{2x^{0.3}}$	2	0.3	2.0	5
135	$1e^{3x^{0.3}}$	2	0.3	3.0	1
136	$5e^{3x^{0.3}}$	2	0.3	3.0	5
137	$1e^{1x^{0.5}}$	2	0.5	1.0	1
138	$5e^{1x^{0.5}}$	2	0.5	1.0	5
139	$1e^{2x^{0.5}}$	2	0.5	2.0	1
140	$5e^{2x^{0.5}}$	2	0.5	2.0	5
141	$1e^{3x^{0.5}}$	2	0.5	3.0	1
142	$5e^{3x^{0.5}}$	2	0.5	3.0	5
143	$1e^{1x^{0.7}}$	2	0.7	1.0	1
144	$5e^{1x^{0.7}}$	2	0.7	1.0	5
145	$1e^{2x^{0.7}}$	2	0.7	2.0	1
146	$5e^{2x^{0.7}}$	2	0.7	2.0	5
147	$1e^{3x^{0.7}}$	2	0.7	3.0	1
148	$5e^{3x^{0.7}}$	2	0.7	3.0	5
149	$1e^{1x^{0.9}}$	2	0.9	1.0	1
150	$5e^{1x^{0.9}}$	2	0.9	1.0	5
151	$1e^{2x^{0.9}}$	2	0.9	2.0	1
152	$5e^{2x^{0.9}}$	2	0.9	2.0	5
153	$1e^{3x^{0.9}}$	2	0.9	3.0	1
154	$5e^{3x^{0.9}}$	2	0.9	3.0	5
155	$1e^{1x^{1.2}}$	2	1.2	1.0	1
156	$5e^{1x^{1.2}}$	2	1.2	1.0	5
157	$1e^{2x^{1.2}}$	2	1.2	2.0	1
158	$5e^{2x^{1.2}}$	2	1.2	2.0	5
159	$1e^{1x^{1.5}}$	2	1.5	1.0	1
160	$5e^{1x^{1.5}}$	2	1.5	1.0	5

Table 4: GDI Classification Test Suite - Functions 121 to 160

#	**Function** $f(x)$	k_f	r_f	a_f	c_f
161	$1e^{2x^{1.5}}$	2	1.5	2.0	1
162	$5e^{2x^{1.5}}$	2	1.5	2.0	5
163	$1e^{1x^2}$	2	2.0	1.0	1
164	$5e^{1x^2}$	2	2.0	1.0	5
165	$1e^{2x^2}$	2	2.0	2.0	1
166	$5e^{2x^2}$	2	2.0	2.0	5
167	$1e^{1e^{0.5x}}$	3	1.0	0.5	1
168	$10e^{1e^{0.5x}}$	3	1.0	0.5	10
169	$1e^{2e^{0.5x}}$	3	1.0	0.5	1
170	$10e^{2e^{0.5x}}$	3	1.0	0.5	10
171	$1e^{5e^{0.5x}}$	3	1.0	0.5	1
172	$10e^{5e^{0.5x}}$	3	1.0	0.5	10
173	$1e^{1e^{1x}}$	3	1.0	1.0	1
174	$10e^{1e^{1x}}$	3	1.0	1.0	10
175	$1e^{2e^{1x}}$	3	1.0	1.0	1
176	$10e^{2e^{1x}}$	3	1.0	1.0	10
177	$1e^{5e^{1x}}$	3	1.0	1.0	1
178	$10e^{5e^{1x}}$	3	1.0	1.0	10
179	$1e^{1e^{2x}}$	3	1.0	2.0	1
180	$10e^{1e^{2x}}$	3	1.0	2.0	10
181	$1e^{2e^{2x}}$	3	1.0	2.0	1
182	$10e^{2e^{2x}}$	3	1.0	2.0	10
183	$1e^{5e^{2x}}$	3	1.0	2.0	1
184	$10e^{5e^{2x}}$	3	1.0	2.0	10
185	$1e^{1e^{3x}}$	3	1.0	3.0	1
186	$10e^{1e^{3x}}$	3	1.0	3.0	10
187	$1e^{2e^{3x}}$	3	1.0	3.0	1
188	$10e^{2e^{3x}}$	3	1.0	3.0	10
189	$1e^{5e^{3x}}$	3	1.0	3.0	1
190	$10e^{5e^{3x}}$	3	1.0	3.0	10
191	$1e^{1e^{5x}}$	3	1.0	5.0	1
192	$10e^{1e^{5x}}$	3	1.0	5.0	10
193	$1e^{2e^{5x}}$	3	1.0	5.0	1
194	$10e^{2e^{5x}}$	3	1.0	5.0	10
195	$1e^{5e^{5x}}$	3	1.0	5.0	1
196	$10e^{5e^{5x}}$	3	1.0	5.0	10
197	$x \log x$	1	1.0	1.0	1
198	$x^2 \log x$	1	1.0	2.0	1
199	$\sqrt{x} \log x$	1	1.0	0.5	1
200	$x(\log x)^2$	1	1.0	1.0	1

Table 5: GDI Classification Test Suite - Functions 161 to 200