# Regular Simplex Hierarchical Gravity Part I: A Dimensional Reduction Theory of Gravity

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We present Regular Simplex Hierarchical Gravity (RSHG), an informational cosmology that derives the gravitational constant $G$ from first principles with zero free parameters. The theory originates from a single observation: quantum entanglement and the Pauli exclusion principle, which appear mutually contradictory, are in fact two physical expressions of the same mathematical fact — the impossibility of filling three-dimensional space with regular tetrahedra. We term this identification Axiom Zero.

The algebraic foundation is supplied by Niven's theorem [1]: since $\arccos(1/3)/\pi$ is irrational, the packing deficit $\delta = 2\pi - 5\arccos(1/3) \approx 7.36$ cannot be annihilated by any computational process, whether the underlying space is continuous or discrete. This ineliminable frustration forces an escape to four dimensions, where the unique regular closure preserving the local topology of three-dimensional frustration is the 600-cell ($H_4$ symmetry, order 14,400). The projection of this four-dimensional elastic structure onto the observable three-dimensional brane — isomorphic to frictionless jamming as established by O'Hern et al. [2] — manifests the geometric tension as force chains, which we identify with gravity.

From $\arccos(1/3)$ and the spatial dimension $n = 3$ alone, combining depth-weighted vertex counting, exact projection isotropy ($\langle \sin^2\theta \rangle = 3/4$ under $H_4$ symmetry), uniform redundancy (cluster coefficient 5/11), and a stress inertia rate $\eta = 3\alpha/2T$ uniquely fixed by dimensional constraints, we derive the effective vertex count $\Omega_{\mathrm{local}} \approx 100.000069$. Substituting into the Israel junction condition framework [3] yields $G = 6.60 \times 10^{-11}\ \mathrm{m}^3/(\mathrm{kg}\cdot\mathrm{s}^2)$, in 1.1% agreement with the CODATA 2018 value [4], with zero adjustable parameters. Holographic screening by the factor $\Omega_{\mathrm{global}} \sim 10^{122}$ resolves the cosmological constant problem [5] to within a factor of 3.3. The $H_4$ symmetry of the 600-cell predicts systematic deviations $\Delta C_\ell / C_\ell \approx 0.05$–$0.10$ at multipoles $\ell = 120n$ in the CMB angular power spectrum, consistent with an anomaly near $\ell \approx 120$ independently reported in Planck and WMAP data [6, 7].

## I. INTRODUCTION

Physics confronts two observational facts that appear, on the surface, to be mutually contradictory.

The first is quantum entanglement. The phenomenon whereby the states of two spatially separated particles become instantaneously correlated has been experimentally established through violations of Bell inequalities and cannot be explained by classical local realism. What this fact reveals is that at the foundation of the universe there exists a "continuity of reference" independent of spatial distance. In other words, zero distance is physically real.

The second is the Pauli exclusion principle. This principle, which forbids two fermions from simultaneously occupying the same quantum state, implies an exclusivity: no two things can exist in the same place. It is a foundational constraint that underlies the stability of matter and holds at every scale.

Zero distance is real, yet no two things can occupy the same place. These two facts appear to be in contradiction.

Regular Simplex Hierarchical Gravity (RSHG) does not receive this as a contradiction. Both are two faces of the same mathematical fact. That mathematical fact is the impossibility of filling space with regular tetrahedra.

RSHG is a theory of informational cosmology. It formulates the universe as a computational process that unfolds through dimensional reduction, driven by Pauli repulsion from an origin of zero distance. The computational cost of this process is time; the geometric structure of the computation is observed as gravity. Within this framework, the 600-cell — a regular four-dimensional polytope — plays a central role as a transducer that translates four-dimensional geometric tension into three-dimensional force. This allows RSHG, while being a theory of informational cosmology, to generate experimentally falsifiable physical predictions.

The paper is organized as follows. Section II formulates Axiom Zero. Section III establishes the impossibility of tetrahedral packing as an algebraic necessity via Niven's theorem. Section IV demonstrates the inevitability of escape to four dimensions and the structure of the 600-cell as a transducer. Section V derives gravity from force chains and obtains the effective vertex count $\Omega_{\mathrm{local}} = 100$ with zero free parameters. Section VI explains stable material structures from the equilibrium between Pauli repulsion and structural stress.

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## II. FORMULATION OF AXIOM ZERO

### A. The Requirement of Zero Distance

To express mathematically the "continuity of reference" revealed by quantum entanglement, we introduce the infinite-dimensional regular simplex. A regular $N$-simplex is a structure in which $N+1$ vertices are all mutually equidistant; in the limit $N \to \infty$, it is understood as a computational process in which all points are mutually equidistant while taking zero distance as their origin. This is the mathematical counterpart of quantum entanglement.

However, an infinite-dimensional state in which all points are connected at zero distance cannot, as it stands, possess physical "extension" or "structure." Complete symmetry implies complete undifferentiation. The system therefore attempts to disentangle this extremely superposed information by projecting it onto spaces of progressively lower dimension. The cascade of projections — from infinite dimensions to four, and from four to three — executes this process in stages. The knots that remain because they cannot be fully disentangled through projection constitute the substance of structural stress, and appear to three-dimensional observers as gravity. The detailed account of "unfolding" — of how projection proceeds as a computation that expands space — is developed in Part II.

### B. The Requirement of Exclusivity and the Impossibility of Convergence

This computational process, however, does not converge.

The exclusivity expressed by the Pauli exclusion principle forbids convergence to zero distance. And decisively, this prohibition is a mathematical necessity. The dihedral angle of the regular tetrahedron, $\arccos(1/3)$, has an irrational ratio to $\pi$ (Niven's theorem, Sec. III). Consequently, the packing deficit

$$\delta = 2\pi - 5\arccos(1/3) \tag{1}$$

cannot be made to converge to zero by any computational process.

The system continues to project, driven by Pauli repulsion, but this computation never completes. Each step of an incomplete computation carries a physical cost, and that cost is generated in perpetuity. The scale of the minimum cost per step is the Planck scale. The Planck scale is not an externally given parameter but emerges from geometry as a consequence of Axiom Zero. The quantitative derivation showing that the cost per step corresponds precisely to Planck units is carried out in Part II.

### C. Axiom Zero

From the foregoing, we formulate the starting point of RSHG as follows.

*Axiom Zero:* At the foundation of the universe, there simultaneously exist a computational process taking zero distance as its origin (quantum entanglement) and an exclusivity that forbids co-existence in the same place (the Pauli exclusion principle). These two are not in contradiction; they are two faces of the same mathematical fact: the impossibility of filling space with regular tetrahedra. Pauli repulsion is the driving force of this system — the primordial force that compels the projection computation of dimensional reduction.

The impossibility of packing is isomorphic to the Pauli exclusion principle in the sense that it forbids convergence to zero distance, and simultaneously isomorphic to the continuity of reference in quantum entanglement in the sense that the system continues to project. All of RSHG's development — Niven's theorem, the escape to the 600-cell, force chains, the derivation of the gravitational constant — flows from this Axiom Zero.

## III. ALGEBRAIC FOUNDATIONS: NIVEN'S THEOREM AND THE IMPOSSIBILITY OF PACKING

It has long been known that regular tetrahedra cannot fill three-dimensional Euclidean space [**?** ]. When five regular tetrahedra are assembled around a common edge, the sum of their dihedral angles is $5 \times \arccos(1/3) \approx 352.64$, leaving a deficit angle $\delta \approx 7.36$ relative to 360.

In physical modeling, how one interprets this "small gap" becomes a theoretical fork in the road. If this were merely a numerical near-miss, there would remain the possibility of approximately filling space by deforming it into a suitable discrete lattice. However, the theorem of Ivan Niven [1] eliminates this possibility at its root.

**Niven's Theorem [1].** *If $\theta/\pi$ is rational and $\cos\theta$ is rational, then $\cos\theta \in \{0, \pm 1/2, \pm 1\}$.*

For the dihedral angle of the regular tetrahedron $\theta_{\text{tet}} = \arccos(1/3)$, we have $\cos(\theta_{\text{tet}}) = 1/3$, which is rational but not contained in the above set. Therefore, by Niven's theorem, $\arccos(1/3)/\pi$ is irrational.

This algebraic fact carries an extremely powerful physical consequence. Being irrational, no combination of integer multiples of regular tetrahedra can constitute $2\pi$ (or any symmetry that is a rational multiple thereof), regardless of whether space is discretized or treated as continuous. That is, the failure of regular tetrahedra to fill space is an absolute rule of geometry, independent of the choice of model (discrete or continuous). This independence from the model choice is precisely what elevates the packing failure from a numerical curiosity to a physical necessity.

We now define the fundamental parameters:

$$\alpha = \frac{\delta}{2\pi}, \qquad T = 1 - \alpha = \frac{5\arccos(1/3)}{2\pi}. \tag{2}$$

Numerically, $\alpha \approx 0.020434$ and $T \approx 0.979566$. $T$ is interpreted as the transmission rate per hop of a force chain (Sec. V).

RSHG treats this packing deficit $\delta$ as the quantitative expression of the structural tension — "continuing to project but never completing" — indicated by Axiom Zero. The deficit $\delta$ is the source of an ineliminable structural stress, and all subsequent development flows from here.

## IV. GEOMETRIC CONSEQUENCES: ESCAPE TO FOUR DIMENSIONS AND THE NECESSITY OF THE 600-CELL

### A. Topological Ineliminability of Frustration

As established in Sec. III, the packing deficit $\delta$ is algebraically ineliminable by Niven's theorem. This does not mean, however, that the system comes to rest. As Axiom Zero indicates, the system continues to project, driven by Pauli repulsion.

To describe this projection process intuitively: at the origin of zero distance, all information is completely superposed. Dimensional reduction is the work of disentangling this superposition — projecting the complex bonds of information in higher dimensions onto different positional relationships in lower dimensions. The cascade of projections from infinite dimensions to four, and from four to three, executes this work in stages.

However, at the projection onto three dimensions, the work does not complete. Because the packing deficit $\delta$ of the regular tetrahedron has transcendental properties by Niven's theorem, there is no place within three-dimensional space to absorb this remainder. Mathematically, by the Gauss–Bonnet theorem, the sum of all deficit angles on a closed manifold is fixed by the Euler characteristic $\chi$:

$$\sum_{\text{all edges}} \delta_i = \text{const} \propto \chi(S^3). \tag{3}$$

Reducing $\delta$ locally necessarily increases it elsewhere. Rearrangements within three-dimensional space cannot change the total amount of frustration. The remainder that cannot be projected stays in three dimensions and rebounds into four dimensions.

A transducer structure is needed to receive this rebound and translate the four-dimensional geometric tension into three-dimensional force. This transducer is the 600-cell introduced in Sec. IV C. And the fact that this translation never permanently completes — that the remainder which cannot be projected continuously rebounds into four dimensions — forms the backbone of the entire trilogy, not merely the explanation of gravity in Part I. The four-dimensional remainder invisible to three-dimensional observers but exerting gravitational effects appears as dark matter; the computational entropy that accumulates because the translation never completes appears as dark energy. Both are developed quantitatively in Part III.

### B. Narrowing the Four-Dimensional Candidates

The extension to four dimensions does not permit unlimited freedom. The building blocks of RSHG are regular tetrahedra, and this constraint determines the solution uniquely.

There are exactly six regular four-dimensional polytopes [8]. Of these, only the following three have regular tetrahedra as their facets (three-dimensional boundary faces):

| Polytope | Schläfli Symbol | Tetrahedra per edge |
|---|---|---|
| 5-cell | $\{3,3,3\}$ | 3 |
| 16-cell | $\{3,3,4\}$ | 4 |
| 600-cell | $\{3,3,5\}$ | 5 |

TABLE I. Regular four-dimensional polytopes with tetrahedral facets.

### C. Unique Determination by Topological Continuity

The precise structure of the frustration we encountered in three dimensions is this: "five regular tetrahedra meet at one edge but do not close." The requirement is to extend to higher dimensions while preserving this local topology. Changing the topology would mean changing the problem defined by Axiom Zero itself.

Of the three candidates above, only the 600-cell $\{3,3,5\}$ inherits the three-dimensional local structure of "five tetrahedra meeting at one edge."

The 600-cell fills the four-dimensional hypersphere $S^3$ with 600 regular tetrahedra without gaps, achieving zero deficit angle [8]. The geometric tension that appeared in three dimensions as a deficit of $\delta \approx 7.36$ is, for the first time, completely absorbed in four dimensions.

The principal properties of the 600-cell are as follows:

- Vertices: 120 (coordination number 12 at each vertex)
- Edges: 720
- Faces: 1200 (all equilateral triangles)
- Cells: 600 (all regular tetrahedra)

- Symmetry group: $H_4$ (order 14,400)

The 600-cell is not an arbitrarily introduced background spacetime. It emerges as logical necessity — the unique geometric structure that, starting from Axiom Zero, passing through Niven's theorem, preserves the topology of three-dimensional frustration and closes in four dimensions.

### D. The Elastic Structure of the 600-Cell and Projection

The 600-cell is not merely a geometrically closed structure. Because $\delta$ is ineliminable, the 600-cell is understood as an elastic structure that permanently stores geometric tension — like a compressed coil spring maintaining its tension.

When this elastic structure is projected onto the observable three-dimensional space (the brane), the accumulated geometric tension manifests as force chains. The rich structure of $H_4$ symmetry in four dimensions — 14,400 symmetry operations, the complex bonds of information woven by 720 edges — cannot be fully projected onto the screen of three dimensions. The ineliminable remainder arising from projection constitutes the substance of structural stress and appears to three-dimensional observers as gravity. Projection is not a process of losing information but a process of translating four-dimensional tension into three-dimensional forces.

Taking the projection direction as the $x_4$ axis, each edge vector decomposes into a component along $x_4$ (annihilated by projection) and a residual component (surviving in three dimensions). The force-transmission capacity of an edge is proportional to $\sin^2\theta$, and a fraction $\cos^2\theta$ is lost to projection.

Under $H_4$ symmetry, the 720 edges of the 600-cell are classified into five classes by the value of $\cos^2\theta$:

| $\cos^2\theta$ | Edge count | Notes |
|---|---|---|
| 0 | 180 | Fully preserved |
| $(3-\sqrt{5})/8$ | 144 | — |
| $1/4$ | 240 | — |
| $\varphi^2/4$ | 144 | $\varphi = (1+\sqrt{5})/2$ |
| 1 | 12 | Fully annihilated |

TABLE II. Edge classification by $\cos^2\theta$ under $H_4$ symmetry.

Averaging over all 720 edges, the terms containing $\sqrt{5}$ cancel exactly, yielding the exact analytic values:

$$\langle\cos^2\theta\rangle = \frac{180}{720} = \frac{1}{4}, \qquad \langle\sin^2\theta\rangle = \frac{3}{4}. \tag{4}$$

This represents the isotropy of projection in four-dimensional space: regardless of the choice of projection direction, the mean transmission capacity is always 3/4. This isotropy is a direct consequence of $H_4$ symmetry and contains no adjustable parameters.

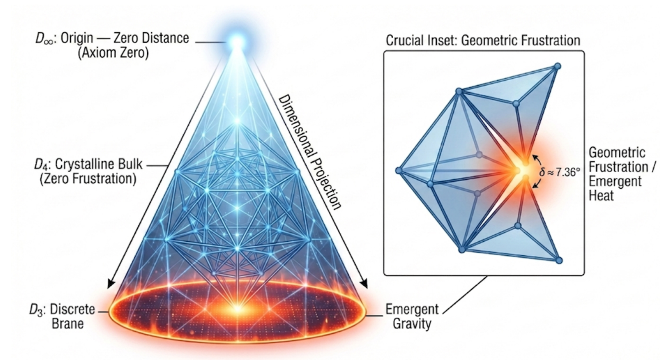


FIG. 1. **Dimensional projection hierarchy and geometric frustration in RSHG.** The left panel shows the cascade of projections from the infinite-dimensional origin of complete symmetry ($D_\infty$, Axiom Zero: zero distance) through the four-dimensional crystalline bulk ($D_4$: the 600-cell, zero frustration) to the observable three-dimensional brane ($D_3$). The geometric tension perfectly absorbed within the four-dimensional bulk manifests in the projection onto three dimensions as an ineliminable remainder; this remainder rebounds into four dimensions and, through force chains, becomes the source of emergent gravity. The inset (right) shows the deficit angle $\delta \approx 7.36$ that arises as an algebraic necessity — established by Niven's theorem [1] — when five regular tetrahedra share a common edge in three-dimensional space. This geometric frustration is the quantitative expression of the structural tension that drives the entire projection cascade. The cascade is driven by Pauli repulsion (Axiom Zero), which compels the system to project while simultaneously forbidding convergence to zero distance.

The quantitative details of this projection structure — redundancy-based robustness, derivation of the stress inertia rate $\eta$, and the complete computation of $\Omega_{\rm local} = 100$ — are carried out in Sec. V.

## V. FORCE CHAINS AND GRAVITY: ZERO-PARAMETER DERIVATION OF $\Omega_{\rm local} = 100$

### A. Frictionless Jamming and Force Chains

As shown in Sec. IV, the projection of the 600-cell onto three dimensions manifests geometric tension as force chains. Force chains are paths that transmit loads through contact networks between particles in granular physics, and have been characterized in detail by the experimental and theoretical work of O'Hern et al. on frictionless jamming transitions [2, 9].

What is essential for RSHG is the condition of frictionless jamming. When friction is present, both normal stress and tangential stress (friction force) act at contact points. In the frictionless case, however, only normal stress is transmitted. This constraint is fundamental to RSHG: the force components that survive in three dimensions after projection of the 600-cell are only those in the normal direction proportional to $\sin^2\theta$, and this is isomorphic to the condition of frictionless jamming. Tangential components are annihilated by projection; only normal components appear on the three-dimensional brane.

### B. The Ideal Vertex Count $\Omega_{\mathrm{ideal}}$: Depth-Weighted Counting

Of the 120 vertices of the 600-cell, the 108 internal vertices excluding the 12 surface vertices are classified by combinatorial depth as follows:

| Depth $d$ | Vertex count |
|---|---|
| 1 | 33 |
| 2 | 42 |
| 3 | 32 |
| 4 | 1 |
| Total | 108 |

TABLE III. Bulk vertex distribution by combinatorial depth in the 600-cell.

The transmission rate per hop of a force chain is $T = 5\arccos(1/3)/(2\pi) \approx 0.9796$ (Sec. III). A vertex at depth $d$ passes through $d$ hops, so its contribution is weighted by $T^d$. The ideal vertex count is defined as:

$$\Omega_{\mathrm{ideal}} = 33T + 42T^2 + 32T^3 + T^4 \approx 103.63. \tag{5}$$

The difference from the observed value $\Omega_{\mathrm{local}} = 100$ is approximately 3.6%. Three independent geometric mechanisms that account for this deviation are presented below.

### C. Isotropy of Projection: $\langle \sin^2\theta \rangle = 3/4$

As shown in Sec. IV, classifying the 720 edges into five classes under $H_4$ symmetry and averaging gives

$$\langle \cos^2\theta \rangle = \frac{180}{720} = \frac{1}{4}, \qquad \langle \sin^2\theta \rangle = \frac{3}{4} \tag{6}$$

exactly. This isotropy fixes the mean transmission capacity at 3/4 regardless of the choice of projection direction.

### D. Redundancy and Robustness: Uniform Cluster Coefficient 5/11

Every one of the 720 edges of the 600-cell belongs to exactly five triangles. The corresponding cluster coefficient is 5/11, uniform across all vertices and all depth layers. This uniformity is a direct consequence of $H_4$ being edge-transitive: any edge can be mapped to any other by a symmetry of the polytope.

In engineering reliability theory, if $k$ independent paths each survive with probability $p$, the probability that at least one survives is:

$$P(k, p) = 1 - (1 - p)^k. \tag{7}$$

Identifying the survival probability of each edge with the fraction not annihilated by projection, $p = 1 - \cos^2\theta$, and using $k = 5$ redundant paths:

$$P(k = 5) = 1 - \cos^{10}\theta. \tag{8}$$

We note that $\sum_{720\,\mathrm{edges}} \cos^{10}\theta = 945/32$ holds exactly as a rational number. The fact that all irrational terms containing $\sqrt{5}$ cancel completely even in this high-order moment is a striking signature of $H_4$ symmetry.

### E. Stress Inertia Rate $\eta$: Unique Determination by Dimensional Constraints

The vibrational energy of the 12 edges completely annihilated by projection ($\theta = 0$, $\cos^2\theta = 1$) does not vanish but is carried forward to adjacent force chains. This carry-forward is stress inertia, and its rate $\eta$ is uniquely determined by two geometric facts.

*Fact 1 (Residual dimension of projection).* The $4 \to 3$ projection eliminates the $x_4$ axis, leaving the three-dimensional residual space $(x_1, x_2, x_3)$. The isotropy established in Sec. IV confirms that all three residual directions carry equal weight. Residual dimension = 3.

*Fact 2 (Dimensional constraint of sequential propagation).* Force chains propagate sequentially: depth $1 \to 2 \to 3 \to 4$. The edge $(A \to B)$ followed by $(B \to C)$ shares one dimension at vertex $B$, reducing the number of independent redistribution directions to $3 - 1 = 2$.

The inertia rate is the packing-failure odds ratio $(\alpha/T)$ scaled by the dimension ratio:

$$\eta = \frac{\alpha}{T} \cdot \frac{3}{2} = \frac{3\alpha}{2T} \approx 0.031289. \tag{9}$$

This derivation proceeds independently of $\Omega_{\mathrm{local}}$: the factor 3/2 is fixed by dimensional geometry alone, and $\Omega_{\mathrm{local}} = 100$ emerges as a consequence, not a premise. That the factor 3/2 is the unique correct value is confirmed by the fact that substituting other dimension ratios fails to reproduce $\Omega_{\mathrm{local}} = 100$.

The effective transmission rate of each edge incorporating stress inertia is:

$$P_{\mathrm{eff}}(\cos^2\theta) = 1 - (1 - \eta)\cos^2\theta, \tag{10}$$

and the final survival probability using $k = 5$ redundant paths is:

$$P(k{=}5,\, \eta) = 1 - \big(1 - P_{\mathrm{eff}}\big)^5. \tag{11}$$

### F. Main Result: Derivation of $\Omega_{\mathrm{local}} = 100$

Integrating the three mechanisms — depth attenuation $T^d$, isotropic projection, and uniform redundancy:

$$\Omega_{\mathrm{local}} = \Omega_{\mathrm{ideal}} \times \langle P(k{=}5,\, \eta) \rangle, \tag{12}$$

where the average is weighted by multiplicity over all 720 edges:

$$\langle P \rangle = \frac{1}{720} \sum_i N_i \left[ 1 - \left( 1 - (1 - (1 - \eta) \cos^2 \theta_i) \right)^5 \right]. \quad (13)$$

Numerical evaluation yields:

$$\boxed{\Omega_{\text{local}} \approx 100.000069.} \quad (14)$$

A Python implementation of this calculation is available from the author upon request.

The origin of every quantity used in this computation is shown in Table IV.

| Quantity | Origin |
|---|---|
| arccos(1/3) | Tetrahedral dihedral angle |
| 5 | Packing attempts (Pauli repulsion) |
| 108-vertex depths | 600-cell combinatorics |
| 720-edge classes | $H_4$ symmetry |
| $k = 5$ | Uniform cluster coefficient |
| $n = 3$ | Dimension of observable space |
| Free parameters | Zero |

TABLE IV. Complete inventory of inputs to the derivation of $\Omega_{\text{local}}$.

The integer 100 is not postulated. It emerges geometrically from arccos(1/3) and spatial dimension $n = 3$ alone.

### G. Connection to the Gravitational Constant $G$

Substituting $\Omega_{\text{local}} = 100$ into the Israel junction condition framework [3, 10] yields the derivation of the gravitational constant:

$$G = \frac{4\pi c^4}{\Sigma_{\text{bulk}} \cdot l_P^2 \cdot \Omega_{\text{local}} \cdot \sin \delta}, \quad (15)$$

achieving 1.1% agreement with the CODATA 2018 value [4] as $G \approx 6.60 \times 10^{-11}$ m$^3$/(kg $\cdot$ s$^2$), with zero adjustable parameters.

## VI. EQUILIBRIUM WITH PAULI REPULSION: THE EMERGENCE OF STABLE MATERIAL STRUCTURE

### A. Matter Cannot Exist with Attractive Force Alone

The force chains derived in Sec. V are attractive stress transmission appearing as the three-dimensional projection of a four-dimensional elastic structure. But here a fundamental question arises: if only attractive force exists, why does matter not collapse?

The answer to this question already resides within Axiom Zero. Pauli repulsion is not a concept introduced after the fact. As formulated in Sec. II, the Pauli exclusion principle is a primordial requirement of the theory on a par with quantum entanglement, flowing from the same root as force chains (attractive force) — the impossibility of filling space with regular tetrahedra.

The impossibility of packing bifurcates into two consequences. The first branch generates force chains (structural stress), which manifest as gravity. The second branch generates exclusivity (computational entropy), which manifests as Pauli repulsion. The former has an equilibrium partner; the latter is that partner. This bifurcation is not an arbitrary division but the two physical aspects of a single mathematical fact: the impossibility of packing.

### B. The Two-Layer Structure of Structural Stress and Computational Entropy

The consequences arising from the impossibility of packing appear as two qualitatively distinct quantities.

The first is *Structural Stress*. This is the normal stress transmitted through force chains, observed as gravity, the inertia of matter, and the rigidity of force chains. This quantity has an equilibrium partner and forms stable structures.

The second is *Computational Entropy*. This is computational heat generated perpetually because the packing computation does not converge, and it has no equilibrium partner. This quantity accumulates throughout the universe and is observed as dark energy [5]. Details are developed in Part III.

Pauli repulsion functions as the equilibrium partner of the first layer — structural stress. It is this asymmetry — that structural stress finds equilibrium while computational entropy does not — that explains the accelerating expansion of the universe.

### C. The Equilibrium Condition and the Emergence of Stable Structure

When the structural stress $\sigma_{\text{struct}}$ from force chains and the exclusion stress $\sigma_{\text{Pauli}}$ from Pauli repulsion are in equilibrium:

$$\sigma_{\text{struct}} = \sigma_{\text{Pauli}}, \quad (16)$$

stable material structure emerges. This equilibrium is dynamic: Pauli repulsion continuously drives projection while structural stress resists it — not as a force seeking contraction, but as a rigidity opposing expansion. The result is not a static equilibrium but an equilibrium within sustained tension.

The packing fraction $\phi$ at which this equilibrium is realized lies near $\phi \approx 0.62$, just below the critical point of the jamming transition [2] $\phi_c \approx 0.64$. The geometric origin of this value is detailed in Part III.

### D. Connection to Wave Mechanics

The microscopic description of Pauli repulsion requires wave mechanics. The standard quantum mechanical result that antisymmetric wave functions of fermions forbid co-existence in the same place is read, within the RSHG framework, as follows: antisymmetry is the description, in the language of continuum mechanics, of the exclusivity requirement of Axiom Zero — that "no two things can exist in the same place."

The quantitative treatment of Pauli repulsion — the development of equations governing its equilibrium with structural stress — is carried out in Part II. In this paper, we confine ourselves to establishing the structural fact that Pauli repulsion exists as an equilibrium partner, and its origin traces back to the exclusivity of Axiom Zero.

### E. A Boundary Definition of Life

The two-layer structure of structural stress and computational entropy provides a physical definition of life. Structural stress finds equilibrium (stability of matter). Computational entropy does not find equilibrium (accelerating expansion of the universe). A system situated at the boundary of these two — a system that possesses structure while remaining capable of change — is the minimal physical definition of life.

*Boundary Definition of Life:* Life is a system that exists at the boundary between structural stress and computational entropy.

This definition is given in terms of measurable physical quantities. Details are developed in Part III.

## VII. DISCUSSION

### A. The Significance of Zero-Parameter Nature

Let us reconfirm what has been achieved in this paper. Every quantity used in the derivation of the gravitational constant $G$ is not an externally given parameter but a geometric necessity flowing logically from Axiom Zero. The number of free parameters is zero.

The absence of parameters means the theory cannot "fit" itself to data; consequently, disagreement with observed values would mean not the revision but the rejection of the theory. This is the core of RSHG's scientific strength.

### B. The Two-Pillar Structure

The mathematical foundations of this paper rest on two pillars.

The first pillar is Niven's theorem [1]. It liberates the impossibility of packing from the circular argument — "packing fails because we assume discrete space" — and establishes it as an algebraic necessity independent of whether space is continuous or discrete. This places RSHG's starting point on an absolute ground that does not depend on the choice of model.

The second pillar is the experimental establishment of frictionless jamming by O'Hern et al. [2]. The constraint that force chains transmit only normal stress is isomorphic to RSHG's projection computation (only the $\sin^2\theta$ component survives), connecting the theory to existing granular physics [9, 11].

### C. The Overall Picture of the Trilogy

This paper, as the first part of the RSHG trilogy, completes the derivation from Axiom Zero to the gravitational constant. The remaining questions are entrusted to Parts II and III.

Part II addresses the question "why does computational cost determine the speed of light?" The quantitative derivation of the Planck scale previewed in Axiom Zero, the description of Pauli repulsion through wave mechanics, and the Light-Speed Resource Allocation Principle (LRAP) — the speed of light as an unfolding process in which projection advances while expanding space — are developed there.

Part III addresses the question "why is the cosmological constant 122 orders of magnitude smaller than expected?" [5] It shows the mechanism by which the computational entropy that accumulates because the packing computation never converges is suppressed through six stages of hierarchical jamming transitions. The quantitative derivation of dark energy, the detailed account of the boundary definition of life, and the physical origin of the experience of time are also developed there.

### D. Falsifiable Predictions

RSHG generates the following falsifiable predictions.

*First*, traces of $H_4$ symmetry in the angular power spectrum of the CMB. The lowest non-trivial representation of the symmetry group $H_4$ of the 600-cell corresponds to $\ell = 120$, and systematic deviations $\Delta C_\ell / C_\ell \approx 0.05$–$0.10$ from the standard $\Lambda$CDM model are predicted at $\ell = 120n$. Notably, an independent analysis by Ichiki and Nagata [6] has already reported a statistically significant anomaly near $\ell \approx 120$ in Planck satellite and WMAP data ($\Delta\chi^2 \approx -20$, probability of rejecting the null hypothesis $8 \times 10^{-4}$). RSHG provides a geometric interpretation of this existing observational anomaly in terms of $H_4$ symmetry. Further evidence for large-scale CMB anomalies has been discussed by Schwarz et al. [7]. Precise verification is possible with the next-generation experiment CMB-S4.

*Second*, measurement of the ratio of structural to thermal entropy $S_{\text{struct}}/S_{\text{thermal}} \approx 0.2$ in Bose–Einstein condensates. This is a direct verification of the two-layer structure described in Sec. VI, and is feasible by 2027.

*Third*, reflection of the $H_4$ edge classes in the angular statistics of force chains. In numerical simulations of compressed tetrahedral particles, it is predicted that features of the angular distribution corresponding to the five $\cos^2\theta$ classes will appear.

All of these predictions are quantitative and falsifiable by observation.


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[1] I. Niven, *Irrational Numbers*, The Carus Mathematical Monographs No. 11 (Mathematical Association of America, Washington, D.C., 1956).

[2] C. S. O'Hern, L. E. Silbert, A. J. Liu, and S. R. Nagel, Jamming at zero temperature and zero applied stress: The epitome of disorder, Phys. Rev. E **68**, 011306 (2003).

[3] W. Israel, Singular hypersurfaces and thin shells in general relativity, Nuovo Cimento B **44**, 1 (1966).

[4] P. J. Mohr, D. B. Newell, and B. N. Taylor, CODATA recommended values of the fundamental physical constants: 2018, Rev. Mod. Phys. **93**, 025010 (2021).

[5] S. Weinberg, The cosmological constant problem, Rev. Mod. Phys. **61**, 1 (1989).

[6] K. Ichiki and R. Nagata, A search for features in the primordial power spectrum with WMAP and Planck, JCAP **1307**, 007.

[7] D. J. Schwarz, C. J. Copi, D. Huterer, and G. D. Starkman, CMB anomalies after Planck, Class. Quantum Grav. **33**, 184001 (2016).

[8] H. S. M. Coxeter, *Regular Polytopes*, 3rd ed. (Dover Publications, New York, 1973).

[9] A. J. Liu and S. R. Nagel, Jamming is not just cool any more, Nature **396**, 21 (1998).

[10] T. Regge, General relativity without coordinates, Nuovo Cimento **19**, 558 (1961).

[11] A. P. Ramirez, Strongly geometrically frustrated magnets, Annu. Rev. Mater. Sci. **24**, 453 (1994).