

# The Truth of Riemann Conjecture Revealed in Three Dimensional Space

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**Abstract** This article reveals that the Riemann Hypothesis can be attributed to the combined effect of two geometric projections within three-dimensional real space: the projection of the function's non-trivial zeros onto the complex plane, and the projection of the singularity at the point  $S=1$  on the real axis onto the real number plane.

**Key words:** singularities ; non-trivial zeros; Infinite Projection

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## 1. Introduction

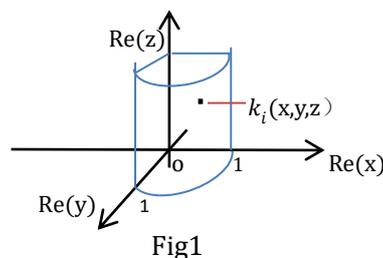
The Riemann hypothesis posits that all non-trivial zeros of the Riemann Zeta function  $\zeta(s)$  lie on the critical line  $\text{Re}(s)=1/2$ . While computational verification supports this claim for over  $10^{13}$  zeros [1], a rigorous mathematical proof remains elusive.

We study non-trivial zeros on the complex plane. However, the possibility that they are points in three-dimensional real space cannot be ruled out. If non-trivial zeros exist as such points (as illustrated in Figure 1), then we must consider their influence on a given complex plane within this real space.

Since the non-trivial zeros of the Riemann zeta function are intimately related to its singularities, and the key singularity lies at  $S=1$  on the real axis, our analysis can be focused on the relationship between this singularity and the zeros on the complex plane. By considering the infinite iterative projection of the singularity's influence onto the critical axis, we can derive a constrained region for the real parts of the non-trivial zeros on that axis. This, in conjunction with the defining property of the non-trivial zeros, leads to the conclusion that all such zeros must lie on the line  $\text{Re}(s) = 1/2$ .

## 2. The Form of Existence of Non-trivial Zeros

We study non-trivial zeros on the complex plane. However, the possibility that they are points in three-dimensional real space cannot be ruled out. If non-trivial zeros exist as such points (as illustrated in Figure 1), because the projection of a point in three-dimensional real space onto a complex plane is the real part of the zero point, then we must consider their influence on a given complex plane within this real space. ( Any  $\{ \text{Re}(z) \circ \text{Re}(x) \}$  plane in three-dimensional space may be designated as a complex plane)



According to the domain of non trivial zeros, there are:  $0 \leq x < 1, 0 \leq y < 1, 0 \leq z < \infty$ . So the Non-trivial Zeros is inside the blue fan-shaped cylinder, if set  $\{ \text{Re}(z) \circ \text{Re}(x) \}$  is complex plane, So after projecting  $k_i$  onto the complex plane, the real part of  $k_i$  can be determined on the  $\text{Re}(x)$  -axis

### 3. Projection of S=1

The simulated three-dimensional spatial coordinate diagram is shown below.  $\text{Re}(z)$  coincides with  $\text{Im}$ .

Because the projection is performed in the  $\{\text{Re}(x) \circ \text{Re}(y)\}$  plane, it is independent of the complex plane. The  $\text{Im}$  axis in Figure 3 only indicates the presence of complex planes and singular points. The projection of the  $K$  point onto the complex plane gives its real coordinate  $k_i$  on the  $\text{Re}(x)$ .

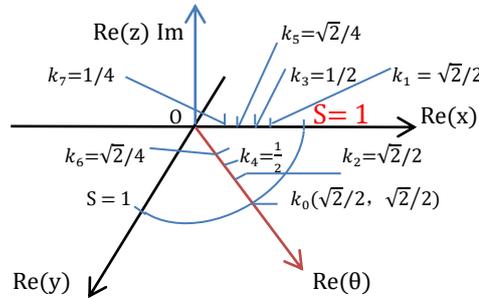


Fig2

In Fig2, set the  $\text{Re}(\theta)$  axis through the origin  $O$ ,  $\text{Re}(\theta)$  on real number “ $\text{Re}(x)$ - $\text{Re}(y)$ plane”,  $\text{Re}(\theta)$  average segmentation angle  $\pi/2$ ,  $\text{Re}(\theta)$

intersects with the arc at point  $k_i$  Fig2

$$\because Ok_i = 1, \therefore k_0(x, y) = k_0(\sqrt{2}/2, \sqrt{2}/2).$$

#### 3.1 Infinite projection of singularity

Since in the “ $\text{Re}(x)$ - $\text{Re}(y)$  plane,” all points within the quadrant containing the  $\text{Re}(\theta)$ -axis satisfy the domain of definition for both the poles and the non-trivial zeros of the Riemann Zeta function, we can, with the aid of the  $\text{Re}(\theta)$ -axis, project  $S=1$  infinitely within the interval from  $0$  to  $S=1$ . See Fig2  $k_1, k_2, k_3 \dots$

Since on  $\text{Re}(x)$ , the segments  $k_1=\sqrt{2}/2 \sim S=1$  are all projections of  $S=1$  on  $\text{Re}(x)$ , that is in the segments  $k_1=\sqrt{2}/2 \sim S=1$ , all points are equivalent to pole, and  $S=1$  is Points where the Zeta function is Undefined or not parsed, [1] if these points are meaningful on  $\text{Re}(x)$ , then they are also meaningful on the arc, when  $S=1$ , This is obviously contradictory to the non trivial zero critical zone  $\text{Re}(x) < 1$ . So here cannot be any 'non trivial zeros' within segment:

$$k_1=\sqrt{2}/2 \sim S=1 \text{ of “Im - Re}(x)\text{plane”}.$$

This defines nested **zero-free regions (ZFR)**

#### 3.2 Boundaries of (ZFR)

Since these ZFR are interrelated, so corresponding to, of  $k_1=\sqrt{2}/2 \sim S=1$  on  $\text{Re}(x)$ , there should also be a (ZFR) on  $\text{Re}(\theta)$ . it is  $k_2=\sqrt{2}/2 \sim S=1$  in Fig2.

Project  $k_2$  from  $\text{Re}(\theta)$  onto  $\text{Re}(x)$  and mark it as  $k_3$ ,  $k_3 = 1/2$ .

$1/2 \sim \sqrt{2}/2$  is a newly added (ZFR) on  $\text{Re}(x)$  and the newly added the (ZFR) on  $\text{Re}(x)$  will also affect the (ZFR) on  $\text{Re}(\theta)$ .

This forms a connection between two (ZFRs)'s, where  $k_1 = \sqrt{2}/2$  is their Boundaries. since the angle between  $\text{Re}(x)$  and  $\text{Re}(\theta)$  is  $\pi/4$ , so when projecting from  $\text{Re}(x)$  onto  $\text{Re}(\theta)$ . each time the projection will reduce the projection value by  $\sqrt{2}$  times.

$$\text{The mathematical expression is } k_n = \sqrt{2}^{-n}, \text{ when } n \rightarrow \infty, \sqrt{2}^{-n} = \sqrt{2}^{-\infty} = 0.$$

That is to say, after infinite projection  $\sqrt{2}/2 \sim S=1$  (ZFRs) between  $\text{Re}(x)$  and  $\text{Re}(\theta)$ , in  $0 < \text{Re}(x) < 1$  is composed of countless different (ZFR).

Because the junction of two ZFR segments is a projection of a singularity, so non trivial zeros can only

appear on the boundary where two (ZFR) intersect. For example, the first boundary line is  $\sqrt{2}/2$ .

In  $0 < \text{Re}(x) < 1$ , we can get boundary line :  $\sqrt{2}/2, 1/2, \sqrt{2}/4, 1/4, \sqrt{2}/8 \dots \sqrt{2}^{-n}$ ,  $n=1,2,3,4\dots$

Since the non trivial zeros of the Zeta function in  $0 < \text{Re}(x) < 1$ , [2] and the range between 0 and 1 is (ZFR), so the non trivial zero point of the Zeta function is located on the boundary between two (ZFR) in  $0 < \text{Re}(x) < 1$ .

Since in  $\zeta(s) = 2^s \pi^{s-1} \sin \frac{\pi s}{2} \Gamma(1-s) \zeta(1-s)$ ,  $\zeta(s) = \zeta(1-s)$  [3] Symmetry constrains the position of

the zero point, For the critical line  $1/2$ , there is only a  $\sqrt{2}/2$  boundary line on the right side,

but  $\frac{\sqrt{2}}{2} - \frac{1}{2} = \frac{\sqrt{2}-1}{2}$ ,  $(\frac{1}{2} - \frac{\sqrt{2}-1}{2}) = \frac{2-\sqrt{2}}{2}$  is not the boundary of (ZFR), so in critical line  $1/2$ , Unable to find symmetrical boundary lines.

## 5. Conclusion

On the complex plane, no matter how one calculates the real part of a zero, it cannot escape the constraints set by the boundary lines generated from projecting the singularities onto the real plane.

The infinite projection of  $S=1$  in  $\text{Re}(x)$  generates ZFRs whose boundaries coincide with the critical line  $\text{Re}(s)=1/2$ . This provides a geometric rationale for the Riemann hypothesis.

So, the Riemann hypothesis is correct.

**Prove completion**

1. The authors declare that there are no conflicts of interest regarding the publication of this paper.
2. All references cited in this paper are from publicly available online sources.

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